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Phase space description of the production of quark gluon plasma in heavy-ion collisions

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Abstract. A new source term is proposed to describe the production of quark gloun plasma in heavy-ion collisions.

Keywords. Quark gluon plasma; relativistic heavy-ion collision; source term.

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1. Introduction

The primary interest in theoretically studying ultra relativistic heavy-ion collision (URHIC) has been to understand the dynamics of quark gluon plasma (QGP), which is a deconfined state of hadronic matter. This requires an understanding of the production mechanism, in a manner in which (i) the non-Markovian nature of the production [1–3], (ii) the dynamical nature of the vacuum [1], and (iii) the quasi-particle nature of the excitations [4] are inherent. The mechanism should be further based on non-perturbative aspects of QCD, as the recent lattice studies reveal [5]. To the extent that we are aware, none of the mechanisms, based on the Schwinger mechanism [6], or the perturbative QCD [7], incorporates all these features in a single framework. A robust framework exists for studying the evolution of QGP, owing to the pioneering work of Bjorken [8] which was itself based on an earlier prescient work of the Landau school [9]. Here, one employs a semi-classical formulation [10–12] of the problem where the evolution of QGP is studied via an appropriate transport equation. The transport equation for a parton distribution function f, which typically has the form

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial r_i} + F_i \frac{\partial f}{\partial p_i} + \dot{Q}_a \frac{\partial f}{\partial Q_a} = \Sigma + \mathscr{C} \tag{1}$$

is defined in an extended phase space involving the color part, in addition to the standard position and momentum variables. The dynamical nature of the charge is reflected in the last term in the lhs, while F_i obeys the analog of Lorentz equation $F_i = Q_a\left(E_i^a + (\vec{\nabla} \times \vec{B^a})_i\right)$, with $\vec{E_a}$ and $\vec{B_a}$ obeying Yang–Mill's equations. $\mathscr{C}$ is the collision term. The most important term for us is the source term Σ, which we shall try to model below. Ideally speaking $\vec{E_a}$, $\vec{B_a}$, f, $\mathscr{C}$ and Σ need to be determined in a self-consistent manner [10,11], since each of them is a functional of the other quantities. Pending a detailed analysis, we merely indicate how to fix the source term once the space-time history of the fields is known.

2. The source term

We model the non-perturbative aspects using the color flux tube model [13], which provides a natural setting for discussing quark confinement [14], in terms of color strings – which are chromoelectric flux tubes terminating on two partons. After the two nuclei collide and start receding from each other, color strings are formed between them. These strings merge to form a color rope. Consequently the production process reduces to the instability of the QCD vacuum in the presence of a classical chromoelectric field (CEF) which is, in general, space-time dependent. Consider the gluon production first; this case has no counterpart in QED. Expand the gauge potential as a sum of classical values and their fluctuations: $A^a_\mu = C^a_\mu + \phi^a_\mu$, where $C^a_\mu = \langle A^a_\mu \rangle$. Expanding the Y–M Lagrangian, we find terms responsible for gluon production to be

$$L_{2g} = -\frac{g}{2} f^{abc} \big[(\partial_\mu C^a_\nu - \partial_\nu C^a_\mu) \phi^{\mu b} \phi^{\nu c} + (\partial_\mu \phi^a_\nu - \partial_\nu \phi^a_\mu)(C^{\mu b} \phi^{\nu c} + \phi^{\mu b} C^{\nu c}) \big] + \mathscr{O}(g^2), \tag{2}$$

where we have kept the terms quadratic in the fluctuations. Keeping in mind the model and invoking (local) gauge covariance of CEF, we take the form of the gauge potential to have an 'abelian' structure: $C^a_\mu = \delta_{\mu,0} \sum_i C_i(t,\vec{r}) \delta_{a,i}$ where the summation is restricted to the diagonal generators of the gauge group. For all such configurations, the magnetic field vanishes. We now evolve the state in the Fock space as a function of time, and project it onto the two gluon state $|gg\rangle \equiv |\vec{p}_1, \vec{p}_2; s_1, s_2; c_1, c_2\rangle$ labelled by momentum, spin and color quantum numbers respectively; the on-the-mass shell condition is imposed as a subsidiary condition. In the leading order which we consider for simplicity, the amplitude is given by

$$\langle gg|T(t)|0\rangle = \frac{ig}{(2\pi)^3} \frac{(E_2 - E_1)}{2\sqrt{E_1 E_2}} \vec{\varepsilon}^{s_1}(\vec{p}_1) \cdot \vec{\varepsilon}^{s_2}(\vec{p}_2) f^{ac_1c_2} \cdot \tilde{C}^{0,a}(E_1 + E_2; \vec{p}_1 + \vec{p}_2; t), \tag{3}$$

where $T(t) \equiv U(t,0) - 1$. Further,

$$\tilde{C}^{0,a} = \int_0^t \mathrm{d}t_1 \mathrm{e}^{-i(E_1+E_2)t_1} \int \mathrm{d}^3\vec{r} \exp(i(\vec{p}_1 + \vec{p}_2) \cdot \vec{r}) C^{0,a}(t_1, \vec{r})$$

is the incomplete Fourier transform of the gauge field and E_i are the energies carried by the gluons. The corresponding expression for the $q\overline{q}$ production is given by

$$\langle q\overline{q}|T(t)|0\rangle = \frac{g}{(2\pi)^3} \frac{m}{\sqrt{E_1 E_2}} \tilde{C}^{0,a} T^a_{c_1,c_2} u^\dagger_{s_1}(p_1) v_{-s_2}(-\vec{p}_2), \tag{4}$$

with $|q\overline{q}\rangle \equiv |\vec{p}_1, \vec{p}_2; s_1, s_2; c_1, c_2\rangle$. T^a are the generators of the gauge group in the fundamental representation, while u, v are the usual Dirac spinors. The probability that a pair is produced *any time* during the interval $(0,t)$ is given by $|\langle \phi|T(t)|0\rangle|^2$, ϕ standing for either gg or $q\overline{q}$. The production rate at any time t is thus given by its derivative at that instant; clearly, the rate takes both positive and negative values, from which one concludes that the particles acquire a (time-dependent) finite life-time. Equivalently, vacuum has a dynamical role – both as a source and a sink. Further, the rate at any time depends not merely on the field configuration at that instant – it has a highly non-Markovian character, being determined by the entire history. In short, quantum interference effects dominate the production mechanism, and this cannot be accessed by the approaches based on Schwinger mechanism and pQCD.

3. Results

We will now illustrate the results both in momentum space as well as in configuration space with the help of a model example drawn from its behavior expected in real situations [8]. Let CEF be given by $\mathscr{E}_i^a = \delta_{i,z}\mathscr{E}_0(\delta_{a,3}+\delta_{a,8})\exp((|z|-t)/t_0)\theta(t)\theta(t-|z|)$ with the initial field strength given by $\mathscr{E}_0$ and $\theta(x)$ being the Heaviside function. The field is characterised by the single time-scale t_0, in units of which all the observables will be expressed. Indeed, t_0 gives the decay time of the field or equivalently, the overall 'production time' of the partons, while g is related to the string tension. The initial energy density $\mathscr{U}$, t_0 and g are not entirely independent of each other: $g\mathscr{U}t_0^4 \sim 1$. We display the two particle rates in momentum space and production probabilities (PP) in configuration space for gluons as contour diagrams. Consider the rates in momentum space first. We fix for the gluon pair: $\sqrt{s} = 0.6,\ p_{\mathrm{T}} = 0.1$. This configuration admits two sets of solutions for $p_{\mathrm{L}_{1,2}}$ – the longitudinal momenta of the two gluons; in one case they have the same sign, and in the other, opposite. We denote the two channels by $\uparrow$ and $\downarrow$ respectively. Figures 1 and 2 exhibit the main features, where x and y-axis stand for longitudinal momentum $p_{1\mathrm{L}}$ of one of the particles and time t respectively. The rate is a highly fluctuating function of all the variables involved, changing signs rapidly, both with the momenta as well as time. It oscillates over a much larger range for smaller values of p_{T} and $\sqrt{s}$. For example, it oscillates roughly between $(-500, 500)$ at $\sqrt{s} = 0.6,\ p_{\mathrm{T}} = 0.1$ (figures 1, 2). In contrast, the range is restricted to $\sim(-10, 10)$ at $\sqrt{s} = 1.0,\ p_{\mathrm{T}} = 0.4$ (not shown here). Notice also

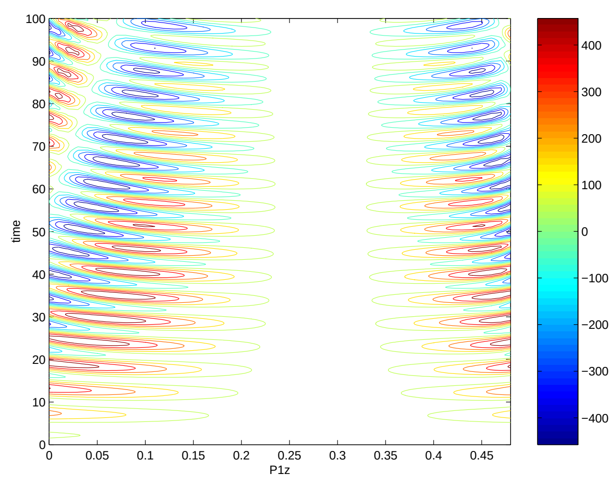

Figure 1. Momentum space rates for gluon production in the $\uparrow$ channel for $\sqrt{s} = 0.6,\ p_{\mathrm{T}} = 0.1$.

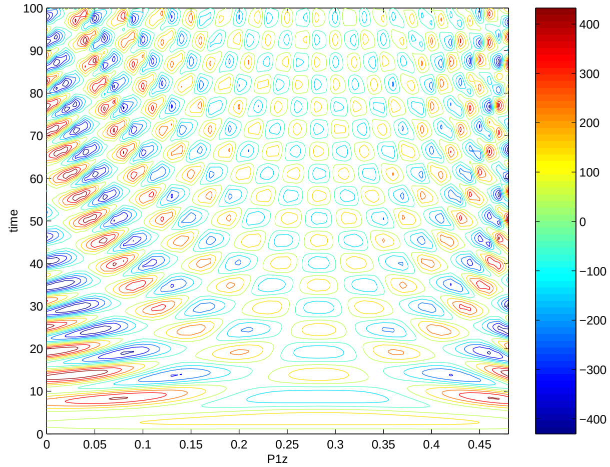

Figure 2. Momentum space rates for gluon production in the ↓ channel for $\sqrt{s} = 0.6,\ p_{\mathrm{T}} = 0.1$.

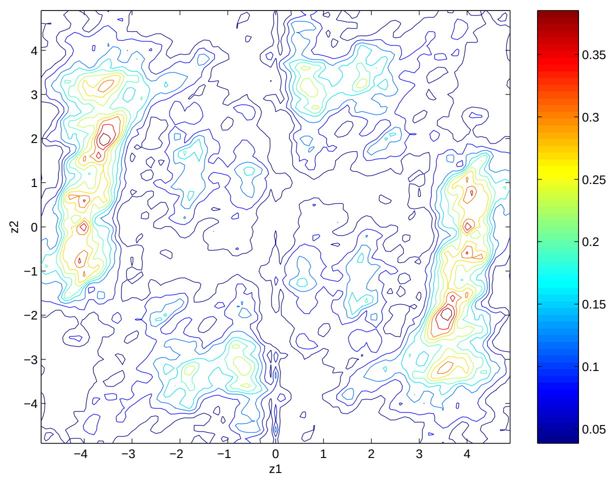

Figure 3. Configuration space production probability for gluons at $t = 2.0,\ \rho = 6.0$ in $z_1 - z_2$ section.

that for a given $\sqrt{s}$ and p_{T}, the fluctuations are more dominant for very small and very large values of longitudinal momenta. There is also an increase in the number of oscillations as we increase $\sqrt{s}$. It may also be seen from the figures that the ↑ channel is relatively placid in comparison with the ↓ channel. Most importantly, these figures exhibit in a vivid manner the production and the absorption of the pairs, and the correlation between them, in the momentum space. The correlations are of vital importance in getting the screening lengths as well as in studying the hadronization. Coming to the quark production, it is sufficient to mention that they are equally rich in features differing only in the topology of the distributions. We do not show them here.

We have done some preliminary computations of configuration space distribution of gluons in QGP by performing a Fourier transform over the momentum space amplitude. The results are strikingly non-trivial; there are clear indications of finding gluons in the classically forbidden region (i.e. where field is absent). At any given time CEF is non-zero only in the range $[-t, t]$ along the longitudinal axis. Under adiabatic assumptions it is expected that production and subsequent evolution of quarks and gluons will be confined between the plates (nuclei). Contrary to this our results show that prodution probability is non-zero outside the plates and quite often it is more than a decaying tail indicating that the size of the source is not limited by the nuclear volume.

The configuration space probabilities are functions of longitudinal position of two gluons z_1 and z_2 and their relative transverse separation ρ. We have shown the probabilities in the contour diagrams below on the $z_1 - z_2$ section keeping time t and transverse seperation ρ fixed. We describe the general features here. There is an overall increase in production probability with increase in time but it decreases with the increase in transverse separation. The activity is widespread in the $z_1 - z_2$ plane over the range of the order ρ. Therefore for $\rho > t$ predominant activity is found for $|z_1|, |z_2| > t$. This is clearly demostrated by figure 3.

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