PRAMANA
— journal of
physics

Vol. 61, No. 5
November 2003
pp. 1021–1025

Collective flow in relativistic heavy-ion collisions

R S BHALERAO
Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India

Abstract. A brief introduction is given to the field of collective flow, currently being investigated experimentally at the Relativistic Heavy-Ion Collider, Brookhaven National Laboratory. It is followed by an outline of the work that I have been doing in this field, in collaboration with Nicolas Borghini and Jean-Yves Ollitrault.

Keywords. Quark–gluon plasma; relativistic heavy-ion collisions; collective flow.

PACS Nos 12.38.Mh; 25.75.-q; 25.75.Ld

1. Introduction

It is important to investigate whether the quark–gluon matter formed in relativistic heavy-ion collisions, attains thermal equilibrium before it hadronizes. One can claim the formation of a new state of matter, namely quark–gluon plasma, only if the thermalization is demonstrated unambiguously. Observation of a strong collective flow of outgoing particles is an unmistakable signature of thermalization. Let us see, why this is so. (Readers interested in detailed reviews of this field, may see refs [1–4].)

1.1 *Directed and elliptic flows*

In this talk I shall focus mostly on the non-central collisions. These are the collisions which have non-zero impact parameter. The reaction plane is defined as the plane determined by the impact parameter vector and the collision axis. (Obviously, in central or head-on collisions no reaction plane can be defined.) Thus if the two nuclei are approaching each other parallel to the z-axis and the impact parameter vector is parallel to the x-axis, then the xz plane is the reaction plane. The xy plane is the azimuthal plane. A relativistic collision of two nuclei results in a state with hundreds of particles leaving the reaction zone. Let $f(\phi)$ be the distribution of the particles in the azimuthal plane. Here ϕ is the azimuthal angle between the trajectory of an outgoing particle and the reaction plane. $f(\phi)$ is an even and periodic function of ϕ. Hence the Fourier expansion of $f(\phi)$ reads

$$f(\phi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\phi,$$

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\phi)\cos n\phi \; \mathrm{d}\phi.$$

Obviously, $a_n = a_0\langle\cos n\phi\rangle$ where the expectation value of $\cos n\phi$ is taken with respect to the distribution $f(\phi)$. The flow v_n is defined as

$$v_n \equiv \langle\cos n\phi\rangle, \quad n = 1,2,3,\ldots .$$

Thus v_n is essentially the nth harmonic coefficient of $f(\phi)$. The first two harmonics $v_1 \equiv \langle\cos\phi\rangle$ and $v_2 \equiv \langle\cos 2\phi\rangle$ are the *directed* and *elliptic* flows, respectively. It is useful to note that v_n can also be written as

$$v_n = \langle \mathrm{e}^{in\phi}\rangle, \quad n = 1,2,3,\ldots .$$

In central collisions, there is an azimuthal isotropy, i.e., $f(\phi)$ is constant. Hence all Fourier harmonics $a_n(n \neq 0)$ vanish, and $v_n(n = 1,2,\ldots) = 0$. Thus there is no flow as defined above. However, in central collisions, one can define what is called a *radial* flow. In the rest of this talk, I shall consider only non-central collisions.

1.2 *Importance of flow measurements*

In a non-central collision the overlap region of two nuclei is lens-shaped. Thus the initial state is characterized by a *spatial anisotropy* in the azimuthal plane. If a non-vanishing flow is observed experimentally, then it follows that the azimuthal momentum distribution $f(\phi)$ is anisotropic or non-flat. Thus the final state is characterized by a *momentum anisotropy* in the azimuthal plane. The initial spatial anisotropy gives rise to the final momentum anisotropy, on account of multiple interparticle collisions. If either of the two ingredients, namely initial spatial anisotropy and rescatterings, is missing, there is no flow. Thus in non-central collisions, the flow v_n provides a measure of rescatterings. In other words, the flow is sensitive to the number of interactions and parton–parton scattering cross-sections. But these are precisely the issues which have a bearing on the degree of thermalization.

For the reasons stated above, the study of the collective flow has emerged as an important area of research in the context of the wealth of data produced at SPS, CERN and the new and upcoming data from RHIC, BNL. Flow is a signature of pressure at early times. Flow provides information on the equation of state of the matter produced in collisions. Study of the flow also provides important constraints on theoretical models.

How is the flow measured? Recall $v_n = \langle\exp(in\phi)\rangle$, where the azimuthal angle ϕ is measured with respect to the reaction plane. If ϕ is measured with respect to a fixed direction in the laboratory, then $v_n = \langle\exp(in(\phi - \phi_{\mathrm{R}}))\rangle$, where ϕ_{R} is the azimuthal angle of the reaction plane. Note that the average is done first over all particles in one (collision) event and then over a large number of events with nearly equal multiplicities M. Different events in a sample, in general correspond to different ϕ_{R}s which are neither known nor easy to determine. Hence the flow is usually measured with the help of correlations among particles.

1.3 *Correlations*

What is the origin of interparticle correlations in the azimuthal plane? Correlations arise due to

- Particle or resonance decay, e.g., $\rho \to 2\pi$, $\omega \to 3\pi$, $\pi^0 \to 2\gamma$,
- Momentum conservation,
- Final-state interactions (Coulomb, strong),
- Quantum correlations,
- (Mini)jets, etc.

But these are the *direct* or *non-flow* correlations which have nothing to do with the orientation of the reaction plane. Hence we are *not* interested in these correlations.

Correlations among particles also arise indirectly, because the trajectory of each outgoing particle is correlated with the orientation of the reaction plane. These are the *indirect* or *flow* correlations which we are looking for.

How do correlations help in the measurement of the flow v_n? Consider two-particle correlations:

$$\begin{aligned}\langle \mathrm{e}^{in(\phi_1-\phi_2)}\rangle &= \langle \mathrm{e}^{in(\phi_1-\phi_\mathrm{R})}\mathrm{e}^{in(\phi_\mathrm{R}-\phi_2)}\rangle \\ &= \langle \mathrm{e}^{in(\phi_1-\phi_\mathrm{R})}\rangle\langle \mathrm{e}^{in(\phi_\mathrm{R}-\phi_2)}\rangle \\ &= v_n^2, \end{aligned} \tag{1}$$

where the second equality is obtained assuming that the direct or non-flow correlations are either absent or somehow eliminated from the data sample. Similarly, the flow can be measured with the help of 4-particle correlations too:

$$v_n^4 = \langle \mathrm{e}^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle. \tag{2}$$

Note that in eqs (1) and (2), the knowledge of ϕ_R is unnecessary.

Standard flow analyses rely on two-particle correlations. Flow measurements based on cumulants of multi-particle correlations were proposed by Borghini *et al* [5].

1.4 *Cumulants*

Wherever statistical analyses are done, whether in physics, biology or psychology, correlation functions play an important role; see, e.g., [6]. In an obvious notation these may be denoted by $\rho_1(x_1)$, $\rho_2(x_1,x_2)$, $\rho_3(x_1,x_2,x_3)$, etc. A knowledge of correlation functions to all orders provides complete information of the statistical system. However, the above correlation functions contain uncorrelated parts which have to be subtracted out to get to the 'true' correlations:

$$\begin{aligned}C_2(x_1,x_2) &= \rho_2(x_1,x_2) - \rho_1(x_1)\rho_1(x_2)\\ C_3(x_1,x_2,x_3) &= \rho_3(x_1,x_2,x_3) - \sum_{(3)}\rho_1(x_1)\rho_2(x_2,x_3) + 2\rho_1(x_1)\rho_1(x_2)\rho_1(x_3),\end{aligned}$$

where (3) indicates that there are three terms of this type. These reduced quantities are called cumulants. They are constructed precisely in such a way as to vanish whenever any one or more of the points x_i becomes statistically independent of the others.

The 2-particle cumulant in our problem is defined as [5]

$$\langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle \equiv \langle e^{in(\phi_1-\phi_2)}\rangle - \langle e^{in\phi_1}\rangle\langle e^{-in\phi_2}\rangle.$$

Higher-order cumulants are defined similarly.

1.5 *Generating functions*

Generating functions produce multiparticle correlations and cumulants in an elegant way.

A typical generating function for correlations reads as [5]

$$G_n(z) \equiv \prod_{j=1}^{M}\left[1+\frac{i}{M}\left(z^* e^{in\phi_j}+z e^{-in\phi_j}\right)\right],$$

where $z = x + iy$ and M is the multiplicity of the event. Averaging $G_n(z)$ over events and expanding $\langle G_n(z)\rangle$ in powers of z and z^* generates azimuthal correlations of arbitrary orders. This is the reason why $G_n(z)$ is called a 'generating function'.

The generating function for cumulants is given by

$$\mathscr{C}_n(z) \equiv \ln\langle G_n(z)\rangle. \tag{3}$$

Expanding $\mathscr{C}_n(z)$ in powers of z and z^* generates cumulants $c_{k,l}$ of arbitrary orders:

$$\mathscr{C}_n(z) = \sum_{k,l}\frac{z^{*k}z^l}{k!\,l!}c_{k,l}, \tag{4}$$

where

$$c_{k,l} = \langle\langle e^{in(\phi_1+\cdots+\phi_k-\phi_{k+1}-\cdots-\phi_{k+l})}\rangle\rangle.$$

2. Flow from large-order correlations

As we have seen above, the flow v_n can be extracted from 2-, 4-, 6-,... particle correlations. It is advantageous to consider higher-order correlations because the systematic error due to non-flow correlations becomes smaller as the order increases [5]. However, numerical and analytical efforts increase very rapidly as the order increases. Hence, past studies have focussed mainly on 2- and 4-particle correlations. Nevertheless, the genuine collective behaviour induces correlations of arbitrarily large order. Hence it is important to study correlations of a large, in fact asymptotically large number of particles with respect to the reaction plane, in order to learn about the collective motion of the fireball.

With this as our motivation, we have been working on a new method [7] to extract the genuine collective flow involving an asymptotically large number of particles. Consider the series for $\mathscr{C}_n(z)$, eq. (4). We realized that the asymptotic behaviour of the cumulants is determined by the radius of convergence of this series, i.e., by the singularities of $\mathscr{C}_n(z)$ in the complex z plane. According to eq. (3), these singularities are either the singularities of $\langle G_n(z)\rangle$ or its zeros. Since $\langle G_n(z)\rangle$ is a polynomial, it has no singularities, but it has zeros. The asymptotic behaviour of the cumulants is given by the zero which is closest to the origin. The flow v_n can be determined, once the zero of $\langle G_n(z)\rangle$ is identified. Thus ours is a direct method; it does not require calculation of the cumulants. It is formally analogous to the Yang–Lee theory of phase transitions [8], which is based on the zeros of the grand partition function.

Although the above idea is simple, there are several technical issues (such as systematic and statistical errors arising due to a variety of causes) which need to be probed. Ultimately we would like to propose a set of recipes which can be used by experimentalists, in a straightforward manner, to obtain integrated and differential, directed and elliptic flows from experimental data. This work is in progress.

References

[1] W Reisdorf and H G Ritter, *Ann. Rev. Nucl. Part. Sci.* **47**, 663 (1997)
[2] N Herrmann, J P Wessels and T Wienold, *Ann. Rev. Nucl. Part. Sci.* **49**, 581 (1999)
[3] J-Y Ollitrault, *Nucl. Phys.* **A638**, 195c (1998); e-print nucl-ex/9802005
[4] S A Voloshin, talk given at the *16th International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Quark Matter 2002 (QM 2002),* Nantes, France, 18–24 July 2002; e-print nucl-ex/0210014
[5] N Borghini, P M Dinh and J-Y Ollitrault, *Phys. Rev.* **C64**, 054901 (2001)
[6] H C Eggers, P Lipa, P Carruthers and B Buschbeck, *Phys. Rev.* **D48**, 2040 (1993)
[7] R S Bhalerao, N Borghini and J-Y Ollitrault, *Nucl. Phys.* **A727**, 373 (2003); e-print nucl-th/0310016; nucl-th/0307018
[8] C N Yang and T D Lee, *Phys. Rev.* **C87**, 404 (1952)