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Higgs bosons in the standard model, the MSSM and beyond

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Abstract. I summarize the basic theory and selected phenomenology for the Higgs boson(s) of the standard model, the minimal supersymmetric model and some extensions thereof, including the next-to-minimal supersymmetric model.

Keywords. Higgs bosons; supersymmetry; extended Higgs sectors.

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1. Introduction

We are nearing the 40th anniversary of the introduction of the idea of electroweak symmetry breaking via an elementary Higgs field [1,2]. We are still awaiting direct experimental confirmation or refutation. This brief review summarizes some key properties of the most fully studied theories based on a Higgs sector: the standard model (SM); the minimal supersymmetric model (MSSM); and the next-to-minimal supersymmetric model (NMSSM) and related extensions. More complete summaries can be found in several recent reviews [3,4]. See also [5]. In the SM case, I will review current constraints and basic phenomenology but focus most of our attention on the problems that suggest a supersymmetric extension. In particular, the MSSM solves the naturalness/hierarchy issues while yielding coupling unification and radiatively-induced electroweak symmetry breaking. I summarize basic tree-level features and radiative corrections. I then discuss the very attractive NMSSM extension in which a single additional singlet Higgs superfield is added to the MSSM structure. Implications of adding still more singlets are then considered. Finally, I review the motivations for a left–right symmetric extension of the MSSM.

2. The SM Higgs boson

The SM employs just a single doublet (under $SU(2)_{\rm L}$) complex scalar field to give masses to all particles. Given the mass, $m_{h_{\rm SM}} = \frac{1}{2}v^2\lambda$ (where λ is the quartic self-coupling strength), all couplings of the $h_{\rm SM}$ are determined.

$$g_{h_{\rm SM}f\overline{f}} = \frac{m_f}{v}, \quad g_{h_{\rm SM}VV} = \frac{2m_V^2}{v}, \quad g_{h_{\rm SM}h_{\rm SM}VV} = \frac{2m_V^2}{v^2},$$
$$g_{h_{\rm SM}h_{\rm SM}h_{\rm SM}} = \frac{3}{2}\lambda v = \frac{3m_{h_{\rm SM}}^2}{v}, \quad g_{h_{\rm SM}h_{\rm SM}h_{\rm SM}h_{\rm SM}} = \frac{3}{2}\lambda = \frac{3m_{h_{\rm SM}}^2}{v^2}, \qquad (1)$$

where $V = W$ or Z and $v = 2m_W/g = 246$ GeV. The couplings and $m_{h_{\rm SM}}$ determine the branching ratios and total width. The $h_{\rm SM}$ is very narrow until $m_{h_{\rm SM}} > 2m_W$, at which point the VV decay modes start to take over and the width increases rapidly, reaching $\Gamma^{\rm tot}_{h_{\rm SM}} \sim \frac{1}{2}m_{h_{\rm SM}}$ for $m_{h_{\rm SM}} \sim 900$ GeV. The branching ratios and width appear in figure 1. Note that $B(h_{\rm SM} \to \gamma\gamma)$ is substantial for $m_{h_{\rm SM}} \sim 120$ GeV – this is important for the LHC $\gamma\gamma$ final state discovery mode for a light $h_{\rm SM}$.

The most immediate goal of the present and future colliders will be to discover the SM Higgs (or a SM-like Higgs) if it exists and then to measure its branching ratios, total width, self-coupling, spin, parity and CP. This will not be possible without having both the LHC and a future LC. The exact strategies will depend on

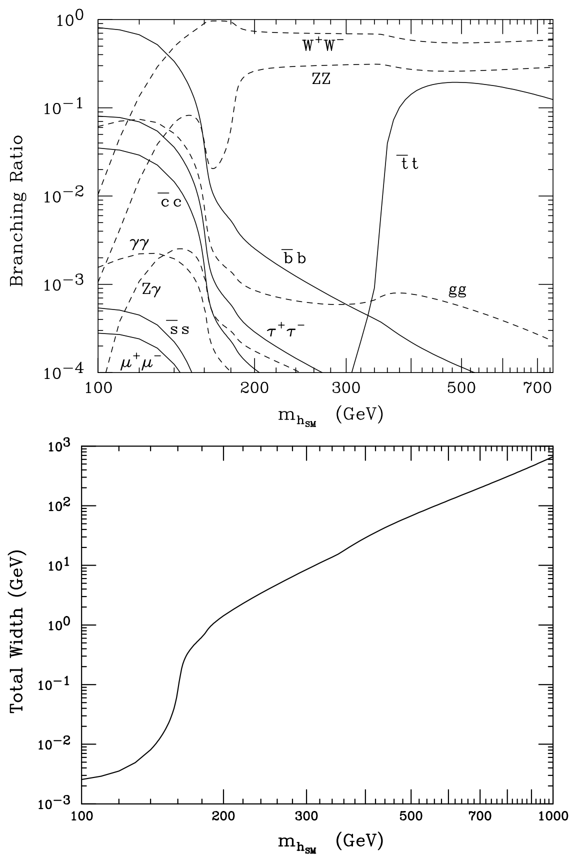

Figure 1. The SM Higgs branching ratios and total width.

the still unknown value of $m_{h_{\rm SM}}$. However, we do have some hints regarding $m_{h_{\rm SM}}$ from current data. Precision electroweak constraints [6] give $m_{h_{\rm SM}} < 211$ GeV at 95% confidence level (CL), with a preferred central value of $m_{h_{\rm SM}} = 91^{+58}_{-37}$ GeV, below the 95% CL LEP bound of $m_{h_{\rm SM}} \geq 114.4$ GeV [7].

It is important to assess the constraints on the SM Higgs sector related to the scale Λ at which new physics beyond the SM first emerges. The two basic theoretical constraints are: (1) the Higgs self coupling should not blow up below scale Λ – this leads to an upper bound on $m_{h_{\rm SM}}$ as a function of Λ; (2) the Higgs potential should not develop a new minimum at large values of the scalar field of order Λ – this leads to a lower bound on $m_{h_{\rm SM}}$ as a function of Λ. Together, these two constraints imply that the SM can be valid all the way up to $M_{\rm Pl}$ if $130 \lesssim m_{h_{\rm SM}} \lesssim 180$ GeV, as illustrated in the left-hand half of figure 2 taken from [8].

The precision electroweak constraints can also be somewhat modified if we allow for new physics operators (there are two important ones – one contributing to the S parameter and the other to the T parameter) characterized by some scale Λ. If the coefficients of these two operators are tree-level in size, then the above upper bound on $m_{h_{\rm SM}}$ can be considerably weakened, as illustrated by the blue line in the right-hand half of figure 2 [9]. However, the survival of the SM as an effective theory all the way up to $M_{\rm Pl}$ is unlikely due to the problem of 'naturalness' and the associated 'fine-tuning' issue. We should impose the additional condition that $m_{h_{\rm SM}} \sim m_Z$ is not a consequence of extreme fine-tuning. Recall that after including the one-loop corrections we have

$$m^2_{h_{\rm SM}} = \mu^2 + \frac{3\Lambda^2}{32\pi^2 v^2}(2m^2_W + m^2_Z + m^2_{h_{\rm SM}} - 4m^2_{\rm t}), \tag{2}$$

where $\mu^2 = -2\lambda v^2 \sim \mathcal{O}(m^2_Z)$ is a fundamental parameter of the theory. These two terms have entirely different sources, and so a value of $m_{h_{\rm SM}} \sim m_Z$ should not arise by fine-tuned cancellation between the two terms. There are then two

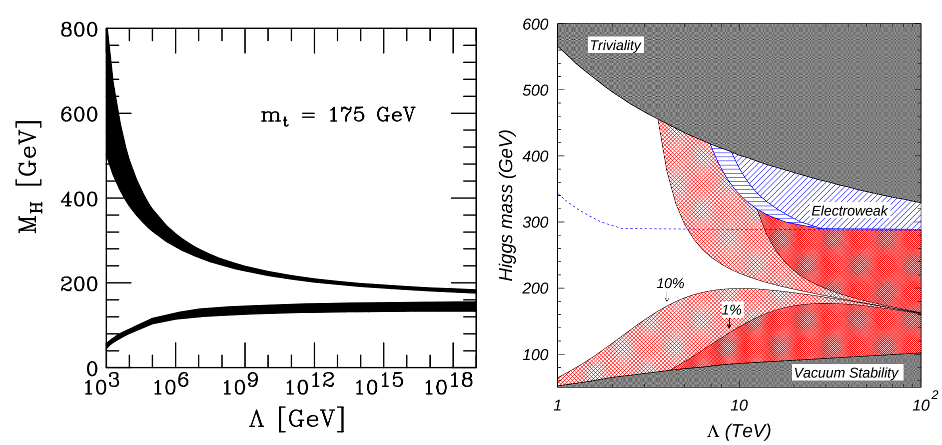

Figure 2. Left: Triviality and global minimum constraints on $m_{h_{\rm SM}}$ vs. Λ. Right: fine-tuning constraints on Λ.

possible solutions: Λ should be restricted to values $\lesssim$1 TeV; or $m_{h_{\rm SM}}$ should obey the 'Veltman' condition

$$m_{h_{\rm SM}}^2 = 4m_t^2 - 2m_W^2 - m_Z^2 \sim (317\ \text{GeV})^2\,. \tag{3}$$

In fact, this latter is a bit too simple and is somewhat modified in a Λ-dependent way by going to the next order in the loop calculations. This leads to an $m_{h_{\rm SM}}(\Lambda)$ solution to the no-fine-tuning 'Veltman' condition. However, just as we do not want to have a fine-tuned cancellation of the two terms in eq. (2), we also do not want to insist on too fine-tuned a choice for $m_{h_{\rm SM}}$ (in the SM there is no symmetry or theory that can predict this value). This implies that this solution cannot be employed out to too high a Λ. In practice, it is appropriate to allow a certain percentage (e.g. 1% or 10%) amount of fine-tuning in the cancellation between μ^2 and the loop contributions or in the choice of $m_{h_{\rm SM}}(\Lambda)$. These combined conditions are illustrated by the indicated bands in the right-hand half of figure 2.

The two-Higgs-doublet model (2HDM) is an example of new physics that could weaken the precision EW bound, but not cure the naturalness/hierarchy problem without additional new physics above a TeV. For example [10], let us consider the CP-conserving 2HDM with Higgs bosons h, H, A and $H^\pm$. Suppose all except A are heavy ($\sim$1 Tev) and that h is SM-like. The h diagrams will contribute a large $\Delta S > 0$ and large $\Delta T < 0$. The predicted S and T values would lie far outside the usual 95% CL ellipse. However, one can get back inside the ellipse if there is large additional $\Delta T > 0$, and this can arise from a small mass difference $m_{H^\pm} - m_H$. Algebraically, for large m_h and m_H,

$$\Delta\rho = \frac{\alpha}{16\pi m_W^2 c_W^2}\left\{\frac{c_W^2}{s_W^2}\frac{m_{H^\pm}^2 - m_{H^0}^2}{2} \right. \\ \left. -3m_W^2\left[\log\frac{m_{h^0}^2}{m_W^2} + \frac{1}{6} + \frac{1}{s_W^2}\log\frac{m_W^2}{m_Z^2}\right]\right\}. \tag{4}$$

By taking $m_{H^\pm} - m_H \sim$ few GeV with $m_h \sim m_H \sim m_{H^\pm} \sim 1$ TeV, the net $\delta\rho$ (equivalently ΔT) and ΔS will be solidly inside the 95% CL ellipse. As a possible side benefit of this model, if m_A is small and $\tan\beta$ (the ratio v_u/v_d of the vacuum expectation values of the neutral members of the H_u and H_d Higgs doublets that give mass to up and down type quarks respectively) is large, the resulting contribution to the anomalous magnetic moment of the muon, a_μ, can explain part of the observed deviation relative to the SM prediction [11,12].

Still, if we want a consistent effective theory all the way up to $M_{\rm Pl}$ without fine-tuning, we must have some new physics at a scale $\Lambda \sim$ 1–10 TeV. The prime candidate is supersymmetry (SUSY). The parameter Λ above would be identified with the scale of SUSY breaking, suggesting low energy SUSY with new particles at a mass scale of order 1 TeV. The prototype SUSY model is the minimal supersymmetric standard model (MSSM) which comprises SM particles, their sparticle partners and a two-doublet Higgs sector with constraints such that the only free purely Higgs sector parameters are $\tan\beta$ and m_A. (SUSY-breaking parameters influence the Higgs sector strongly at the one-loop level.) At large m_A, there is a light SM-like h. The great success of the MSSM is the fact that the combination of a 1 TeV scale for SUSY-breaking and the two-doublet Higgs sector results in rather precise coupling constant unification. Since it is hard to view this as an accident, one should take the MSSM and its extensions that preserve gauge coupling unification very seriously.

In general, it is clear that there will be many scenarios in which the SM is the effective theory up to some scale $\Lambda \gtrsim 1$ TeV. Thus, it is important to assess our ability to discover the $h_{\rm SM}$ or a SM-like Higgs, in the mass range from 114.4 GeV up to ~700 GeV or so, and perform precision measurements of its properties.

2.1 *Production/detection modes for the $h_{\rm SM}$ at hadron colliders*

A list of the most important modes for $h_{\rm SM}$ detection is: $gg \to h_{\rm SM} \to \gamma\gamma$, $gg \to h_{\rm SM} \to VV^{(*)}$, $q\overline{q} \to V^{(*)} \to h_{\rm SM}V$ with $h_{\rm SM} \to b\overline{b}, VV^{(*)}$, $qq \to qqV^{(*)}V^{(*)} \to qqh_{\rm SM}$ with $h_{\rm SM} \to \gamma\gamma$, $\tau^+\tau^-, VV^{(*)}$, and $qq, gg \to t\bar{t}h_{\rm SM}$ with $h_{\rm SM} \to b\overline{b}, \gamma\gamma, VV^{(*)}$.

Some NLO and higher corrections for these production processes have been computed. Generally, the 'K' factors are >1 but not always. (In particular, $K(t\bar{t}h_{\rm SM}) < 1$ at the Tevatron.) There are many references in which the important cross-sections for $h_{\rm SM}$ at the Tevatron and LHC can be found; for detailed references and some graphs, see [3]. The resulting prospects for detecting $h_{\rm SM}$ at the Tevatron and LHC are summarized in figure 3. Since the Tevatron will accumulate no more than 15 fb^{-1} (probably more like 5 fb^{-1}) of integrated luminosity before the LHC is in full swing, $h_{\rm SM}$ discovery at the Tevatron is on the edge except at low masses. The LHC energy and luminosity parameters were chosen, in large part, to guarantee $h_{\rm SM}$ detection. Once the LHC has accumulated of order 100 fb^{-1} to 300 fb^{-1}, figure 3 shows that there should be a substantial signal in several modes. In fact, some moderately precise checking of Higgs properties (ratios of branching ratios, some partial widths) will be possible. However, really precise measurements must await the linear collider (LC). For more details and references, see [4].

2.2 *Precision measurements at the LC*

At the LC, the primary production modes are:

$$e^+e^- \to Z^* \to Zh_{\rm SM}, \quad e^+e^- \to \nu\overline{\nu}W^*W^* \to \nu\overline{\nu}h_{\rm SM}, \quad e^+e^- \to t\bar{t}h_{\rm SM}.$$

The predicted cross-sections are such that $L = 200$–300 fb^{-1} will result in thousands of Higgs produced in the $Zh_{\rm SM}$ and $WW \to h_{\rm SM}$ modes at $\sqrt{s} = 500$ GeV. The $t\bar{t}h_{\rm SM}$ cross-section is smaller and $\sqrt{s} = 800$ GeV is required for substantial production rate.

The $Zh_{\rm SM}$ mode is *very* important as it allows one to observe the $h_{\rm SM}$ as a bump in the M_X spectrum of the $e^+e^- \to ZX$ final state, independent of how the $h_{\rm SM}$ decays. This allows a model-independent determination of $g^2_{ZZh_{\rm SM}}$, using which all $B(h_{\rm SM} \to F)$ can be extracted: $B(h_{\rm SM} \to F) = \sigma(e^+e^- \to Zh_{\rm SM} \to ZF)/\sigma(e^+e^- \to Zh_{\rm SM})$. A determination of $\Gamma^{\rm tot}_{h_{\rm SM}}$ is needed to compute $\Gamma(h_{\rm SM} \to F) = B(h_{\rm SM} \to F)\Gamma^{\rm tot}_{h_{\rm SM}}$. One technique employs the W-fusion cross-section:

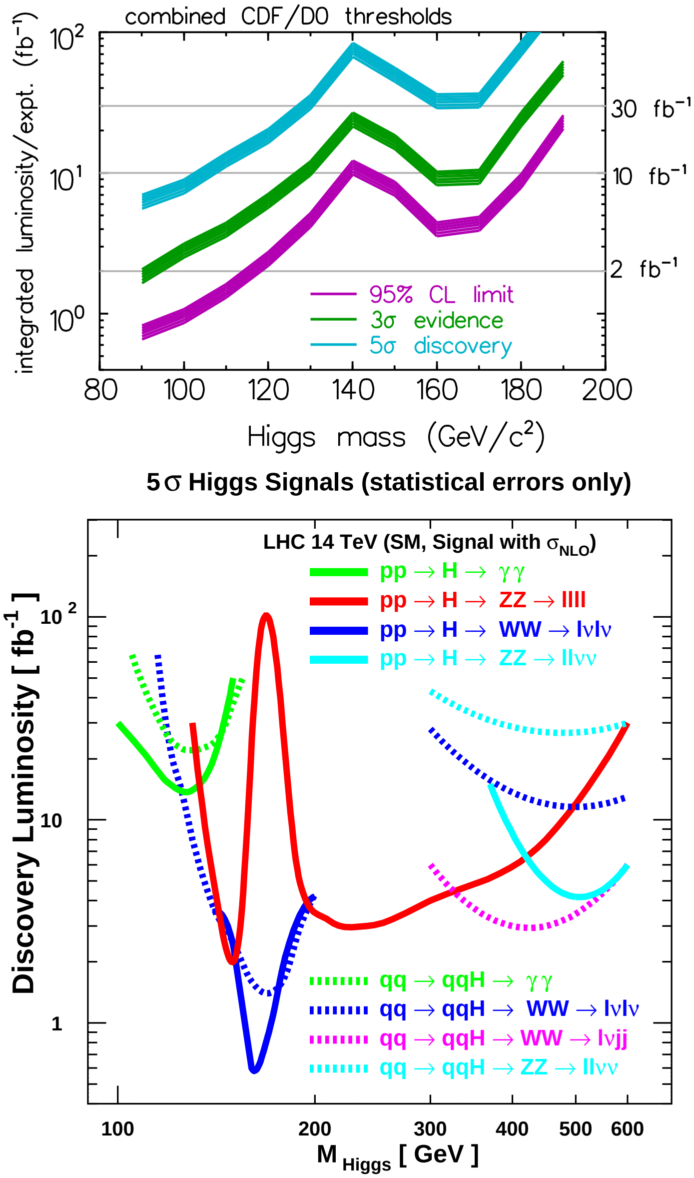

Figure 3. SM Higgs discovery at Tevatron and LHC. $h_{\rm SM}$ detection is guaranteed at the LHC.

$$\Gamma(h_{\rm SM} \to WW) \propto \frac{\sigma(e^+e^- \to \nu\bar{\nu}h_{\rm SM} \to \nu\bar{\nu}WW)}{B(h_{\rm SM} \to WW)_{Zh_{\rm SM}}},$$
$$\Gamma^{\rm tot}_{h_{\rm SM}} = \frac{\Gamma(h_{\rm SM} \to WW)}{B(h_{\rm SM} \to WW)}\,. \tag{5}$$

Table 1. Measurement precisions for the properties of a SM-like Higgs boson, $h_{\rm SM}$, for a range of Higgs boson masses. Unless otherwise noted (see footnotes below the table), we assume $\sqrt{s} = 500$ GeV and $L = 500$ fb^{-1}.

$\Delta m_{h_{\rm SM}}$	$\simeq 120$ MeV (Recoil against leptons from Z) $\simeq 50$ MeV (Direct reconstruction)				
$m_{h_{\rm SM}}$ (GeV)	120	140	160	200	400–500
$\sqrt{s}$ (GeV)			500		800
$\Delta\sigma(Zh_{\rm SM})/\sigma(Zh_{\rm SM})$	4.7%	6.5%	6%	7%	10%
$\Delta\sigma(\nu\bar{\nu}h_{\rm SM})B(b\bar{b})/\sigma B$	3.5%	6%	17%	–	–
$\delta g_{h_{\rm SM}xx}/g_{h_{\rm SM}xx}$ (from Bs)					
$t\bar{t}$	6–21%†	–	–	–	10%
$b\bar{b}$	1.5%	2%	3.5%	12.5%	–
$c\bar{c}$	20%	22.5%	–	–	–
$\tau^+\tau^-$	4%	5%	–	–	–
$\mu^+\mu^-$	15%‡	–	–	–	–
WW^*	4.5%	2%	1.5%	3.5%	8.5%
ZZ^*	–	–	8.5%	4%	10%
gg	10%	12.5%	–	–	–
$\gamma\gamma$	7%	10%	–	–	–
$g_{h_{\rm SM}h_{\rm SM}h_{\rm SM}}$	20%§	–	–	–	–
$\Gamma^{\rm tot}_{h_{\rm SM}}$ ††	10.1%	8.2%	12.9%	10.6%	22.3%

†The $h_{\rm SM}t\bar{t}$ coupling errors are from $e^+e^- \to t\bar{t}h_{\rm SM}$, with $\sqrt{s} = 500$–800 GeV and 1 ab^{-1} of data.

‡Based on $\sqrt{s} = 800$ GeV and 1 ab^{-1} of data.

§Based on $\sqrt{s} = 500$ GeV and 1 ab^{-1} of data.

††Indirect determination from $\Gamma(VV^*)/B(VV^*)$, $V = W, Z$.

A rough determination of $g_{h_{\rm SM}h_{\rm SM}h_{\rm SM}}$ is possible using sensitivity of $\sigma(e^+e^- \to Zh_{\rm SM}h_{\rm SM})$ coming from the sub-graph described by $e^+e^- \to Zh^*_{\rm SM}$ with $h^*_{\rm SM} \to h_{\rm SM}h_{\rm SM}$. All the other graphs contributing to the same $Zh_{\rm SM}h_{\rm SM}$ final state are the primary 'background' to the one sub-graph of interest. The accuracies with which various branching ratios and couplings can be determined at the LC are given in table 1 from [4]. For more details see [13].

2.3 *Checking the SM Higgs boson quantum numbers*

Of particular ultimate importance will be the determination of the quantum numbers of a presumed Higgs resonance. The spin-0 nature of the $h_{\rm SM}$ can be checked by looking at the threshold rise of the $Zh_{\rm SM}$ cross-section [14,15], which is much more rapid for $J = 0$ than for $J = 1$ or $J = 2$. More difficult is the determination of the CP of the $h_{\rm SM}$. It appears that the best approach to this determination, especially for $m_{h_{\rm SM}}$ such that $\Gamma^{\rm tot}_{h_{\rm SM}}$ is small, is to employ the $\gamma\gamma$ collider option at the LC. Most other techniques will yield very poor accuracy. For example, angular leptonic distributions in $Zh_{\rm SM} \to \ell^+\ell^- h_{\rm SM}$ production and/or $h_{\rm SM} \to Z^* Z^* \to 4\ell$ only check that $h_{\rm SM}$ has a substantial CP $= +$ component. Indeed, since any CP $= -$ component couples only at one loop, one could have up to 80% CP-odd without seeing an alteration in the angular distribution. Of course, the $Zh_{\rm SM}$ cross-section would be smaller than anticipated, but such a reduction could arise from other sources than CP-mixing. In particular, the observed Higgs boson could be just one of the several CP-even Higgs bosons that share the ZZ-Higgs coupling-squared. One can employ $e^+e^- \to Zh_{\rm SM}$ with $h_{\rm SM} \to \tau^+\tau^-$ and use the self-analysing decays $\tau^+ \to \rho, \pi + \nu$, but this is quite hard and the accuracy of the CP determination is not wonderful [16–19]. At the $\gamma\gamma$ collider, the approach [18,20,21] is to transversely polarize the laser photons (yielding partially transversely polarized back-scattered photons) and use the fact that the CP-even part couples to transversely polarized photons as $\vec{\epsilon} \cdot \vec{\epsilon}\,'$ while the CP-odd part couples as $\vec{\epsilon} \times \vec{\epsilon}\,'$. As a result, it is easy to isolate the CP-even from the CP-odd Higgs components by comparing rates for parallel vs. perpendicular transverse polarizations. The asymmetries expected are large because both the CP-even and CP-odd components of a Higgs boson couple strongly to $\gamma\gamma$ (via the top-quark loop for the CP-odd part), which is to be contrasted with the ZZ or WW coupling which only arises at one-loop level for the CP-odd component. Recent studies [22] show that one can check that the $h_{\rm SM}$ is CP $= +$ with an accuracy of $\sim 11\%$ for the sample case of $m_{h_{\rm SM}} \sim 120$ GeV.

3. Beyond the SM Higgs boson

There are many possible extensions of the SM Higgs sector. Some are very highly motivated, while others fall into the 'why not?' category. First, one can easily imagine extending only the Higgs sector of the SM to include extra Higgs representations. For example, one can add one or more singlet (under $SU(2)_{\rm L} \times U(1)$) Higgs fields, one or more extra doublet Higgs representations (which can allow for CP violation through the Higgs sector), one or more triplet representations (as in left–right symmetric models), and so forth.

It is worth noting that when analysing data from LEP assuming only the SM $h_{\rm SM}$, there is a broad interval of $m_{h_{\rm SM}} > 100$ GeV in which the measured $1 - CL_b$ value is significantly below the value predicted by pure background. In particular, there are 'weak' signals in the vicinity of $m_h \sim 115$ GeV and $m_h = 97$ GeV in hZ production [7] and at $m_h + m_A = 187$ GeV in hA production. These are consistent with a more complicated Higgs sector with multiple CP-even Higgs bosons sharing the ZZ coupling and the possible presence of a CP-odd Higgs boson, A, or of mixing between a set of CP-even and CP-odd Higgs bosons.

One motivation for extra Higgs representations is the fact that appropriate choices will result in exact coupling constant unification [23]. For example, two-doublets plus one $Y = 0$ triplet gives coupling unification at $M_U = 1.7 \times 10^{14}$, which is acceptable (i.e. would not result in proton decay) if there is no group unification (as is the case in certain types of string models). The inclusion of $Y = 2$

triplets in the left–right symmetric models will give rise to the see-saw mechanism for neutrino masses and can also give coupling unification (albeit at low values of M_U).

However, none of the Higgs-sector-only extensions provide a solution to the hierarchy/fine-tuning problem. The various approaches that attempt to address this issue are well-known. One set of choices includes technicolor, top-assisted technicolor, and little Higgses. Generically, all these have difficulties with precision electroweak data, although very careful construction can ameliorate the custodial symmetry violation. The second very popular approach is to avoid the fine-tuning and naturalness issues by allowing for large extra-dimensions with extra dimension sizes set by the TeV scale. In this kind of approach, coupling unification at the inverse dimension size or apparent unification at a large scale of order M_U are both possible, but are not particularly well motivated.

My view is that supersymmetry with exactly two Higgs doublets (the MSSM) or two Higgs doublets with one or more Higgs singlets is the best motivated. (a) The naturalness, fine-tuning and hierarchy issues are resolved if the scale of supersymmetric particle masses is $m_{\rm SUSY} \sim 1$ TeV–10 TeV. (b) Coupling unification at $M_U \sim$ few $\times 10^{16}$ GeV is excellent for $m_{\rm SUSY} \sim 1$ TeV–10 TeV. (c) Electroweak symmetry breaking starting from generically large (possibly universal) scalar masses at M_U is 'automatic' as a result of the H_u scalar mass-squared being driven negative under renormalization group evolution by the large top-quark Yukawa coupling. (d) The predicted value of M_U is large enough to allow for true group unification without unacceptable proton decay. (e) Excellent agreement with precision electroweak data is nearly automatic. Thus, the remainder of this review will focus on Higgs bosons in the context of supersymmetry, beginning with the MSSM.

3.1 *The Higgs bosons of the MSSM*

This section is based on references and materials found in [3–5]. This minimal SUSY model contains exactly two Higgs doublets, one with $Y = +1$ (Φ_u) and one with $Y = -1$ (Φ_d). Φ_u (Φ_d) is required for giving masses to up-quarks (down-quarks and leptons). Further, two doublets with opposite Y (or more generally an even number) are needed for anomaly cancellation. A model with more than two doublets is disfavored in view of the fact that coupling unification fails badly.

The MSSM Higgs sector is CP-conserving (CPC) at tree-level. However, corrections involving complex soft-SUSY-breaking parameters can introduce CP-mixing at the one-loop level [24,25]. For the CPC case, the Higgs mass eigenstates are: the CP-even h, H; the CP-odd A; and the charged Higgs pair $H^\pm$. We will focus on this case for the next part of the discussion.

3.1.1 *Tree-level Higgs masses and diagonalization.* At tree-level, all Higgs masses and couplings are determined by just two parameters. $\tan\beta = v_u/v_d$ (where $v_u = \sqrt{2}\langle\Phi_u^0\rangle, v_d = \sqrt{2}\langle\Phi_d^0\rangle$) and m_A. The CP-even eigenstates are obtained by diagonalizing a 2×2 matrix using a rotation angle α:

$$\begin{aligned} h &= -(\sqrt{2}\,{\rm Re}\,\Phi_d^0 - v_d)\sin\alpha + (\sqrt{2}\,{\rm Re}\,\Phi_u^0 - v_u)\cos\alpha, \\ H &= (\sqrt{2}\,{\rm Re}\,\Phi_d^0 - v_d)\cos\alpha + (\sqrt{2}\,{\rm Re}\,\Phi_u^0 - v_u)\sin\alpha. \end{aligned} \tag{6}$$

At tree-level, $m_h \le m_Z|\cos 2\beta| \le m_Z$, due to the fact that all Higgs self-coupling parameters of the MSSM are related to the squares of the electroweak gauge couplings. A particularly useful relationship between α and β is: $\cos^2(\beta-\alpha) = m_h^2(m_Z^2-m_h^2)/m_A^2(m_H^2-m_h^2)$.

3.1.2 *Tree-level couplings.* Three-point Higgs boson–vector boson couplings are conveniently summarized by listing the couplings that are proportional to either $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$, and the couplings that are independent of α and β:

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$	Angle-independent
HW^+W^-	hW^+W^-	
HZZ	hZZ	
ZAh	ZAH	ZH^+H^-, γH^+H^-
$W^\pm H^\mp h$	$W^\pm H^\mp H$	$W^\pm H^\mp A$

All vertices that contain at least one vector boson and exactly one non-minimal Higgs boson state (H, A or $H^\pm$) are proportional to $\cos(\beta-\alpha)$. The couplings of the neutral Higgs bosons to $f\bar{f}$ relative to the standard model value, $gm_f/2m_W$, are given by (below, γ_5 indicates pseudoscalar coupling):

$$h b\bar{b} \quad (\text{or } h\tau^+\tau^-): -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,, \tag{7}$$

$$ht\bar{t}: \quad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,, \tag{8}$$

$$Hb\bar{b} \quad (\text{or } H\tau^+\tau^-): \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,, \tag{9}$$

$$Ht\bar{t}: \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)\,, \tag{10}$$

$$Ab\bar{b} \quad (\text{or } A\tau^+\tau^-): \quad \gamma_5\tan\beta\,, \tag{11}$$

$$At\bar{t}: \quad \gamma_5\cot\beta. \tag{12}$$

3.1.3 *The decoupling limit at tree-level.* In the decoupling limit of $m_A \gg m_Z$ [26],

$$m_h^2 \simeq m_Z^2\cos^2 2\beta, \quad m_H^2 \simeq m_A^2 + m_Z^2\sin^2 2\beta, \quad m_{H^\pm}^2 = m_A^2 + m_W^2, \tag{13}$$

$$\cos^2(\beta-\alpha) \simeq \frac{m_Z^4\sin^2 4\beta}{4m_A^4}\,. \tag{14}$$

Thus, $m_A \sim m_H \sim m_{H^\pm}$ up to terms of order m_Z^2/m_A, and $\cos(\beta-\alpha)=0$ up to corrections of order m_Z^2/m_A^2. Further, the h couplings are all SM-like. This means that the effective low-energy theory below scales of order m_A is the SM. However, one should note that at large $\tan\beta$, the $hb\overline{b}$ coupling could have significant deviations from the SM value if $\tan\beta\cos(\beta-\alpha)$ is not small. This is sometimes called 'delayed decoupling'. The couplings of the heavy Higgs bosons include: HAZ and $W^\pm H^\mp Z$ at maximal strength and $Ht\bar{t}, At\bar{t} \propto \cot\beta$ and $Hb\overline{b}, Ab\overline{b} \propto \tan\beta$.

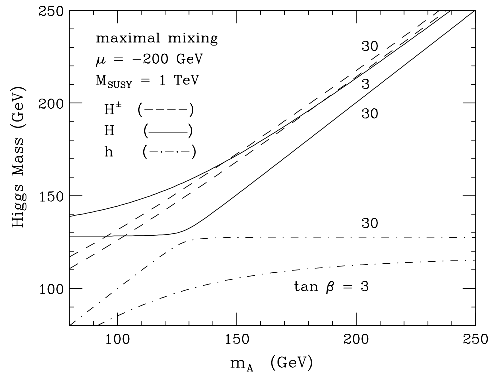

Figure 4. Higgs masses as a function of m_A for maximal mixing with $m_{\text{SUSY}} = M_Q = M_U = M_D = 1$ TeV.

3.1.4 *Radiative corrections to* m_h. There are top and stop loop contributions to the mass-matrix. These do not cancel completely since SUSY is broken. The crucial parameters are the average of the two top-squark squared-masses, $M_S^2 \equiv \frac{1}{2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)$and the parameter $X_{\rm t} \equiv A_{\rm t} - \mu\cot\beta$ that enters into stop-mixing. ($A_{\rm t}$ describes trilinear soft-SUSY-breaking and μ appears in the $\mu\widehat{H}_u\widehat{H}_d$ term of the superpotential.) The radiatively-corrected upper bound on the lightest CP-even Higgs mass is approximately given by

$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_{\rm t}^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_{\rm t}^2}\right) + \frac{X_{\rm t}^2}{M_S^2}\left(1 - \frac{X_{\rm t}^2}{12M_S^2}\right)\right]. \tag{15}$$

This reaches a maximum for $X_{\rm t} \sim \sqrt{6}M_S$. Also, $m_h^{\max}$ rises only very slowly with M_S once $M_S \gtrsim 1$ TeV. Finally, m_h tends to increase slowly with $\tan\beta$. Figure 4 illustrates the Higgs boson masses for several values of $\tan\beta$.

3.1.5 *Radiative corrections to couplings* [27]. The dominant corrections for Higgs couplings to vector bosons arise from radiative corrections to $\cos(\beta-\alpha)$ (which we shall shortly discuss). For Yukawa couplings there are additional (non-decoupling) one-loop vertex corrections defined by

$$\begin{aligned} -\mathcal{L}_{\rm eff} = \epsilon_{ij}[&(h_b + \delta h_b)\bar{b}_{\rm R}\mathbf{\Phi}_{\mathbf{d}}^i Q_{\rm L}^j + (h_{\rm t} + \delta h_{\rm t})\bar{t}_{\rm R} Q_{\rm L}^i \mathbf{\Phi}_{\mathbf{u}}^j] \\ &+ \Delta h_{\rm t}\bar{t}_{\rm R} Q_{\rm L}^k \mathbf{\Phi}_{\mathbf{d}}^{k*} + \Delta h_b \bar{b}_{\rm R} Q_{\rm L}^k \mathbf{\Phi}_{\mathbf{u}}^{k*} + {\rm hc}, \end{aligned}$$

implying a modification of the tree-level relations between $h_{\rm t}$, h_b and $m_{\rm t}$, m_b as follows:

$$m_b = \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b), \quad m_{\rm t} = \frac{h_{\rm t} v}{\sqrt{2}}\sin\beta(1+\Delta_{\rm t}). \tag{16}$$

The dominant contributions to Δ_b are $\tan\beta$-enhanced, with $\Delta_b \simeq (\Delta h_b/h_b)\tan\beta$. In the same limit, $\Delta_{\rm t} \simeq \delta h_{\rm t}/h_{\rm t}$, with the additional contribution of $(\Delta h_{\rm t}/h_{\rm t})\cot\beta$ providing a small correction. The most important point is that Δ_b does not vanish in the limit of large values of the supersymmetry breaking masses if, for example, μ, $M_{\tilde{g}}$ and $M_{\tilde{b}_{1,2}}$ remain of similar size. Indeed, $\Delta_b \sim \pm 1$ is possible for large $\tan\beta$ regardless of the size of these mass parameters. The corresponding $\Delta_\tau \ll \Delta_b$ because the magnitude of Δ_b is proportional to $\alpha_{\rm s}$ and $h_{\rm t}$ while Δ_τ is proportional to only the weak gauge couplings.

3.1.6 *Radiative corrections to* $\cos(\beta-\alpha)$. In terms of the radiative corrections $\delta\mathcal{M}^2_{11}, \delta\mathcal{M}^2_{22}, \mathcal{M}^2_{12}$ to the 2×2 CP-even mass matrix, we obtain a correction to our earlier computation of $\cos(\beta-\alpha)$. For $m_A \gg m_Z$, one finds [27]

$$\cos(\beta-\alpha) = c\left[\frac{m_Z^2\sin 4\beta}{2m_A^2} + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right)\right],$$
$$c \equiv 1 + \frac{\delta\mathcal{M}^2_{11} - \delta\mathcal{M}^2_{22}}{2m_Z^2\cos 2\beta} - \frac{\delta\mathcal{M}^2_{12}}{m_Z^2\sin 2\beta}.$$

In the generic $c \neq 0$ cases, we get rapid decoupling for $m_A \gg m_Z$, just as at tree-level. However, $\cos(\beta-\alpha) = 0$ can be achieved also by choosing the MSSM parameters (that govern the $\delta\mathcal{M}^2_{11,22,12}$) such that $c=0$. That is,

$$2m_Z^2\sin 2\beta = 2\,\delta\mathcal{M}^2_{12} - \tan 2\beta\left(\delta\mathcal{M}^2_{11} - \delta\mathcal{M}^2_{22}\right). \tag{17}$$

Note that eq. (17) is independent of the value of m_A. For a typical choice of MSSM parameters, eq. (17) yields a solution at large $\tan\beta$: $\tan\beta \simeq [2m_Z^2 - \delta\mathcal{M}^2_{11} + \delta\mathcal{M}^2_{22}]/[\delta\mathcal{M}^2_{12}]$. For this value of $\tan\beta$, $\cos(\beta-\alpha)=0$ independently of the value of m_A. We shall refer to this phenomenon as m_A-independent decoupling. Explicit solutions to eq. (17) depend on ratios of MSSM parameters and are insensitive to the overall supersymmetric mass scale, modulo a mild logarithmic dependence on $M_S/m_{\rm t}$.

3.1.7 *More on the decoupling limit.* After combining the various radiative corrections and working to first order in $\cos(\beta-\alpha)$, we find that at large m_A the deviation of the $hb\bar{b}$ coupling from its SM value vanishes as m_Z^2/m_A^2 for all values of $\tan\beta$. Then, if we keep only the leading $\tan\beta$-enhanced radiative corrections we have

$$\frac{g^2_{hVV}}{g^2_{h_{\rm SM}VV}} \simeq 1 - \frac{c^2 m_Z^4\sin^2 4\beta}{4m_A^4}, \quad \frac{g^2_{htt}}{g^2_{h_{\rm SM}tt}} \simeq 1 + \frac{c m_Z^2 \sin 4\beta\cot\beta}{m_A^2},$$
$$\frac{g^2_{hbb}}{g^2_{h_{\rm SM}bb}} \simeq 1 - \frac{4c m_Z^2\cos 2\beta}{m_A^2}\left[\sin^2\beta - \frac{\Delta_b}{1+\Delta_b}\right]. \tag{18}$$

From these results we see that the approach to decoupling is fastest for the h couplings to vector bosons and slowest for its couplings to down-type quarks. As before, if $c=0$, as possible for large $\tan\beta$, then we have m_A-independent decoupling. If c is not suppressed, the deviations of $\Gamma(h\to b\bar{b})$ from the SM prediction are such that 5% deviations might be visible for m_A as large as 1 TeV, but, if $\tan\beta$ is such that $c\sim 0$, we might also see no deviations even if m_A is small. This means that in order to interpret deviations, a knowledge of soft-SUSY-breaking parameters is needed.

For loop-induced decays/couplings such as ggh or $\gamma\gamma h$ there are really two decoupling issues. (1) Is $m_A \gg m_Z$? (2) Is $m_{\mathrm{SUSY}} \gg m_Z$? If only the first holds, then SUSY loops (of colored or charged particles, respectively) can still yield deviations with respect to SM expectations.

3.1.8 *Branching ratios and widths of MSSM Higgs bosons.* Once $m_A \gtrsim$ 120–130 GeV, h is SM-like in its decays, with little dependence on $\tan\beta$. However, the branching ratios of H, A and $H^\pm$ are quite complex when $\tan\beta$ is $\lesssim 5$. At high $\tan\beta$, if m_A is in the decoupling regime then the A and H decay to $b\bar{b}$ and $\tau^+\tau^-$, while $H^\pm \to \tau^\pm\nu$ (tb) for $m_{H^\pm} < m_{\mathrm{t}} + m_b$ $(m_{H^\pm} > m_{\mathrm{t}} + m_b)$, respectively. Regarding widths, h is always narrow, whereas $A, H, H^\pm$ can acquire widths of order 1 to 10 GeV for large $\tan\beta$ values and large masses.

3.1.9 *MSSM Higgs cross-sections.* At hadron colliders, the important cross-sections are:

$$\begin{aligned}&gg \to \phi, \quad qq \to qqV^*V^* \to qqh, qqH,\\ &q\bar{q} \to V^* \to hV/HV, \quad gg, q\bar{q} \to \phi b\bar{b}/\phi t\bar{t},\end{aligned}$$

where $\phi = h$, H or A. At the LC, the most important cross-sections are

$$\begin{aligned}&\text{Higgs-strahlung:} \quad e^+e^- \to Zh, \quad e^+e^- \to ZH,\\ &\text{Pair production:} \quad e^+e^- \to hA, \quad e^+e^- \to HA, \quad e^+e^- \to H^+H^-,\\ &\text{Yukawa radiation:} \quad e^+e^- \to t\bar{t}\phi, \quad e^+e^- \to b\bar{b}\phi\,. \end{aligned} \tag{19}$$

3.1.10 *Some remarks on Higgs discovery and measurements in the MSSM.* LEP limits on the MSSM Higgs sector are really rather substantial, especially for the minimal-mixing scenario that is in many respects the 'cleanest' model. The unexcluded domains lie at high m_A (for which decoupling is setting in) and high $\tan\beta$.

Regarding future discovery, if the Tevatron reaches L = 10–25 fb^{-1}, then it will be able to discover h in most cases [28]. Further, at very high $\tan\beta$ the detection of $b\bar{b}H/A$ will be possible for a range of modest m_A at the Tevatron. The LHC is guaranteed to find at least one MSSM Higgs boson. This is illustrated by the 'standard' $(m_A, \tan\beta)$ plot of figure 5. But, as one pushes further into the decoupling region, there is an increasingly large 'wedge' of parameter space (covering a range of moderate $\tan\beta$) in which only h will be detectable. A LC will certainly detect $e^+e^- \to Zh$; $e^+e^- \to HA$ will be observable if $m_A \lesssim \sqrt{s}/2$ (e.g. $\lesssim$300 GeV for $\sqrt{s} = 600$ GeV). But, above this the LC wedge is even bigger than the LHC wedge [31]. Thus, if SUSY is observed at the LHC and/or LC and if h is seen, then one will know that there are (at least) $H, A, H^\pm$ to be discovered. However, these will not be detected if the MSSM parameters are in the 'wedge'. There are then only two options for their direct discovery: (a) increase $\sqrt{s}$ past $2m_A$ – of course, m_A may not be known; (b) operate the LC in the $\gamma\gamma$ collider mode. In fact, as illustrated in figure 6, $\gamma\gamma$ collisions will allow H, A discovery precisely in the 'wedge' region up to $m_A \sim m_H \lesssim 0.8\sqrt{s}$, even if there is no prior knowledge of m_A [22,32]. Thus, the $\gamma\gamma$ option would become a priority at a certain point, and one could simultaneously have a very interesting overall $\gamma\gamma$ physics program. This scenario is especially likely if there is a substantial time-gap between a 500 to 800 GeV LC and a TeV-scale LC.

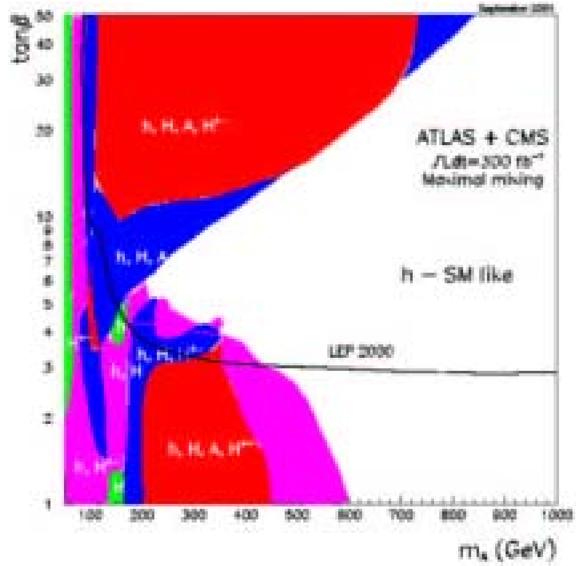

Figure 5. 5σ discovery regions at the LHC (from [29], see also [30]).

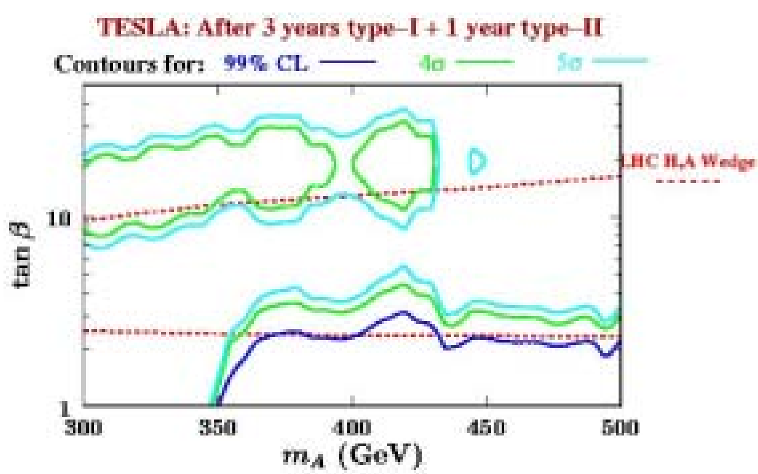

Figure 6. Contours for discovery and 99% CL exclusion after four years of TESLA $\gamma\gamma$ running. The roughly horizontal lines indicate the upper and lower limits of the 'wedge' region where neither the LHC nor the LC could detect H, A or $H^{\pm}$.

3.1.11 *CP-violation in the MSSM Higgs sector induced at one-loop* [24]. If the soft-SUSY-breaking parameters are complex, then δh_b, Δh_b, δh_t and Δh_t can all be complex. In fact, it is possible to find parameter choices consistent with EDM limits, and so forth, that give large CP-violation in the Higgs sector. This would have five crucial consequences. (1) The h, H and A all mix together and one has simply three neutral eigenstates $h_{1,2,3}$. (2) The fermionic couplings of $h_{1,2,3}$ will all have a mixture of $a + i\gamma_5 b$ couplings, where a is the CP-even part and b is the CP-odd part. (3) The $h_{1,2,3}$ will share the VV coupling strength squared, generalizing the usual sum rule to $\sum_{i=1,2,3} g^2_{h_i VV} = g^2_{h_{\rm SM} VV}$. (4) The $h_{1,2,3}$ could at the same time have somewhat similar masses, perhaps overlapping within the experimental resolution in certain channels. (5) In some regions of parameter space, one h_i can have substantial VV coupling (which is the usual requirement for easy discovery), but instead of decaying in the usual way, decays to a pair of lighter $h_j h_j$ or $h_j h_k$ or to Zh_k. All of this make Higgs discovery more difficult. There is even a region of parameter space such that there is a fairly light Higgs boson ($\lesssim 50$ GeV) that would not have been seen at LEP.

3.2 *Remarks regarding a general CP-violating 2HDM*

All the features discussed just above regarding a CP-violating MSSM Higgs sector apply even more forcefully to a general CP-violating two-Higgs-doublet model (2HDM), and can potentially lead to some real problems for Higgs detection and analysis, as we enumerate. (1) The Tevatron could fail to see any of the h_i signals simply because all are weaker than predicted for the case where there is a single SM-like Higgs. The same could be true of the LHC. A particular problem is that the $\gamma\gamma$ decay mode of any h_i is rapidly suppressed when the $h_i VV$ coupling is not of full strength, which also suppresses the $WW \to h_i$ fusion cross-section. (2) Even a Higgs with good production cross-section might not be detectable since it decays to two other Higgs bosons, each of which decays to $b\bar{b}$ (for example). (3) A future LC would be guaranteed to find at least one of the Higgs bosons, provided there is no precision electroweak 'conspiracy' discussed earlier. This is because: (a) the precision electroweak data require significant $g^2_{ZZh_i}$ weight for $m_{h_i} \lesssim 200$ GeV; (b) the Zh_i and $W^*W^* \to h_i$ cross-sections cannot all be suppressed for h_i in this mass region; and (c) the LC can probe to very small $g^2_{ZZh_i}$. In the case of the MSSM, the renormalization group equations and constraints guarantee that one of the h_i will be light and have substantial $g^2_{ZZh_i}$.

There is still a decoupling limit in the MSSM context. If $m_{H^\pm} \gg m_Z$, then h will become pure CP-even. The H and A will be heavy and can still mix strongly, but at least discovery of h would be guaranteed.

3.3 *Determination of Higgs CP properties*

From the above discussion, it should be apparent that there are many important situations in which we will need a way of precisely measuring the CP properties of one or more Higgs bosons. These include: (1) separating the H and A of the MSSM; (2) determining the CP admixture of a given mass eigenstate in the case of a CPV Higgs sector; and (3) resolving overlapping Higgs resonances of different or mixed CP character. In many respects, the $\gamma\gamma$ collider is clearly the best machine for accomplishing these tasks, and in many cases it would be the only way. At the $\gamma\gamma$ collider [18,20,21], one uses maximally polarized (either transverse or circular, depending upon whether the Higgs sector appears to be CPC or CPV, respectively) laser photon beams and looks at various rate asymmetries. Signal–background interference in the $t\bar{t}$ final state can also probe the CP of a Higgs boson with large $t\bar{t}$ branching ratio [33].

3.4 *Phenomenological indications for the decoupling limit*

There are many observations that suggest that the Higgs sector parameters may be in a decoupling limit. First, we have already noted that LEP limits tend to push in that direction in the MSSM context; they require large $\tan\beta$ and m_A in the minimal-mixing scenario, for example. Second, allowing the most general fermionic coupling structure in, for example, a general 2HDM leads to FCNC. However, in the decoupling limit this is not a problem since the FCNC couplings of the surviving light Higgs are suppressed by the small value of $\cos(\beta-\alpha)$ [26]. The MSSM is a particular example of this; it can be shown that all CP-violating couplings of the SM-like h vanish as $\cos(\beta-\alpha)\to 0$ in the true decoupling limit. Of course, H and A (in the 2HDM for example) will generally have FCNC and CPV couplings, but their effects are suppressed by a factor of m_h^2/m_A^2 due to the large masses appearing in the propagators of the heavy Higgs bosons [26]. As a result, all FCNC and CPV effects are at the same level for h, H and A and are of order $\cos(\beta-\alpha)\sim m_h^2/m_A^2$.

Thus, we might in general anticipate that the Higgs sector will be in a decoupling limit, unless the model contains other symmetries for suppressing the naturally present FCNC and CPV couplings. In this regard, it is worth noting that SUSY left–right symmetric models can be constructed with the needed symmetries.

3.5 *The NMSSM Higgs sector*

There are many reasons to think that the simple two-doublet MSSM Higgs sector should be extended by including at least one extra singlet (see [5] for a summary). First, it is not easy to avoid having extra singlet superfields in the generic string theory context. At a more phenomenological, model-building level, introducing an extra singlet superfield, $\widehat{S}$, and the superpotential interaction $W\ni\lambda\widehat{H}_1\widehat{H}_2\widehat{S}$ leads to an effective $\mu\widehat{H}_1\widehat{H}_2$ term with $\mu=\lambda s$ when $\langle\widehat{S}_{\text{scalar component}}\rangle=s$. It is natural for s, and hence μ to have a scale of the order of electroweak scale, as is absolutely required for acceptable phenomenology of the MSSM. In the MSSM context, the μ term of the superpotential has no natural source; it is introduced more or less 'by hand'. In general, the NMSSM also includes a superpotential term of the form $W\ni\kappa\widehat{N}^3$. Of course, it is important to remember that adding extra singlets to the two doublets of the MSSM does not affect the success of gauge unification.

Assuming no CP violation, the NMSSM Higgs sector mass eigenstates comprise: 3 CP-even Higgs bosons, $h_{1,2,3}$; 2 CP-odd Higgs bosons, $a_{1,2}$; and a charged Higgs pair $h^{\pm}$. There has been a substantial body of work addressing the phenomenology of the NMSSM Higgs sector, particularly the issue of whether or not there is a 'no-lose' theorem for Higgs discovery at a given type of collider (i.e. a guarantee that at least one of the NMSSM Higgs bosons will be discovered).

Linear collider. Many groups have shown that one can add a singlet and still find a signal [34]. The basic point is that the renormalization group equations and structure of the superpotential for the NMSSM are such that there will always be one of the h_i that both has mass below ∼160 GeV and has $g^2_{ZZh_i} \gtrsim 0.5 g^2_{ZZh_{\rm SM}}$. This h_i will be easily detectable in $e^+e^- \to Zh_i$ production at a LC with $\sqrt{s} > 350$ GeV.

The LHC. Establishing that one of the NMSSM Higgs bosons is guaranteed to be detected at the LHC has proven to be a highly non-trivial task. However, recent work has shown that it may actually be possible [35,36]. I review the situation briefly. First, let us recall the basic parameters of the model required to specify the Higgs sector. They are: λ, κ, μ, $\tan\beta$, A_λ, and A_κ (the latter two are the soft SUSY breaking parameters associated with λ and κ superpotential terms). A scan over all the parameters is performed. Perturbativity for the couplings after evolution up to M_U via the RGE equations is imposed.

The 'standard' Higgs boson discovery modes for the LHC for a multi-doublet + singlet Higgs sector are listed below (with $\ell = e, \mu$):

(1) $gg \to h/a \to \gamma\gamma$;
(2) associated Wh/a or $t\bar{t}h/a$ production with $\gamma\gamma\ell^\pm$ in the final state;
(3) associated $t\bar{t}h/a$ production with $h/a \to b\bar{b}$;
(4) associated $b\bar{b}h/a$ production with $h/a \to \tau^+\tau^-$;
(5) $gg \to h \to ZZ^{(*)} \to 4$ leptons;
(6) $gg \to h \to WW^{(*)} \to \ell^+\ell^-\nu\bar{\nu}$;
(7) $WW \to h \to \tau^+\tau^-$;
(8) $WW \to h \to WW^{(*)}$.

We first surveyed all NMSSM parameter choices such that the decay modes in which one Higgs decays to one or more Higgs boson of lower mass,

$$\begin{aligned} &\text{(i) } h \to h'h' , \quad \text{(ii) } h \to aa , \quad \text{(iii) } h \to h^\pm h^\mp , \quad \text{(iv) } h \to aZ , \\ &\text{(v) } h \to h^\pm W^\mp , \quad \text{(vi) } a' \to ha , \quad \text{(vii) } a \to hZ , \quad \text{(viii) } a \to h^\pm W^\mp , \end{aligned} \tag{20}$$

are forbidden. The outcome is that the statistical significances for detecting the Higgs bosons obtained by combining all the modes (1)–(8), including the absolutely crucial W-fusion modes, are always $\gtrsim 7\sigma$. Thus, NMSSM Higgs boson discovery by just one detector with $L = 300$ fb^{-1} is essentially guaranteed for those portions of parameter space for which Higgs boson decays to other Higgs bosons or supersymmetric particles are kinematically forbidden.

However, if we scan over parameters such that at least one of the modes of eq. (20) is allowed, we find many scenarios in which the signals in modes (1)–(8) are extremely weak. In all such cases (a) there is a light CP-even Higgs boson with mass of order 110 to 130 GeV with substantial doublet content that decays mainly to two still lighter CP-odd Higgs states, $h \to aa$, where m_a can range from ∼5 to ∼50 GeV and (b) all the other Higgs states are either dominantly singlet-like, implying highly suppressed production rates, or relatively heavy, decaying to $t\bar{t}$ or to one of the 'difficult' modes (i)–(viii). In such cases, it seems evident that the best

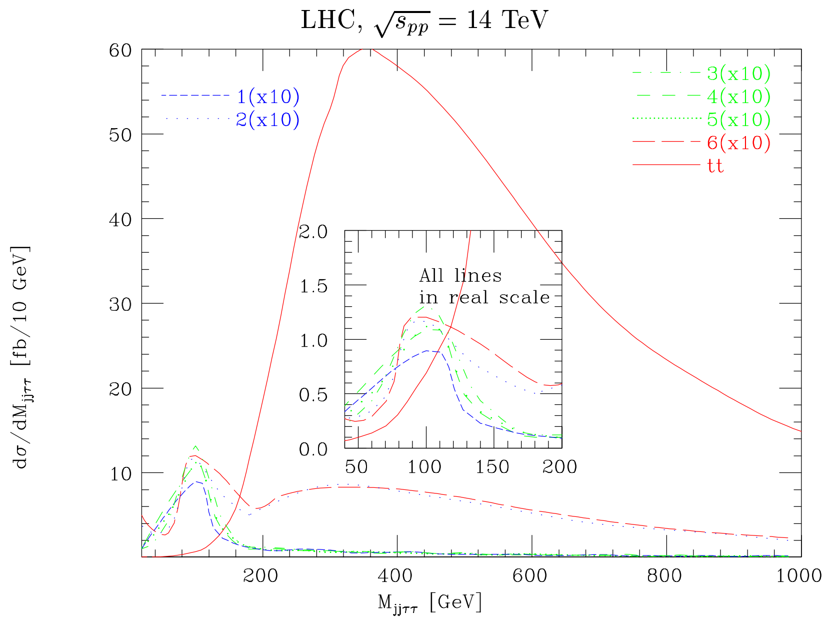

Figure 7. Reconstructed mass of the $jj\tau^+\tau^-$ system for signals and backgrounds after appropriate event selections at the LHC. We plot dσ/d$M_{jj\tau^+\tau^-}$ (fb/10 GeV) vs. $M_{jj\tau^+\tau^-}$ (GeV). Normalization is to the total cross-section after cuts.

opportunity for detecting at least one of the NMSSM Higgs bosons is to employ $WW \to h$ production and develop techniques for extracting a visible signal for the $h \to aa$ final state. We performed a detailed simulation of this signal and found that event selection criteria could be found such that detection of a Higgs signal should be possible in the $WW \to h \to aa \to jj\tau^+\tau^-$ production/decay mode after accumulating 300 fb^{-1} in the ATLAS and CMS detectors. The nominal statistical significance achieved is $>30\sigma$ with $S/B > 1$. However, the signal only emerges on the low end of a tail of a rapidly falling background. The signal is illustrated in figure 7 for six sample cases of the type described.

3.6 *The 'continuum' Higgs possibility*

A still more difficult case for Higgs discovery arises when there is a series of Higgs bosons separated by the mass resolution in the discovery channel(s) [37]. This situation could arise in string models where extra Higgs singlet fields are abundant. (Again, we emphasize that adding extra singlets to the two doublets of the MSSM leaves the success of gauge coupling unification intact.)

This scenario is challenging even for a LC. There, the Higgs signals would overlap if there is one Higgs boson at every $\sim$10 GeV (the detector resolution in the recoil mass spectrum for $e^+e^- \to Z$+Higgs). In general, all the overlapping neutral Higgs bosons could mix with the normal SM Higgs (or the MSSM scalar Higgs bosons) in such a way that the physical Higgs bosons share the WW/ZZ coupling and decay to a variety of channels. In such a case, the only model independent discovery procedure would be to use $e^+e^- \to Z + X$ and look for a broad excess in M_X.

Fortunately, there are some constraints on such a model. Using continuum notation, the important issue is the value of m_C in

$$\int_0^\infty \mathrm{d}m K(m) m^2 = m_C^2\,, \quad \text{where} \quad \int_0^\infty K(m) = 1, \tag{21}$$

where $K(m)(gm_W)^2$ is the (density in Higgs mass of the) strength of the hWW coupling-squared. Constraints include the following: (1) Consistency with precision electroweak data is most easily accommodated if $m_C^2 \lesssim (200\text{–}250\ \text{GeV})^2$. (2) For multiple Higgs representations of any kind in the most general SUSY context, RGE plus perturbativity up to $M_U \sim 2 \times 10^{16}$ GeV also gives $m_C^2 \lesssim (200\text{–}250\ \text{GeV})^2$. Of course one must be cautious in relying too much on such constraints. As we have discussed, many types of new physics at low scale allow evasion of the m_C^2 limits above, e.g. large extra dimensions or appropriate extra Higgs structure.

Ignoring this caveat, let us employ the sum rule with $m_C = 200$ GeV and take $K(m) = \text{constant}$ from $m_A = m_h^{\min}$ to $m_B = m_h^{\max}$, i.e., $K(m) = 1/(m_B - m_A)$. Current LEP constraints imply [38] that $K(m)$ should not be very large for $m < 80$ GeV. To search for the continuum spectrum in the range $m > 80$ GeV, a $\sqrt{s} = 500$ GeV LC is more or less ideal. For $K(m) = \text{constant}$, $m_C = 200$ GeV and $m_A = 70$ GeV we find $m_B = 300$ GeV and $m_B - m_A = 230$ GeV. A fraction $f = 100\ \text{GeV}/230\ \text{GeV} \sim 0.43$ of the continuum Higgs signal lies in the $M_X \in (100, 200)$ GeV region (which region avoids the $M_X = m_Z$ peak region with largest background). After summing $Z \to e^+e^- + \mu^+\mu^-$ events, we find $S \sim 540f$ with a background of $B = 1080$ in the $(100, 200)$ GeV window, assuming $L = 200\ \text{fb}^{-1}$. The result is $(S/\sqrt{B}) \sim 16f(L/200\ \text{fb}^{-1})$ for $m \in (100\text{–}200)$ GeV. This constitutes a very clear signal. With $L \sim 2000\ \text{fb}^{-1}$, it would be possible to determine the magnitude of the signal with reasonable error ($\sim$15%) in each 10 GeV interval.

Detection of this continuum Higgs boson signal would be extremely challenging at a hadron collider such as the LHC. No procedure has been suggested to date for extracting the signal. Even though each Higgs boson would be a very narrow resonance, the only channels with a sufficiently good resolution to resolve the distinct resonances would the $\gamma\gamma$ and 4ℓ final states. However, since each Higgs resonance only couples to ZZ and WW with a fraction of the usual SM strength, both the $\gamma\gamma$ signal (which relies largely on the W-loop contribution to the $\gamma\gamma$ Higgs coupling) and the 4ℓ signal from Higgs$\to ZZ \to 4\ell$ will be greatly suppressed compared to the expectation for the SM Higgs boson. This model thus constitutes a prime example of how a LC would be an absolutely essential complement to the LHC.

3.7 *Left–right symmetric supersymmetric models*

Motivations for high-scale left–right symmetry are very substantial (see [5] for discussion and references). First, the idea that a Higgs field breaks parity at some high scale m_{R} is very attractive. Second, $SO(10)$, which automatically includes ν_{R} fields for neutrino masses as well as the usual $SU(5)$ representation structures, contains the subgroup $SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{\rm B-L} \times SU(3)_{\rm C}$. Third, the see-saw mechanism for neutrino mass generation is easily implemented. Further, if the left–right symmetric model is placed in the supersymmetric context [39–42], the resulting SUSYLR symmetries guarantee that R-parity is conserved. A SUSYLR model also ensures the absence of the strong CP problem as well as the SUSY-CP problem (i.e. the generic problem of SUSY phases giving a large EDM unless cancellations are carefully arranged) at $m_{\rm R}$. It is then a matter of making sure that evolution from $m_{\rm R}$ down to the TeV scale does not destroy these two properties. This can be arranged fairly easily. The primary drawback of LR and SUSYLR models is that gauge unification is not possible without very specific choices of intermediate scale matter.

In the context of Higgs physics, the main reason for bringing up LR and SUSYLR models is that they inevitably contain a very large number of Higgs bosons, often including several 'bi-doublets' (i.e. Higgs representations that transform simultaneously as doublets under $SU(2)_{\rm L}$ and $SU(2)_{\rm R}$) as well as $SU(2)_{\rm L}$ and $SU(2)_{\rm R}$ triplets. However, it is easiest to construct specific models that are phenomenologically and theoretically consistent if there is an effective low-energy theory limit in which all the Higgs bosons other than those equivalent to the usual MSSM Higgs bosons are too heavy to detect at a TeV scale accelerator.

4. Conclusions

The time is approaching when we will have data that should reveal the existence and nature of the Higgs sector. We have developed detailed approaches for exploring and testing the single–doublet standard model Higgs sector as well as the constrained two-Higgs-doublet sector of the MSSM. While the SM Higgs boson will certainly be detected at the LHC, it may take both the LHC and the LC (including $\gamma\gamma$ collider) to fully verify its properties, including quantum numbers, with high precision. Assuming CP conservation in the Higgs sector, at least one of the Higgs bosons of the MSSM will also be observable at the LHC. However, in the decoupling regime only the light SM-like h will be accessible. In this regime, unless $\tan\beta$ is very large, detection of the heavier H, A and $H^\pm$ would require either a very high energy LC or a $\gamma\gamma$ collider facility at a somewhat lower energy LC. CP violation induced by radiative corrections to the Higgs sector might further complicate LHC observation of a Higgs boson, especially when parameters are such that the various neutral Higgs bosons mix strongly with one another. However, a LC would have no trouble detecting and studying the Higgs bosons in such a case.

We must also allow for the possibility that the Higgs sector in either of these contexts could be more complicated, prototypes being the general two-Higgs-doublet extension of the one-doublet SM Higgs sector and the NMSSM extension of the MSSM Higgs sector to include a single extra singlet. In such extensions, it is possible to find parameter choices for which the detection of even one Higgs boson at the LHC or the LC would be quite challenging. In most such cases, if detection is not possible at one machine, it would be possible at the other. For example, in the general 2HDM parameters can be chosen so that the precision electroweak constraints are satisfied even though there is no Higgs sufficiently light to be accessible to a $\sqrt{s} = 600$ GeV LC. But, in such cases there is always a heavy SM-like

Higgs boson that the LHC would easily detect. In the NMSSM case, the converse applies. The only NMSSM Higgs boson signal at the LHC might be quite weak and/or difficult to interpret with certainty, whereas LC detection of at least one CP-even NMSSM Higgs boson is guaranteed. Here, and in a variety of other models, complications due to unexpected decays (e.g. Higgs pair, Z+Higgs, SUSY), CP violation, overlapping signals etc. make attention to multi-channel analysis at the LHC vital. There is enough freedom in the Higgs sector that we should not take Higgs discovery at the Tevatron or LHC for granted, even in the case of the MSSM. To close the gaps, we must keep improving and working on every possible signature.

If no Higgs boson is detected at the LHC, the LHC's ability to determine whether or not the WW sector is perturbative [43] could be quite crucial. If it is perturbative, this will be a strong hint that we missed the signal(s) for a complicated set of light Higgs bosons that share the WW coupling strength squared. In such a case, we will know that the LC is needed to discover and study the Higgs boson(s). More generally, it is important to re-emphasize the complementarity between the LHC and the LC. In particular, as already noted, only the LHC (or a LC with $\sqrt{s} \sim 1$ TeV) would detect a Higgs boson in some versions of a general 2HDM, while the LC would be the only way to probe a continuum of strongly mixed Higgs bosons.

It is also important to emphasize the value of the $\gamma\gamma$ collider option at the LC for detecting the heavy H, A in the 'wedge' region and its unique ability for determining the CP nature of any neutral Higgs boson that is observed. Direct CP determinations could be quite crucial for disentangling any but the simplest SM Higgs sector.

Although not discussed here for lack of space, once observed, the properties and rates for H, A will help enormously in determining important SUSY parameters, especially $\tan\beta$ (see [44]). Exotic Higgs representations, such as an $SU(2)_{\rm L}$ triplet as motivated by the see-saw approach to neutrino masses, will lead to exotic collider signals and possibilities that we have also not attempted to review here.

It is somewhat of a shock to realize that, even after 40 years since the introduction of the elementary Higgs boson possibility, we are still waiting experimental confirmation and still proposing new models that require further development of techniques for ensuring the discovery of Higgs bosons should they exist. That we must persist is clear. The Higgs sector will undoubtedly hold absolutely critical keys to understand the underlying fundamental theory of the Universe in which we live.

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