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On de-globalization in quantum chromodynamics

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Abstract. The recent discovery and resummation of a class of single logarithmic effects (non-global logs), has a significant impact on several QCD observables ranging from the classic Sterman–Weinberg jet definition to currently studied event shapes and rapidity gap observables. The discovery of the above effects overturns, for example, the common wisdom that hadronic energy flow in limited inter-jet regions is dictated *primarily* by the colour flow of the underlying hard partonic subprocess. We discuss some features of non-global logs and the rapid progress being made in estimating and controlling such corrections.

Keywords. Quantum chromodynamics; resummation; jet observables.

Techniques have been in place for well over a decade to deal with the resummation of large logarithms $\alpha_s^n \ln^m 1/\epsilon$, $\epsilon \ll 1$, $m \leq 2n$, arising in perturbative QCD predictions. It was generally believed that the technology to perform such resummations was already in place at least to the extent of accounting for single logarithmic ($m = n$) and more singular contributions ($m > n$) to many observables. However it has recently been discovered that a hitherto unforeseen problem plagues single-log contributions to a wide variety of observables, previously believed to be well-understood at the single-log level.

Broadly speaking the problem appears for any observable which has a kinematic dependence on emissions in only a slice of phase space, hence such observables are termed *non-global*. Since most experimental measurements do place angular cuts on the final state one can expect the class of non-global observables to be fairly widespread. This is indeed the case and one finds in this category event shapes, rapidity gap observables, jet fractions defined through cone-type algorithms, accompanying energy flows in multi-jet events amongst others. For such observables standard methods based on using angular ordering to reduce the problem to that of independent multiple soft emission by jets [1] are insufficient at the single-log level.

We use the Sterman–Weinberg two jet fraction for e^+e^- annihilation to illustrate the point. This is defined as the probability of having all but some small fraction ϵ of hadronic energy flowing inside two narrow cones of half-angle delta ($\epsilon, \delta \ll 1$). One is then left with the task of resumming large logarithms in ϵ and δ in order to recover a meaningful perturbative answer.

To address the above issue one can use the resummation techniques in place prior to the discovery of non-global observables. These use angular ordering (independent

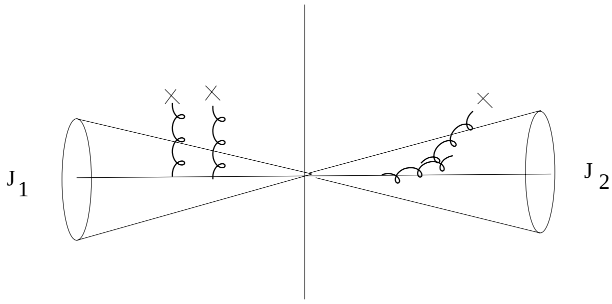

Figure 1. Independent emission (Sudakov) from J_1 along with correlated (non-global) contribution from J_2.

emission approach) to obtain a result proportional to a product of Sudakov form factors (one for each jet), with the jet form factor being (omitting the running of the coupling for simplicity):

$$F_J \sim e^{-2} C_F \alpha_s \Big/ \pi \left(\ln \frac{1}{2\epsilon} - 3/4 \right) \Big) \ln \frac{1}{\delta}. \tag{1}$$

We point out that the technique that leads to the above result is essentially based on a straightforward exponentiation of the single gluon emission probability. The dynamics of subsequent gluon branching is neglected (at single log level) due to collinear and infrared safety of the observable in question, apart from the fact that gluon branching contributes to the running of the coupling in the single emission probability (i.e. a dressing of the 'bare' single emission). It has now been demonstrated that there are a vast number of variables where such a straightforward picture is incomplete at single log level.

Returning to the Sterman–Weinberg jet fraction for instance, it is easy to show that independent emission does not generate the complete single-log structure starting at $\mathcal{O}(\alpha_s^2)$ level.

Let the matrix element squared for ordered two soft gluon emission be denoted by W_{12}^{3a} where 1 and 2 label the hard partons ($q\bar{q}$ pair) and 3 and a the soft gluons with $\omega_3 \gg \omega_a$. Let us consider specifically the configuration where the softer parton a flies outside the jet cones while 3 is contained within. The contribution from this configuration goes as

$$\sigma_{\mathrm{ng}}^2 = \alpha_s^2 \int_{3 \in J_1/J_2} \frac{\mathrm{d}\omega_3}{\omega_3} \mathrm{d}^2 \vec{n_3} \int_{a \notin J_1/J_2} \frac{\mathrm{d}\omega_a}{\omega_a} \mathrm{d}^2 \vec{n_a} W_{12}^{3a} (\Theta(\epsilon - \omega_a)) - 1) \neq 0. \tag{2}$$

The step function represents the constraint on real emission outside the jet cones whilst the -1 accounts for virtual corrections. There is clearly a mismatch between real and virtual pieces which is not accounted for by independent emission formulae which assume the above contribution to vanish in the 'inclusive' configurations where both soft partons fly into or outside jet cones and there is no contribution and the independent emission picture is restored.

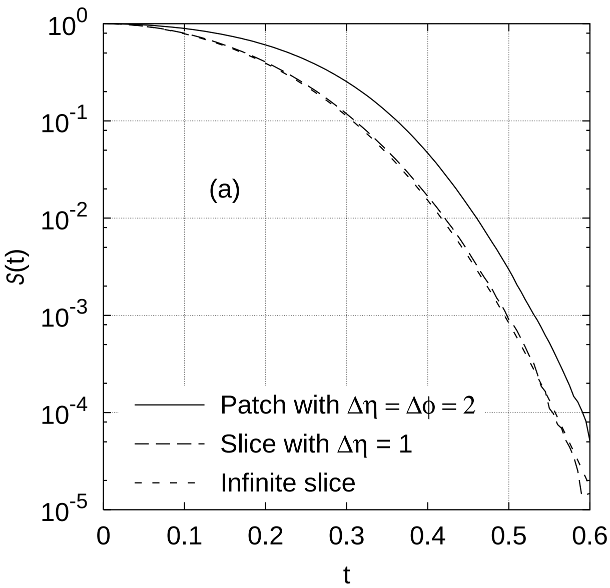

Figure 2. Resummed non-global contribution for energy flow into different regions in the η, ϕ plane.

An explicit computation (using the $C_F C_A$ (correlated rather than independent gluon emission) part of W_{12}^{3a} yields

$$\sigma_{\rm ng}^2 \sim -C_F C_A \frac{2\pi^2}{3} \alpha_{\rm s}^2 \ln^2 \frac{1}{\epsilon}. \tag{3}$$

This demonstrates that the SW jet cross-section is a non-global observable. In general it will receive single log enhancements $\alpha_{\rm s}^n \ln^n \frac{1}{\epsilon}$ for $n \geq 2$ from non-global contributions not included in the independent emission terms. At nth order we will have the non-global log arising due to coherent emission of a single softest gluon outside the jet cone from a clump of $n-1$ energy ordered soft gluons inside the jet cones. The energy ordering of the n soft gluons produces the logarithm to the nth power. Note that non-global logs are to do with multiple gluon emission at relative wide-angles with one another and the emitting hard jets, there are no collinear enhancements in this piece.

The resummation of non-global logarithms is made rather complex due to considerations of colour algebra and geometry of the large-angle gluon ensemble. However, it has been possible to compute a numerical answer in the large N_c limit $S(t)$ plotted against the single-log evolution variable $t = \int_\epsilon^Q \frac{{\rm d}k_t}{k_t} \alpha_{\rm s}(k_t)$ for different geometries into which the emission is restricted (by direct measurement) such as a rapidity slice or a patch in the η, ϕ plane where ϕ is an azimuthal angle. The SW cross-section for narrow cones is essentially the problem of an infinite rapidity slice in which the emission is restricted to be less energetic than ϵ. The most noteworthy feature of the result obtained for $S(t)$, the all orders resummed non-global (large N_c) contribution is the fact that at intermediate to large values of t the behaviour is essentially a geometry independent (apart from overall normalisation) suppression which for $t \leq 0.5$ is a Gaussian fall of e^{-kt^2}, where k is a constant. To explain this universality it has been postulated that the suppression of radiation in some region Ω follows from suppression in a larger buffer zone, making one insensitive to the geometry of Ω. Further study of non-global phenomena is an active and exciting area of research in QCD.

Reference

[1] We use the term loosely, we mean really the hard initiating parton