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High-spin structure of neutron-rich Dy isotopes

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Abstract. In view of recent experimental progress on production and spectroscopy of neutron-rich isotopes of Dy with mass number $A = 166$ and 168, we have made theoretical investigations on the structure of high spin states of $^{164-170}$Dy isotopes in the cranked Hartree–Fock–Bogoliubov (CHFB) theory employing a pairing+quadrupole+hexadecapole model interaction. With the increase of neutron number the rotation alignment of the proton orbitals dominates the structure at high spins, which is clearly reflected in the spin dependence of the rotational g-factors. A particularly striking feature is the difference in the spin-dependent properties of ^{166}Dy as compared to that of ^{164}Dy.

Keywords. Neutron-rich Dy isotopes; high-spin states; g-factors; cranked HFB theory.

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1. Introduction

The ground state structure of most of the stable nuclei is more or less understood, and now there are experimental as well as theoretical attempts to understand the structure of neutron-deficient and neutron-rich nuclei that are far from the line of β-stability. In p-shell region the neutron-to-proton number ratio (N/Z) of exotic nuclei has reached the order of 2 to 3. On the other hand, in the rare-earth region the nuclei with $N/Z \geq 1.5$ are treated as neutron-rich.

Recently there are successful experimental attempts on production and study of spectroscopic properties of neutron-rich dysprosium isotopes ^{166}Dy [1] and ^{168}Dy [2]. While in ^{166}Dy the yrast levels are known up to spin $J = 16$, in ^{168}Dy these are known only up to $J = 4$. In ref. [1] the moment of inertia vs. rotational frequency plot for even–even $^{160-166}$Dy isotopes shows that ^{160}Dy exhibits a sharp up-bend at $J \sim 12$–14. An important feature to be noticed in this figure is the flattening of the curve for ^{166}Dy compared to that for ^{164}Dy marking a clear separation in the behaviour as a function of neutron number. Several years back [3] we studied the variation of g-factors of ^{158,164}Dy and ^{166}Er using pairing + quadrupole model interaction of Baranger and Kumar [4]. A slow decrease of g_J with J (up to $J = 10$) for ^{166}Er was found, later on, to be in good agreement with the

experimental data [5,6]. However, more recent measurements of g-factors of ^{164}Dy [7] indicate only a very small decreasing trend up to $J = 10$.

In ref. [2] the excitation energy ratio $R_4 = E_4/E_2$ for Dy isotopes shows that up to ^{168}Dy it continues to increase rather slowly and smoothly, though E_2 as a function of increasing N shows a small dip(decrease) at N = 98 implying an increase of moment of inertia or deformation parameter β. In the rare-earth region, Z = 66 is the mid-shell point between the magic numbers 50 and 82. Similarly N = 104 is at the mid-point between 82 and 126. Thus, one expects that ^{170}Dy should be the most deformed isotope amongst Dy nuclei. However, we would like to add here the observations made in a recent paper [8], where experimental R_4 vs. N plot is made for several elements (we have also made such a plot which is not being displayed here). The factor R_4 has a maximum (up to the point data are available) for Os, W and Hf at $N = 108$, and for Yb this is at $N = 104$. For Er isotopes R_4 is maximum at $N = 104$, but with almost the same value at $N = 100$ and 102. Thus, maximum in R_4 seems to be shifting to lower N values while going from heavier to lighter elements.

Recently, Sun and Egido [9] have carried out an angular momentum projected shell model type of calculation to study the yrast states of $A = {}^{154-164}$Dy isotopes using a pairing (monopole and quadrupole) plus quadrupole model interaction hamiltonian. The input many-body intrinsic wave functions are the zero quasiparticle BCS and two- and four-quasiparticles excited BCS states. However, though the rotational symmetry of the hamiltonian is restored, these wave functions are not very appropriate in the backbending region of spins, as in this part, rotation alignment is important which induces triaxiality and mixing of K quantum numbers. The results of ref. [9] are only qualitatively good as compared to the experimental data. On the other hand, the CHFB theory contains rotation alignment and possibility of multi-quasiparticle excitations in an inherent manner, and is a variation after projection calculation, but for the fact that angular momentum is conserved only on the average. It may be added that around the same time as the above paper, in 1994, a CHFB calculation [10] was carried out for ^{154}Dy for spins up to $J = 40$ and the results were found to be quite reasonable revealing all the features of the experimental data on backbending and g-factors.

In view of the latest experimental results and our earlier CHFB calculations (with principal x-axis as the cranking axis) in this mass region we have planned to make a systematic investigation of the ground- as well as high-spin-structure of $^{164-170}$Dy isotopes following the same approach with pairing + quadrupole + hexadecapole model interaction hamiltonian. Using the experimental data on the yrast states of ^{164}Dy we can fix, as best as possible, the interaction strengths of the model hamiltonian, and then without any further adjustment of these parameters want to compute the yrast line properties of $A = {}^{166-170}$Dy isotopes.

2. Formalism and calculational details

We employ a quadrupole + hexadecapole + pairing model interaction hamiltonian,

$$H = H_0 - \frac{1}{2}\sum_{\lambda=2,4}\chi_\lambda \sum_\mu \hat{Q}_{\lambda\mu}(-1)^\mu \hat{Q}_{\lambda-\mu} - \frac{1}{4}\sum_{\tau=p,n} G_\tau \hat{P}^\dagger_\tau \hat{P}_\tau \,, \tag{1}$$

where H_0 stands for the one-body spherical part, χ_λ represents the quadrupole and hexadecapole parts with $\lambda = 2,4$ and G_τ represents the proton and neutron monopole pairing interaction. Explicitly, we have

$$\hat{Q}_{\lambda\mu} = \left(\frac{r^2}{b^2}\right) Y_{\lambda\mu}(\theta,\phi), \tag{2}$$

$$\hat{P}^\dagger_\tau = \sum_{\alpha_\tau,\bar{\alpha_\tau}} c^\dagger_{\alpha_\tau} c^\dagger_{\bar{\alpha_\tau}}. \tag{3}$$

In the above, $c^\dagger$ are the creation operators with $\alpha \equiv (n_\alpha l_\alpha j_\alpha m_\alpha)$ as the spherical basis states quantum numbers with $\bar{\alpha}$ denoting the conjugate time-reversed orbital. The standard mean field CHFB equations [11] are solved self-consistently for the quadrupole, hexadecapole and pairing gap parameters. The deformation parameters and pairing gaps are defined in terms of the following expectation values:

$$D_{2\mu} = \chi_2 \langle \hat{Q}_{2\mu} \rangle,\ D_{4\mu} = \chi_4 \langle \hat{Q}_{4\mu} \rangle \tag{4}$$

$$\hbar\omega\beta\cos\gamma = D_{20},\ \hbar\omega\beta\sin\gamma = \sqrt{2}D_{22},\ \hbar\omega\beta_{40} = D_{40}, \tag{5}$$

$$\Delta_\tau = \frac{1}{2} G_\tau \left\langle \hat{P}_\tau \right\rangle. \tag{6}$$

The oscillator frequency $\hbar\omega = 41.0\ \mathrm{A}^{-1/3}$ (MeV), and β, γ and β_{40} are the usual deformation parameters, while Δ_p and Δ_n are the pairing gap parameters for protons and neutrons, respectively. The basis space consists of $N = 4,5$ harmonic oscillator major shells $+\ 0i_{13/2}$ orbitals for protons, and $N = 5,6$ major shells $+\ 0j_{15/2}$ orbitals for neutrons with the assumption of an inert core of protons $Z = 40$ and neutrons $N = 70$. For multipole separable forces with r^2 radial dependence, one should not consider many shells for the basis space [4]. The spherical single particle energies are taken as the spherical Nilsson model single particle energies with A-dependent Nilsson parameters [12]. The upper shell radial matrix elements are reduced by the factors $(N_0 + 3/2)/(N + 3/2)$, as discussed in ref. [4], where N_0 takes the value 4 for protons and 5 for neutrons. Finally the interaction strengths are chosen such that reasonable values of the ground state shape parameters, the first 2^+ excitation energy (~ 100 keV), and the spin-dependent (up to J = 10) g-factors of ^{164}Dy are obtained. We have taken (all in MeV) the following values of the interaction strengths

$$\chi_2 = 60/A^{1.4},\ \chi_4 = 55/A^{1.4},\ G_p = 25.3/A,\ G_n = 21.5/A. \tag{7}$$

3. Results and discussions

As mentioned in the introduction, we have earlier [3] calculated g-factors of ^{164}Dy and ^{166}Er, and the initial decrease of g_J with the increase of J for ^{166}Er was found to be in good agreement with the experimental measurements made later on [5,6]. Recently new measurements [7] on $^{160-164}$Dy are reported which are fairly in agreement with the earlier data. However, for ^{164}Dy even the latest data do show some small, but gradual, decreasing trend from $J = 2$ to 10 which is very consistent with the variation of its moment of inertia

Table 1. Intrinsic shape parameters of ^{166}Dy at J values.

$J\,(\hbar)$	β	γ (deg)	β_{40}	Δ_p (MeV)	Δ_n (MeV)
0	0.353	0.0	0.0160	0.811	0.801
10	0.354	0.90	0.0146	0.575	0.639
20	0.346	1.58	0.0094	0.000	0.346
26	0.338	1.85	0.0042	0.0	0.096

Table 2. Similar to table 1 for ^{170}Dy.

$J\,(\hbar)$	β	γ (deg)	β_{40}	Δ_p (MeV)	Δ_n (MeV)
0	0.344	0.0	−0.0109	0.756	0.814
10	0.346	0.73	−0.0125	0.455	0.657
20	0.338	2.25	−0.0187	0.000	0.340
26	0.330	3.53	−0.0236	0.0	0.000

[1] as a function of J. In the present calculation, we find $g_{10}/g_2 = 0.97$, a decrease of only about 3% which is in very good agreement with the recent data [7] and other theoretical results [9,13]. We discuss more on this a little later.

It is well-known that the CHFB method with only monopole pairing overestimates the effect of rotation alignment. But as far as trend and mechanism are concerned it is very reliable. Here the calculations have been done for $^{164-170}$Dy. But the behaviour of ^{168}Dy being more or less similar to ^{166}Dy, we will not present much of the results for $A = 168$, but rather that of ^{166}Dy to highlight its pronounced differences as compared to ^{164}Dy. In fact, experimentally the best chances are that ^{166}Dy will be the first isotope to be taken up for g factor studies on the heavier mass side. In table 1, we list the values of intrinsic shape parameters for ^{166}Dy at certain values of the spin, $J = 0$, 10, 20, and 26 to indicate the trend. Similarly table 2 lists the values of the shape parameters of ^{170}Dy. In order to see as to which even–even isotope of Dy is the most deformed one in the ground state, we find that $\beta = 0.3528$, 0.3529, 0.3524, 0.3444 and 0.3340; the excitation energy $E_2 = 97$, 98, 91, 98 and 110 keV; and the ratio $R_4 = E_4/E_2 = 3.206$, 3.245, 3.253, 3.235 and 3.191 for $A = 164$–172, respectively. The value of β comes out almost the same for $A = 164$–168, but shows a clear-cut relative decrease for $A = 170$. The relativistic mean field calculation of Lalazissis, Sharma and Ring [14] also shows a saturation of the β values for $^{164-168}$Dy with a good agreement even in the magnitudes. The trend of R_4 is rather consistent with the experimental data up to $A = 168$ [2]. In view of the well-known weaknesses of the CHFB theory, this is a very satisfying result. From these observations we may infer that the $N = 102$ isotope is the most deformed one, rather than that for the mid-shell number $N = 104$. This is also born out from a recent calculation of Regan *et al* [15], where $\beta = 0.292$, 0.288 and 0.281 for $A = 168$, 170 and 172, respectively. In order to see the differences in the behaviour of spin-dependent properties of $A = 164$ and 166 isotopes, it may be worth looking at a plot of the variation of the pairing gaps with spin for these isotopes. This is displayed in figure 1. It is interesting to see that the neutron pairing gap (Δ_n) for $A = 164$ vanishes at $J = 16$, whereas this happens at $J \approx 26$ for $A = 166$ isotope. After the particle number projection there is no early collapse of the pairing gaps, but we do hope that the relative differences will not be washed out.

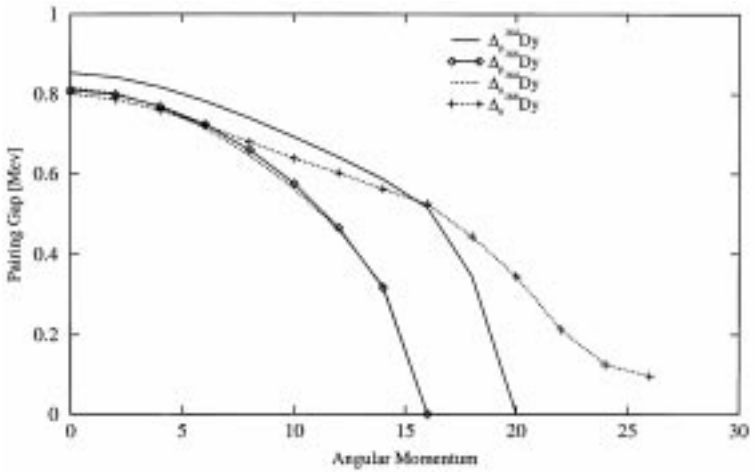

Figure 1. Variation of proton and neutron pairing gaps, Δ_p and Δ_n of ^{164,166}Dy as a function of angular momentum.

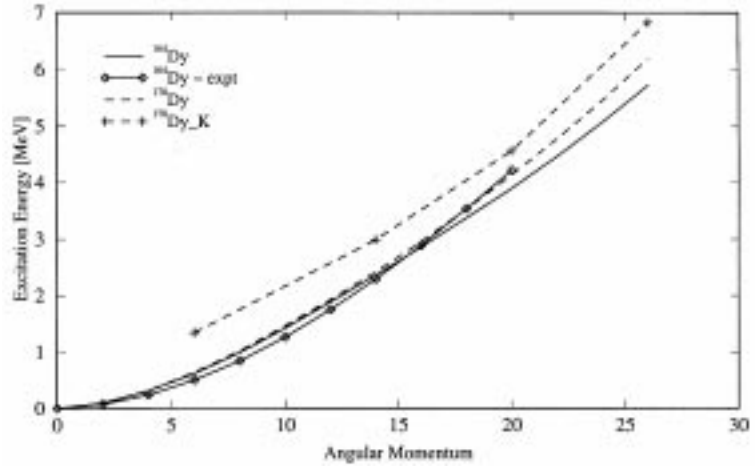

Figure 2. Excitation energy E^* as a function of angular momentum for ^{164}Dy in x-axis cranking (solid) and for ^{170}Dy in x-axis cranking (dotted) as well as z-axis cranking (dotted with (+)) calculations. For comparison, the experimental values for ^{164}Dy are also depicted (solid with ($\diamondsuit$)).

For ^{170}Dy (ratio $N/Z = 1.58$) we have also performed a z-axis cranking (equivalent to putting $\gamma = 120^\circ$ in the x-axis cranking) calculation at certain $J = K$ values, where K is the projection of J on the symmetry axis (z-axis). This is done to check if in such a neutron-rich nucleus, K-bandheads become the yrast line at high spins [16]. A plot of predicted excitation energies E^* as a function of angular momentum is displayed in figure 2. The $K = 6$, 14, 20 and 26 levels lie at about 0.5 MeV above the computed yrast line (x-axis cranking result). The $K = 6$ and 14 states are formed by the deformation alignment of

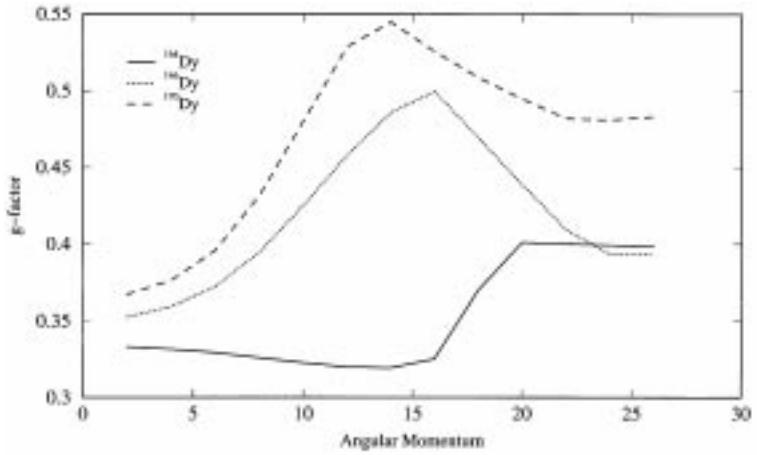

Figure 3. g-Factor as a function of J for ^{164,166,170}Dy, with $g_2 = 0.333$, 0.353 and 0.367, respectively, in standard units.

neutron $1f_{7/2}$, $0h_{9/2}$ and $0i_{13/2}$ quasiparticle orbitals. The $K = 20$ and 26 are generated by further adding 6 and 12 units of angular momentum to 14 due to the alignment of proton multiquasiparticle orbitals $0h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$. From the figure it is clear that the calculated points for ^{164}Dy are not quite in agreement with the experimental ones. The calculated effective moment of inertia (inverse of the slope of the E^* vs. J curve) is smaller in the low-spin region and higher in the high-spin region as compared to the experimental values. But in the CHFB approach only such qualitative agreements are usually expected.

In figure 3 we display the dependence of g-factors on angular momenta for $A = 164$, 166 and 170 isotopes of Dy. The authors of ref. [7] conclude that the g factors of ^{164}Dy are fairly constant for $J = 2$–10. But a careful observation of their figure indicates a slow decrease with the increase of J such that g_{10} may be about 5–10% smaller than g_2. With the interaction strengths employed here the value of $g_2 = 0.333$ is in fairly good agreement with the experimental value, and $g_{10}/g_2 = 0.97$ is also consistent with the data. As seen from figure 3, the ratio $g_{14}/g_2 = 0.96$, has the lowest value for $J = 14$, and then it increases for higher spins up to $J = 20$. This increase is due to the rotation alignment of mainly $ph_{11/2}$ orbitals. It attains almost a constant value of about $Z/A \approx 0.4$ (a 20% increase compared to g_2) for $J = 20$–26. For $^{166-170}$Dy, our calculation predicts an increase of g_J with the increase of J right from $J = 4$ till around the moderate high spins $J = 14$–16 (almost at the spin value where it shows a minimum for ^{164}Dy). The relative decrease at still higher spins is obviously due to the enhanced alignments of neutron orbitals. It may be further added that even at the lowest value of spin ($J = 2$) our predictions are interesting as a function of the neutron number. While Z/A shows a 4% decrease at $A = 170$ compared to that for $A = 164$, we predict, on the contrary, an increase of 10% (see figure 3). Qualitatively this may be attributed to the moving up of the neutron Fermi surface to high-m single particle orbitals so that rotation alignment of the neutron orbitals gets dampened. Then, the relatively larger contribution of protons to the total angular momentum at $J = 2$ in going from $A = 164$ to 170 in Dy isotopes explains the variation of g_2 with A. On the same basis one may expect similar features in other isotopic chains like that of Er and Yb.

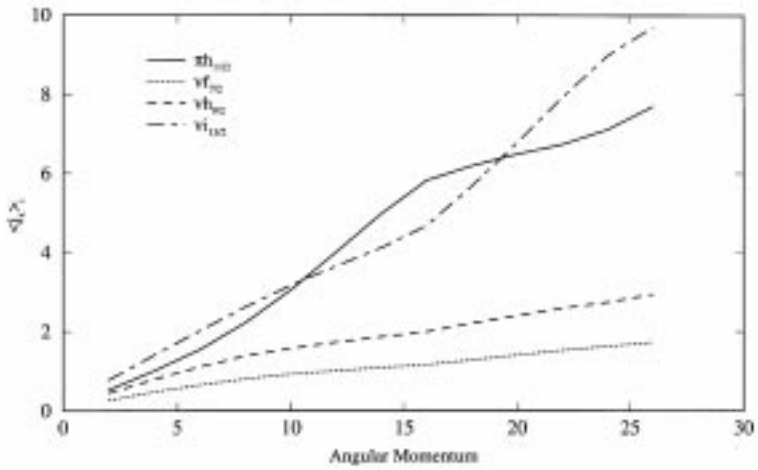

Figure 4. Contributions of a few single particle orbitals to the total angular momentum of ^{166}Dy. As the labels indicate, the neutron $0i_{13/2}$ and proton $0h_{11/2}$ orbitals contribute maximum through their coherent rotation alignment.

A rotation alignment plot for ^{166}Dy for certain important orbitals ($ph_{11/2}$, $nf_{7/2}$, $nh_{9/2}$, $ni_{13/2}$) is shown in figure 4. There is a strong competition of alignment between $ph_{11/2}$ and $ni_{13/2}$ orbitals. As seen from the figure, the proton alignment dominates over that of neutrons between $J = 10$ to 20. Though not shown here, for ^{170}Dy the value of aligned spin from the orbitals $ph_{11/2}$ remains higher than that of $ni_{13/2}$ for the full range of $J = 10$–26. We also find that the high-j orbitals $0i_{13/2}$ for protons and $0j_{15/2}$ for neutrons do not make any significant contributions.

4. Conclusions

High-spin structures of $^{164-170}$Dy have been studied following the standard x-axis CHFB theory employing a separable pairing + quadrupole + hexadecapole model interaction hamiltonian. From these studies, without any free adjustable parameters, we would like to draw the following conclusions:

In the ground state, the $N = 102$ isotope of Dy seems to be the most deformed one, rather than that with $N = 104$ at the mid-shell. The axial component of the hexadecapole deformation β_{40} is small positive for $A = 164$, decreases with increase of A and becomes negative for $A = 170$. For ^{164}Dy we get $\beta_{40} = 0.029$ compared to 0.019 as reported by Stuchbery *et al* [17].

With the present choice of the interaction strengths the high-spin properties, particularly the g-factors, of ^{164}Dy are described very well. Then with the same Hamiltonian the spin-dependent properties of the next even isotope ^{166}Dy are found to exhibit a fairly different behaviour. This striking feature can be established if the g-factors of even a few low-spin states of ^{166}Dy are measured. We also find that at the lowest spin of $J = 2$ the g-factor increases with the increase of neutron number, contrary to the expected trend based on the collective Z/A value.

Finally a few high-K bandheads for ^{170}Dy are predicted to lie only at about 500 keV excitation energy above the yrast line. Because of the K-selection rules these should be relatively long-lived isomeric states.

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