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Phenomenology of the minimal $SO(10)$ SUSY model*

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Abstract. In this talk I define what I call the minimal $SO(10)$ SUSY model. I then discuss the phenomenological consequences of this theory, vis-a-vis gauge and Yukawa coupling unification, Higgs and super-particle masses, the anomalous magnetic moment of the muon, the decay $B_s \to \mu^+ \mu^-$ and dark matter.

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1. Minimal $SO(10)$ SUSY model

Let me first define the minimal $SO(10)$ SUSY model [$\mathrm{MSO_{10}SM}$] [1] and then I will discuss the phenomenological consequences of this theory. In the $\mathrm{MSO_{10}SM}$ the quarks and leptons of one family are contained in a **16** dimensional spinor representation and the two Higgs doublets of the MSSM come from a single **10** dimensional representation. We have

$$\mathbf{16} \supset \left[\mathbf{Q} = \begin{pmatrix} \mathbf{u} \\ \mathbf{d} \end{pmatrix}, \quad \mathbf{L} = \begin{pmatrix} \nu \\ \mathbf{e} \end{pmatrix}, \quad \bar{\mathbf{u}}, \ \bar{\mathbf{d}}, \ \bar{\mathbf{e}}, \ \bar{\nu}\right],$$
$$\mathbf{10_H} \supset [\mathbf{H_u}, \ \mathbf{H_d}, \ \ \mathbf{T}, \ \bar{\mathbf{T}}].$$

For the third generation, there is a unique Yukawa coupling to the Higgs doublets with

$$W \supset \lambda \ \mathbf{16_3} \ \mathbf{10_H} \ \mathbf{16_3}.$$

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As a consequence, the top, bottom, tau and ν_τ Yukawa couplings satisfy Yukawa unification with $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\bar{\nu}_\tau} \equiv \lambda$. Note with a large Majorana mass for $\bar{\nu}_\tau$ we have a see-saw mechanism resulting in a light left-handed neutrino, i.e. $M_{\bar{\nu}}\bar{\nu}_\tau\bar{\nu}_\tau \Longrightarrow m_{\nu_\tau} \sim m_t^2/M_{\bar{\nu}}$. Although I will not discuss Yukawa terms for the first and second generation of quarks and leptons, it is well-known that it is not phenomenologically acceptable for them to receive all their mass via renormalizeable interactions with a single $\mathbf{10_H}$. Nevertheless with effective higher dimensional interactions it is not difficult to obtain realistic fermion masses and mixing angles for all quarks and leptons [2]. Moreover if these mass matrices are hierarchical, we do not significantly affect the results derived from assuming exact Yukawa unification for the third generation.

Finally, the soft SUSY breaking parameters are given by $-\mathcal{L}_{\text{soft}} = m_{16}^2\Sigma_{i=1}^3 16_i^* 16_i + m_{10}^2 10_H^* 10_H - A_0\lambda\ 16_3 10_H 16_3 + M_{1/2}\Sigma_{i=1}^3(\chi_i\chi_i) + \mu B H_u H_d$. All but one of these terms, are the most general consistent with $SO(10)$. A universal scalar mass m_{16} for all three families is an additional assumption. Hence, the soft SUSY breaking parameters are given by

$$m_{16}, m_{10}, A_0, M_{1/2}, \tan\beta.$$

Before continuing we note that one additional soft SUSY breaking parameter is needed, which we discuss next.

1.1 *Radiative EWSB with large* $\tan\beta$ *needs* $m_{H_u}^2 < m_{H_d}^2$

It has been shown that there are two consequences of splitting the two Higgs doublet masses. It reduces the amount of fine tuning for radiative electroweak symmetry breaking (EWSB) [3]. In addition, it permits EWSB in an entirely new region of SUSY parameter space with $m_{16} \gg M_{1/2}$ [4].

We have considered the possibility of both D_X term and 'just so' splitting [1]. In the former, we assume a soft SUSY breaking D term where D_X is the auxiliary field of a $U(1)_X$ gauge interaction defined by $SO(10) \to SU(5) \times U(1)_X$. We then obtain

$$\begin{aligned}
m_{(H_u,H_d)}^2 &= m_{10}^2 \mp 2D_X, \\
m_{(Q,\bar{u},\bar{e})}^2 &= m_{16}^2 + D_X, \\
m_{(\bar{d},L)}^2 &= m_{16}^2 - 3D_X.
\end{aligned}$$

These boundary conditions at the GUT scale generically give the low energy result $m_{\tilde{b}}^2 \le m_{\tilde{t}}^2$ which is *bad* for Yukawa unification.

With 'just so' splitting we have

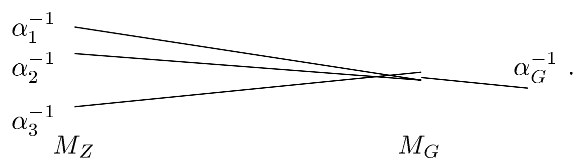
$$\begin{aligned}
m_{(H_u,H_d)}^2 &= m_{10}^2(1 \mp \Delta m_H^2), \\
m_{(Q,\bar{u},\,\bar{e})}^2 &= m_{16}^2, \\
m_{(\bar{d},L)}^2 &= m_{16}^2.
\end{aligned}$$

These boundary conditions give $m_{\tilde{t}}^2 \ll m_{\tilde{b}}^2$ which is *good* for Yukawa unification. This latter case may be motivated by the fact that the Higgs multiplets must be special. They necessarily have a μ term and they also require doublet-triplet splitting. Moreover they have large threshold corrections at the GUT scale due to the tau neutrino.

The $\bar{\nu}_\tau$ contribution to Higgs splitting results from the Yukawa term $(\lambda_{\nu_\tau}\bar{\nu}_\tau L H_u)$ with $\lambda_{\nu_\tau} = \lambda_t = \lambda_b = \lambda_\tau \equiv \lambda$. Since $\bar{\nu}_\tau$ couples only to H_u, this contribution distinguishes H_u and H_d. At one loop we find $\Delta m^2_{H_u} \approx \lambda^2/16\pi^2(2m^2_{16} + m^2_{10} + A^2_0)\log(M^2_{\bar{\nu}_\tau}/M^2_G) + \cdots$. Taking typical GUT values for the parameters $\lambda = 0.7$, $M_{\bar{\nu}_\tau} = 10^{14}$GeV (which gives $(\Delta m^2_\nu)_{\rm atm} \sim 10^{-2}{\rm eV}^2$), $M_G = 3 \times 10^{16}$GeV and $A^2_0 \approx 2m^2_{10} \approx 4m^2_{16}$ we obtain $\Delta m^2_H \equiv \frac{1}{2}\Delta m^2_{H_u}/m^2_{10} \sim 0.07$. This is 'just so' splitting of about the right size.

1.2 *Gauge coupling unification*

Presently, gauge coupling unification provides the only evidence for low energy SUSY [5–7].



Note, when threshold corrections are included, the three gauge couplings $\alpha_i, i = 1,2,3$ do not precisely meet at the GUT scale. Moreover, for consistency, one loop threshold corrections need to be included when using two loop RG running from $M_G \to M_Z$. At one loop there are significant GUT threshold corrections from Higgs and GUT breaking sectors. We now define the GUT scale as the point where $\alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G$. A good fit to low energy data then requires $\epsilon_3 \equiv (\alpha_3(M_G) - \tilde{\alpha}_G)/\tilde{\alpha}_G \sim -4\%$.

2. $SO(10)$ Yukawa unification

Let us now consider the constraint on the soft SUSY breaking parameters resulting from Yukawa unification [1,8,9]. Note, the GUT threshold corrections to Yukawa unification from gauge and Higgs loops is typically insignificant. Weak scale threshold corrections, on the other hand, are proportional to $\tan\beta$ and cannot be ignored [10,11]. The dominant contributions are given by $\delta m_b/m_b = \Delta m_b^{\tilde{g}} + \Delta m_b^{\tilde{\chi}} + \Delta m_b^{\log} + \cdots$ where the first comes from a gluino-sbottom loop, the second from the chargino-stop loop and the third from finite wave function renormalization graphs. Note, in general we have $\Delta m_b^{\tilde{g}} \sim -\Delta m_b^{\tilde{\chi}} > 0$ for $\mu > 0$ (our conventions). The first two contributions are $\tan\beta$ enhanced and can be ~ 50 %, while the typical size of the log contribution is $\sim +6$ %. The contribution to the top quark mass is not $\tan\beta$ enhanced and although the contribution to the tau mass is; nevertheless it is small due to the smaller values of the relevant gauge and Yukawa couplings at the electroweak scale. Finally, good fits to top, bottom and tau masses require $\delta m_b/m_b \lesssim -2\%$.

2.1 *Data favors $\mu > 0$*

We now argue that two pieces of low energy data favor positive values of μ. The first is the rate for the process $b \to s\gamma$ and the second is the anomalous magnetic moment of the muon. In the first case, the chargino term typically gives the dominant SUSY contribution and for $\mu > 0$ it has opposite sign to the standard model and charged Higgs contributions, thus reducing the branching ratio. This is desirable since the SM contribution by itself is a little too large. As a result, trying to fit the data with $\mu < 0$ is problematic. In the second case, the contribution to the muon anomalous magnetic moment due to new physics (beyond the standard model) is measured to be $a_\mu^{\mathrm{NEW}} \times 10^{10} = 33.9$ (11.2) (e^+e^--based) or 16.7 (10.7) (τ-based) [12]. There are two results depending on whether one uses e^+e^- or τ data to determine the hadronic contribution to the amplitude. Note, in either case the sign of a_μ^{NEW} is positive. Moreover in SUSY this sign is directly correlated with the sign of μ [13], again favoring positive μ. Hence we consider only positive μ in our analysis.

2.2 χ^2 *Analysis*

We have performed a χ^2 analysis of the $\mathrm{MSO_{10}SM}$ with 11 input parameters defined at the GUT scale and 9 low energy observables in our χ^2 function [1].

The 11 input parameters at M_G are $[\lambda, \alpha_G, M_G, \epsilon_3;\ m_{10}, A_0, \tan\beta(M_Z),\ D_X$ (D term splitting) (or Δm_H^2 ('just so' Higgs splitting)), $m_{16}, \mu, M_{1/2})$, where the last three parameters are fixed while we vary 8 parameters using the CERN package Minuit to minimize χ^2. The 9 observables (experimental/theoretical uncertainty) $[X_i^{\mathrm{exp}}(\sigma_i)]$ defining $\chi^2 = \Sigma_{i=1}^9 \left[(X_i^{\mathrm{exp}} - X_i^{\mathrm{theory}})^2/\sigma_i^2\right]$ are given by $[G_\mu, \alpha, \alpha_s(M_Z) = 0.118(0.002),\ \rho^{\mathrm{NEW}}, M_Z, M_W,\ M_t = 174.3(5.1),\ m_b(m_b) = 4.20(0.20), M_\tau)$.

2.3 *Bottom line*

The bottom line result of our analysis is that Yukawa unification is possible only in a narrow region of SUSY parameter space. The result is also easy to understand. Since for $\mu > 0$ and $\delta m_b/m_b \lesssim -2\%$ we need $|\Delta m_b^{\tilde\chi}| > \Delta m_b^{\tilde\chi}$. However $\Delta m_b^{\tilde g} \approx (2\alpha_3/3\pi)\ (\mu m_{\tilde g}/m_{\tilde b}^2)\tan\beta,\ \Delta m_b^{\tilde\chi^+} \approx (\lambda_t^2/16\pi^2)\ (\mu A_t/m_{\tilde t}^2)\tan\beta$ and $\Delta m_b^{\log} \approx (\alpha_3/4\pi)\log(\tilde m^2/M_Z^2) \sim 6$ %. In order to enhance the chargino contribution, we can make the numerator larger by making A_t large and negtive. This is accomplished by making A_0 at M_G large and negative, i.e. $A_t \ll 0 \Longleftrightarrow A_0 \ll 0$. This also has the effect of making the denominator for the chargino contribution smaller since the stop mass matrix is of the form $\begin{pmatrix} m_{\tilde t}^2 & m_t\ A_t \\ m_t\ A_t & m_{\tilde t}^2 \end{pmatrix}$. As a consequence we naturally obtain $m_{\tilde t} \ll m_{\tilde b}$; enhancing the chargino, in comparison to the gluino contribution. Of course in order not to have a negative stop mass squared

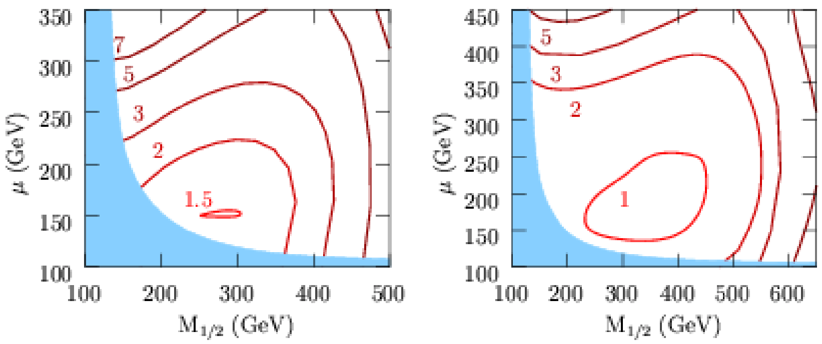


Figure 1. χ^2 contours for $m_{16} = 1500$ GeV (left) and $m_{16} = 2000$ GeV (right). The shaded region is excluded by the chargino mass limit $m_{\chi^+} > 103$ GeV.

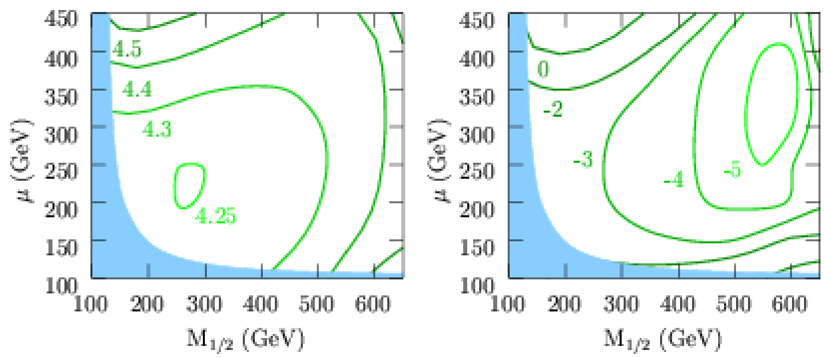


Figure 2. Contours of constant $m_b(m_b)$ (GeV) (left) and δm_b in % (right) for $m_{16} = 2000$ GeV.

we need to make m_{16} large. As a result of the χ^2 analysis we find that good fits require $A_0 \sim -2m_{16}$, $m_{10} \sim \sqrt{2}m_{16}$, $m_{16} \geq 2\,\text{TeV} \gg \mu, M_{1/2}$, and $\Delta m_H^2 \sim 10$ %. In figure 1 we show the χ^2 contours for two different values of m_{16} as a function of μ and $M_{1/2}$. It is clear that χ^2 improves as we increase m_{16}. Note also that the dominant pull for χ^2 is due to the bottom quark mass corrections as can be seen in figure 2.

3. Summary – Minimal $SO(10)$ SUSY model

Before discussing some phenomenological consequences of the MSO$_{10}$SM, let us summarize the main ingredients of the model. We assume a supersymmetric $SO(10)$ GUT with quarks and leptons in **16**s. In addition, we assume that the minimal Higgs content of the MSSM (H_u, H_d) are contained in a single **10**. Finally for the third family we assume the minimal Yukawa interaction with

$$W \supset \lambda\ \mathbf{16_3}\ \mathbf{10_H}\ \mathbf{16_3}.$$

The direct consequences of $MSO_{10}SM$ follow.

- Gauge coupling unification – $\alpha_G, M_G, \epsilon_3 \sim -4\%$;
- Yukawa unification – $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\bar{\nu}_\tau} \equiv \lambda$;
- Soft SUSY breaking parameters [13a] – $m_{16}, m_{10}, A_0, M_{1/2}, \tan\beta \approx 50, \Delta m_H^2$;
- Satisfying – $A_0 \sim -2m_{16}$, $m_{10} \sim \sqrt{2}m_{16}$, $m_{16} \geq 2$ TeV $\gg \mu, M_{1/2}$, and $\Delta m_H^2 \sim 10\%$.

The last condition is required in order to fit the precision low energy electroweak data, including the top, bottom and tau masses. In addition to the above defining properties of the $MSO_{10}SM$, we find two additional direct consequences of the model. The first is a 'natural' inverted scalar mass hierarchy which ameliorates the SUSY flavor and CP problems. Secondly, the rates for proton decay due to dimension 5 operators are decreased. We discuss these two unexpected benefits below.

3.1 *Inverted scalar mass hierarchy*

One way to ameliorate the SUSY flavor and CP problems is to demand that the first and second generation squarks and sleptons are heavy with mass $\gg$ TeV, while the third generation scalars are light with mass $\leq$ TeV. This is easily seen by focusing on the most severe flavor and CP violating processes [14]. The best bounds are for processes involving the two lightest families. For example, we have [14]

- $B(\mu \to e\gamma) < 1.2 \times 10^{-11} \Longrightarrow |(\delta^l_{12})_{LL}| < 2.1 \times 10^{-3}(m_{\tilde{l}}\,(\text{GeV})/100)^2$

  or $|(\delta^l_{12})_{LL}| < 0.8\ (m_{\tilde{l}}\,(\text{TeV})/2)^2$;
- $\Delta m_K <$ Exp. $\Longrightarrow \sqrt{|\text{Re}(\delta^d_{12})^2_{LL}|} < 1.9 \times 10^{-2}(m_{\tilde{q}}\,(\text{GeV})/500)$

  or $\sqrt{|\text{Re}(\delta^d_{12})^2_{LL}|} < 7.6 \times 10^{-2}\ (m_{\tilde{q}}\,(\text{TeV})/2)$;
- $\epsilon_K <$ Exp. $\Longrightarrow \sqrt{|\text{Im}(\delta^d_{12})^2_{LL}|} < 1.5 \times 10^{-3}(m_{\tilde{q}}\,(\text{GeV})/500)$

  or $\sqrt{|\text{Im}(\delta^d_{12})^2_{LL}|} < 6.0 \times 10^{-3}\ (m_{\tilde{q}}\,(\text{TeV})/2)$;
- $d^e_N \sim 2(100/m_{\tilde{l}}\,(\text{GeV}))^2 \sin\Phi_{A,B} \times 10^{-23}$e cm $< 4.3 \times 10^{-27}$e cm $\Longrightarrow$ $\sin\Phi_{A,B} < 4 \times 10^{-4} \times (m_{\tilde{l}}\ (\text{GeV})/100)^2$

  or $\sin\Phi_{A,B} < 0.16 \times (m_{\tilde{l}}\ (\text{TeV})/2)^2$.

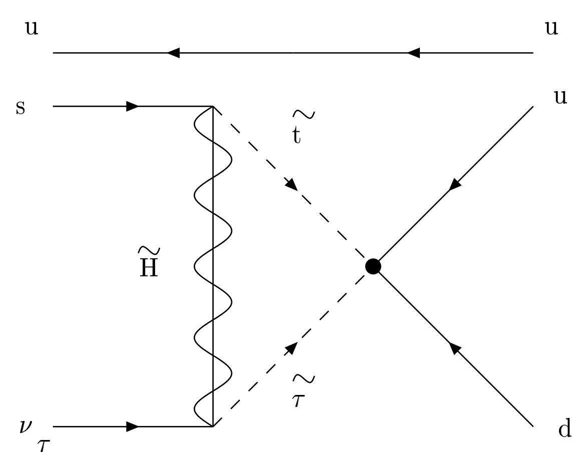
Although a significant degeneracy of the first and second generation squarks and sleptons is still required, it does not require serious fine-tuning. In fact, the flavor and CP problems are now completely amenable to solutions using non-abelian family symmetries. The question one now faces is how to obtain an inverted scalar mass hierarchy with the ratio of scalar masses S satisfying $S \equiv \tilde{m}^2_{1,2}/\tilde{m}^2_3 \gg 1$.

One way of obtaining this inverted scalar mass hierarchy is to assume that it results from Planck/GUT scale physics. However, it was shown that an inverted scalar mass hierarchy can be generated 'naturally' as a consequence of renormalization group running [15]. This latter possibility requires specific soft SUSY breaking boundary conditions at M_G. In particular, it was found that the following boundary conditions can lead to values of $S \geq 400$ [15]. Surprisingly, *these boundary conditions are the same required by Yukawa unification.*

- $m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 \equiv m_{16}^2$;
- $A_t = A_b = A_\tau \equiv A_0$;
- $M_1 = M_2 = M_3 \equiv M_{1/2}$;
- $m_{H_u} = m_{H_d} \equiv m_{10}$; and
- $A_0^2 = 2m_{10}^2 = 4m_{16}^2$ with $m_{16} \gg 1$ TeV.

3.2 *Suppressing proton decay*

Nucleon decay rates are significantly constrained by data from Super-Kamiokande [16]. In particular the decay mode $p \to K^+ + \bar{\nu}_\tau$, due to dimension 5 operators, is typically the dominant decay mode. In the large $\tan\beta$ regime the dominant Feynman diagram is given by



This one loop integral results in a loop factor characteristically of order

$$\text{Loop factor} = \frac{\lambda_t\ \lambda_\tau}{16\pi^2} \frac{\sqrt{\mu^2 + M_{1/2}^2}}{m_{16}^2}.$$

Note that the loop factor is minimized in the limit $\mu, M_{1/2} \ll m_{16}$. This limit is once again consistent with Yukawa unification. Moreover it is only consistent with radiative EWSB with split H_u, H_d masses.

4. Phenomenology

Let us now consider some predictions of the $\mathrm{MSO_{10}SM}$.

4.1 *Light Higgs mass*

First consider the light Higgs mass. In the MSSM the light Higgs mass has an upper bound of order 130 GeV. This upper limit is achieved for large $\tan\beta$. Moreover the large radiative corrections to the Higgs mass are dominated by heavy stop masses. In our case we have $\tan\beta \sim 50$, however we have relatively light stop, sbottom, and stau masses. As a result we find [1]

$$m_h = 114 \pm 5 \pm 3 \text{ GeV}.$$

In figure 3 we show the light Higgs mass contours as a function of $\mu, M_{1/2}$ for two values of m_{16}. For a more detailed analysis of the light Higgs mass prediction, see [1].

4.2 *Muon anomalous magnetic moment*

The anomalous magnetic moment of the muon scales as $(\mu M_{1/2}\tan\beta)/m_{16}^4$. Since we have $m_{16} \geq 2\,\mathrm{TeV}$, we find [1]

$$a_\mu^{\mathrm{SUSY}} \leq 6 \times 10^{-10}.$$

4.3 $\tilde{\chi}^0$ *LSP – Dark matter*

When m_{16} is large, the standard neutralino annihilation channels via squark/slepton exchange diagrams are severely suppressed. This typically leads to an

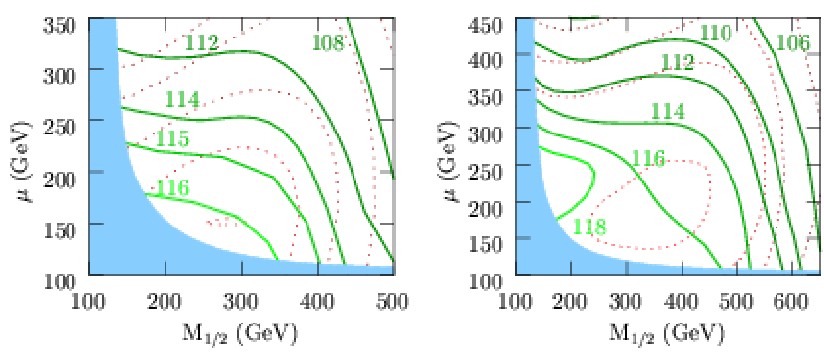


Figure 3. Contours of constant m_h (GeV) (solid lines) with χ^2 contours from figure 1 (dotted lines) for $m_{16} = 1500$ GeV (left) and $m_{16} = 2000$ GeV (right).

excess cosmological abundance of neutralinos with $\Omega_\chi h^2 \gg 0.3$. However, in the large $\tan\beta$ regime the neutralino annihilation channel ($\tilde{\chi}^0\tilde{\chi}^0 \to A^0 \to$ hadrons) is significantly enhanced. In fact, this annihilation channel is so effective that, on resonance, $\Omega_\chi h^2 \ll 0.01$. Thus we find, on the sides of the broad resonance peak, cosmological abundances consistent with dark matter observations [17]. In figure 4 we present an analysis of dark matter abundances in the $\mathrm{MSO_{10}SM}$. The green band is the region with acceptable values of $\Omega_\chi h^2$. Note that we have also included contours of constant branching ratio $B(B_s \to \mu^+\mu^-)$. This is important since this process is extremely sensitive to the value of the CP odd Higgs mass m_A.

4.4 *Large* $\tan\beta$ *and quark flavor violation*

It has been shown that in the large $\tan\beta$ regime there are significant one loop SUSY threshold corrections to CKM matrix elements [11]. Once these corrections are included in an effective two Higgs doublet model below the SUSY breaking scale, the Higgs couplings are no longer flavor diagonal [18,19]. Hence the process $B_s \to \mu^+\mu^-$ can proceed through s-channel CP odd Higgs exchange with a $\tan\beta$ enhanced

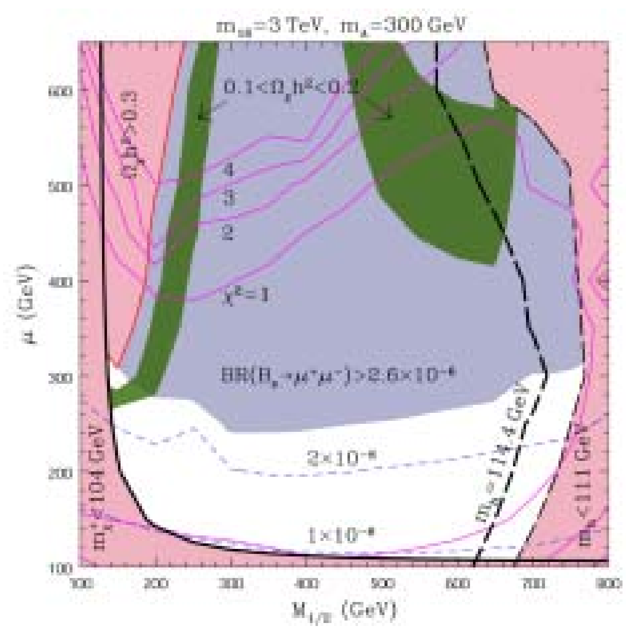


Figure 4. Contours of constant χ^2 for $m_{16} = 3$ TeV and $m_A = 300$ GeV. The red regions are excluded by $m_{\chi^+} < 104$ GeV (below and to the left of a black solid curve), $m_h < 111$ GeV (on the right) and by $\Omega_\chi h^2 > 0.3$. To the right of the black broken line one has $m_h < 114.4$ GeV. The green band corresponds to the preferred range $0.1 < \Omega_\chi h^2 < 0.2$, while the white regions below (above) it correspond to $\Omega_\chi h^2 < 0.1$ ($0.2 < \Omega_\chi h^2 < 0.3$). Also marked are contours of constant $\mathrm{BR}(B_s \to \mu^+\ \mu^-)$. The blue regions in the lower two panels are excluded by $\mathrm{BR}(B_s \to \mu^+\ \mu^-) > 2.6 \times 10^{-6}$.

branching ratio $B(B_s \to \mu^+\mu^-) \propto \tan\beta^4$. The effective two Higgs doublet Yukawa coupling to down quarks is given below. The matrices $\lambda_{di}^{\rm diag}$, $\Delta\lambda_d^{ij}$ ($\delta\lambda_d$) are the zeroth order down quark Yukawa coupling in a diagonal basis and the one loop correction to the Higgs couplings due to gluino (chargino) loops.

$$\mathcal{L}_{\rm eff}^{ddH} = -\bar{d}_{Li}\,\lambda_{di}^{\rm diag}\, d_{Ri}\, H_d^{0*} - \bar{d}_{Li}\,\Delta\lambda_d^{ij}\, d_{Rj}\, H_d^{0*} - \bar{d}_{Li}\,\delta\lambda_d^{ij}\, d_{Rj}\, H_u^0 + \text{h.c.}.$$

As a result of the chargino loop correction, which is proportional to the square of the up quark Yukawa matrix, we must re-diagonalize the down quark mass matrix as

$$m_d^{\rm Diagonal} = V_d^L\ \left[\lambda_d^{\rm diag} + \Delta\lambda_d + \delta\lambda_d \tan\beta\right]\ V_d^{R\dagger}\ \frac{v\cos\beta}{\sqrt{2}}.$$

For large values of $\tan\beta$, this results in a significant correction to the CKM matrix [11]. We then obtain the following couplings to the neutral Higgs mass eigenstates $h,\ H,\ A$ given by [18,19]

$$\begin{aligned}\mathcal{L}_{\rm FV}^{i\neq j} = &-\frac{1}{\sqrt{2}}\bar{d}'_i\left[F_{ij}^h P_R + F_{ji}^{h*} P_L\right] d'_j h\\ &-\frac{1}{\sqrt{2}}\bar{d}'_i\left[F_{ij}^H P_R + F_{ji}^{H*} P_L\right] d'_j H\\ &-\frac{i}{\sqrt{2}}\bar{d}'_i\left[F_{ij}^A P_R + F_{ji}^{A*} P_L\right] d'_j A,\end{aligned}$$

where

$$\begin{aligned}F_{ij}^h &\simeq \delta\lambda_d^{ij}(1+\tan^2\beta)\cos\beta\ \cos(\alpha-\beta),\\ F_{ij}^H &\simeq \delta\lambda_d^{ij}(1+\tan^2\beta)\cos\beta\ \sin(\alpha-\beta),\\ F_{ij}^A &\simeq \delta\lambda_d^{ij}(1+\tan^2\beta)\ \cos\beta.\end{aligned}$$

It is the flavor-violating coupling F_{23}^A which gives the direct $B_s A^0$ coupling [19]. Note, the branching ratio $B(B_s \to \mu^+\mu^-)$ in the cosmologically allowed region is close to the CDF bound (see figure 4). In [17] we show that the process $B_s \to \mu^+\mu^-$ may soon be observed.

5. Two loose ends

5.1 *Fine tuning?*

We have been considering large squark and slepton masses with $m_{16} \geq 2$ TeV. Since we have a 'natural' inverted scalar mass hierarchy, the third generation squarks and sleptons are typically lighter than a TeV. Hence the radiative corrections to the Higgs mass, in the effective low energy theory, are not large. For example, the radiative corrections at the electroweak scale are of order $\delta m_h^2 \propto (\lambda_\tau^2/16\pi^2) m_{\tilde{\tau}}^2$ and they are safe for $m_{\tilde{\tau}} \leq 1$ TeV.

However, there is still the question of whether radiative EWSB requires significant fine tuning. It has been shown that the Z mass is most sensitive to the value of the gluino mass, M_3 [20]. For example, with $\tan\beta = 35$, the following relation was obtained [20]

$$M_Z^2 = -1.5\mu^2 + 5.0M_{1/2}^2 + 0.2A_0^2 + 1.5m_{16}^2 - 1.2m_{H_u}^2 - 0.08m_{H_d}^2 + \cdots.$$

But, recall we have $\mu, M_{1/2} \ll m_{16}$. Thus this problem is ameliorated somewhat, although it is not completely eliminated. (See also the talk by Pokorski, *Pramana – J. Phys.* **62**, 369 (2004).)

Finally, as discussed earlier the fine tuning for EWSB in the regime of large $\tan\beta$ is of order $1/\tan\beta$ when one has Higgs mass splitting [3].

5.2 *SUSY breaking mechanism?*

We have found that Yukawa unification in the $\mathrm{MSO_{10}SM}$ is only consistent with the low energy data in a narrow region of soft SUSY breaking parameter space. It is clear that this idea would be considerably strengthened if there was a mechanism which 'naturally' broke SUSY in this way. Unfortunately, this is not the case for the known SUSY breaking mechanisms.

For example, gauge mediated SUSY breaking (GMSB) gives $A_0 = 0$ at the messenger scale. This is *bad*. Yukawa deflected GMSB can have non-zero A_0 proportional to Yukawa couplings. Perhaps this might work, however the standard gauge contribution would have to be suppressed. Moduli-dominated string SUSY breaking may be possible [15]. The generic formula for stringy SUSY breaking is given by [21]

$$\begin{aligned}
&m_\alpha^2 = (1 + 3\overrightarrow{n_\alpha} \cdot \overrightarrow{\Theta^2})\ m_{3/2}^2,\\
&M_{1/2} \sim 0,\\
&A_{\alpha\beta\gamma} = \pm\sqrt{3}[1 + n_\alpha^i + n_\beta^i + n_\gamma^i - Y_{\alpha\beta\gamma}^i]\ m_{3/2},
\end{aligned}$$

where n_α is the modular weight of the field α; T_i, $(i = 1, \ldots, 6)$ are moduli; Θ_i parametrize the direction of the Goldstino in the T_i field space, and $Y_{\alpha\beta\gamma}^i = 2(\mathrm{Re}\ T_i)\ \partial_{T_i} \ln h_{\alpha\beta\gamma}$, where $h_{\alpha\beta\gamma}$ are the dimensionless Yukawa couplings. With the following modular weights [15]: $n_{Q,U,D,L,E,N} = (0, -1/2, -1/2, 0, 0, 0)$; $n_{H_u,H_d} = (-1/2, -1/2, 0, 0, 0, 0)$, and assuming $(\Theta^2)^i = (0, 0, 1/3, 2/3, 0, 0)$, and $Y_{\alpha\beta\gamma}^i \sim 0$, one finds $m_{3/2}^2 = 2m_{16}^2$ and

$$A_0 = \pm 2m_{16}, \quad m_{10} = \sqrt{2}m_{16}.$$

The only problem with this idea is the absence of any existing string model with these properties. Thus the problem of a 'natural' SUSY breaking mechanism consistent with $\mathrm{MSO_{10}SM}$ Yukawa unification is the most urgent open theoretical question requiring further work.

6. Summary

In this talk I have defined the minimal $SO(10)$ SUSY model and discussed some of its phenomenological consequences. The model predicts:

- Gauge coupling unification with $\alpha_G,\ M_G,\ \epsilon_3 \sim -4\%$;
- Yukawa unification with $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\bar{\nu}_\tau} \equiv \lambda$, and
- Soft SUSY breaking parameters given by [21a] $m_{16},\ m_{10},\ A_0,\ M_{1/2},\ \tan\beta,\ \Delta m_H^2$.

As a result of a χ^2 analysis [1] we find that the low energy precision electroweak data, including the top, bottom and tau masses, only gives good fits for soft SUSY breaking parameters satisfying:

- $A_0 \sim -2m_{16},\ m_{10} \sim \sqrt{2}m_{16},\ m_{16} \geq 2$ TeV $\gg \mu, M_{1/2}$, and $\Delta m_H^2 \sim 10\%$.

This region of parameter space has the virtue of giving:

- a 'natural' inverted scalar mass hierarchy which ameliorates the SUSY flavor and CP problems, and in addition
- suppresses proton decay via dimension 5 operators.

The $\mathrm{MSO}_{10}\mathrm{SM}$ makes the following predictions:

- It gives $\tan\beta \sim 50$ and a light stop. As a consequence we find [1] $m_h = 114 \pm 5 \pm 3$ GeV;
- The decay $B_s \to \mu^+\ \mu^-$ is enhanced and may be observable in the near future [19];
- The SUSY contribution to the muon anomalous magnetic moment is suppressed with $a_\mu^{\mathrm{SUSY}} < 6 \times 10^{-10}$ [1]; and
- Finally, it gives cosmologically acceptable abundances of neutralino dark matter [17].

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