PRAMANA
— journal of
physics

Vol. 60, No. 1
January 2003
pp. 143–150

An empirical fit to estimated neutron emission cross sections from proton induced reactions

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MS received 20 May 2002; revised 16 September 2002

Abstract. Neutron emission cross section for various elements from ^{9}Be to ^{209}Bi have been calculated using the hybrid model code ALICE-91 for proton induced reactions in the energy range 25 MeV to 105 MeV. An empirical expression relating neutron emission cross section to target mass number and incident proton energy has been obtained. The simple expression reduces the computation time significantly. The trend in the variation of neutron emission cross sections with respect to the target mass number and incident proton energy has been discussed within the framework of the model used.

Keywords. Neutron yield; pre-equilibrium; equilibrium; empirical fit.

PACS Nos 24.10.-i; 25.40.-h; 29.85.+c

1. Introduction

There is a growing interest in neutron emission data from intermediate energy (from about 20 MeV to about 100 MeV) proton induced reactions in various target elements because accelerators in this energy region are being used increasingly in different areas of research, medicine and industry. Radiation environments in these particle accelerators are mainly dominated by neutrons spanning a wide range of energy from slow neutrons to several tens of MeV [1]. Moreover, protons in the energy region below 200 MeV are constituents of cosmic rays (solar protons) which directly or through secondary neutrons induce radiation damage to the electronic components in airplanes, satellites etc [2].

It is not feasible to carry out experimental measurements for all possible combinations of target elements and proton energies. Such experiments, moreover, face difficulties due to the presence of mixed radiation fields of gammas and neutrons, pulsed nature of the radiation field, radio frequency interference etc. in accelerators.

The inadequate experimental data are supplemented by calculated results based on nuclear reaction model codes. However, availability and complexity of such computer codes pose problems for easy and quick estimation of neutron emission data particularly for thick target neutron yield distributions. In such cases neutron emissions need to be calculated several times for different proton energies and different target elements of mixed elemental composition. In the present work, we attempt to develop a simple but reliable empirical expression to calculate neutron emission from proton induced reactions in the energy range 25–105 MeV. By replacing time consuming nuclear model calculations with any simple expression one can achieve a substantial saving of computation time in large scale simulation codes.

In the energy range considered, two basic reaction mechanisms play important roles, namely, pre-equilibrium (PEQ) and compound nuclear or equilibrium (EQ) reactions. We have estimated the total neutron emission cross section, PEQ and EQ emission cross sections for neutrons from various target elements using the hybrid model code ALICE-91 [3] which was earlier [4] found to give fairly accurate results in this energy range. ALICE-91 calculates PEQ emissions using the hybrid model and EQ emissions using the Weisskopf–Ewing formalism. The neutron emission cross sections for various elements obtained from ALICE-91 have been studied for any discernible trend with target mass. An empirical relation is established between the calculated neutron emission cross section and the target mass number. In §2 we discuss the findings of our analysis followed by a conclusion in §3.

2. Results and discussions

We have calculated the total, PEQ and EQ neutron emission cross sections from 32 target elements ranging from ^{9}Be to ^{209}Bi for proton induced reactions in the incident energy range $E_{\mathrm{p}} = 25$–105 MeV using the hybrid model code ALICE-91. Hybrid model [5] assumes that the relaxation of the target + projectile composite system proceeds through a cascade of two-body interactions when the excitation energy is shared between the interacting particles. Each stage of the relaxation process is described by the number of particles + holes (excitons, n) present in it. This model explicitly evaluates the pre-emission energy distribution of the ejectile at each stage n in terms of appropriate intermediate state densities.

In the calculation we have considered reaction channels where ejectiles other than neutrons (e.g. protons, deuterons and alphas) are also emitted. Our calculations include possible neutron emissions from various excited residual nuclei formed after the emission of different types of particles from the primary target–projectile interaction. For proton projectile the absorption cross section σ_{abs} is calculated using the optical model and the initial exciton configuration of $n_0 = 3$ (2 particle +1 hole) is chosen. The number of excited neutrons and protons are calculated considering the p–n interaction to be three times stronger than the p–p or n–n interaction. For neutron ejectiles the inverse cross section is calculated using the optical model. The Gilbert–Cameron level density option has been used in our calculation.

We have plotted the calculated total neutron emission cross section against mass number A at different projectile energies in figure 1. The emission cross section shows a definite increasing trend with A almost throughout the entire mass range and for all proton energies considered. For very large values of A the rate of increase, however, becomes much smaller and becomes negative for a few high mass targets.

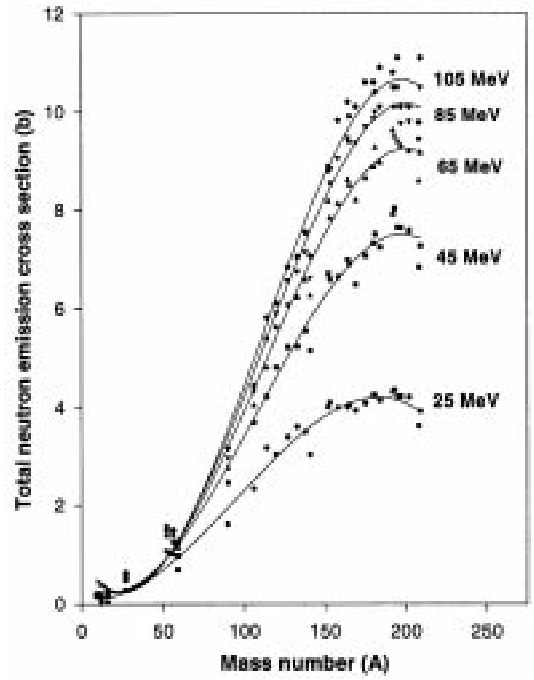

Figure 1. Total neutron emission cross sections from 32 different target elements for proton induced reactions at 25, 45, 65, 85 and 105 MeV. The symbols are the calculated values while the solid lines are the regression curves based on third degree polynomials.

2.1 *Empirical formula for neutron emission cross section*

We have found that the total neutron emission cross sections $\sigma_{\text{emission}}(E_{\text{p}},A)$ for the incident proton energies E_{p} can be fitted with a third order polynomial in A as

$$\sigma_{\text{emission}}(E_{\text{p}},A) = b_0(E_{\text{p}}) + b_1(E_{\text{p}})A + b_2(E_{\text{p}})A^2 + b_3(E_{\text{p}})A^3. \tag{1}$$

The values of the coefficients b_i for the total neutron emission cross sections are given in table 1 for different incident proton energies E_{p} along with the values of r^2 (square of the correlation coefficient) which when equals to 1 indicates the best fit. Our analysis shows that b_i's vary smoothly with the incident proton energies E_{p}. After analyzing these data we find that the coefficients b_i in eq. (1) can be given by an empirical expression of the second order in E_{p} as

$$b_i = c_{i0} + c_{i1}E_{\text{p}} + c_{i2}E_{\text{p}}^2. \tag{2}$$

The values of c_{ij} are listed in table 2.

In order to test the performance of the present empirical formula, we have calculated, using these values of c_{ij} as well as eqs (2) and (1), total neutron emission cross sections for 20 different target elements at 50 MeV and 75 MeV incident proton energies. These

Table 1. Coefficients b_i's at different incident proton energies.

E_p (MeV) =	25	45	65	85	105
b_0	0.1289	0.4374	0.6782	0.8147	0.9353
b_1	−6.9275e-3	−0.0258	−0.0418	−0.051	−0.0601
b_2	4.4051e-4	8.0422e-4	1.0611e-3	1.2127e-3	1.3564e-3
b_3	−1.5335e-6	−2.4903e-6	−3.1882e-6	−3.6172e-6	−4.0624e-6
r^2	0.98	0.99	0.99	0.99	0.99

Table 2. Coefficients c_{ij}'s for different b_i's.

	b_0	b_1	b_2	b_3
c_0	−0.3415	0.0217	−8.657e-5	−1.7805e-7
c_1	0.0211	−1.2689e-3	2.3858e-5	−6.0917e-8
c_2	−8.5732e-5	4.6986e-6	−9.7357e-8	2.3072e-10
r^2	0.9988	0.9986	0.9978	0.9977

cross sections are compared with those calculated by ALICE-91. Figure 2 is a plot of the percentage deviation between the two calculations. We can see that the empirical formula given by eq. (1) reproduces the total neutron emission cross section with reasonable accuracy except for a few low mass targets (e.g. ^{9}Be, ^{12}C, ^{27}Al, ^{52}Cr).

The above expression reduces the computation time by a factor of about 50.

2.2 *Trend of the neutron emission cross section with mass number and incident proton energy*

Replacement of nuclear reaction model algorithms by simple expressions in complex computer codes (particularly those utilizing the Monte Carlo technique) has to be done very carefully and to do that a detailed understanding of neutron emission calculations in nuclear reaction models is necessary. To analyze the neutron emission process in more details we have studied the variation of σ_{abs} against the mass number A of the target. We have observed that σ_{abs} increases as A increases and the rate of increase is much slower in the heavy mass region. This trend is directly reflected in the total neutron emission cross section. To eliminate the effect of σ_{abs} we have calculated total neutron emission multiplicity M as

$$M = \frac{\sigma_{\text{emission}}}{\sigma_{\text{abs}}}. \tag{3}$$

The total neutron multiplicity M also shows a similar increasing trend with A as the emission cross section. This indicates that the trend of increasing neutron emission with mass number is inherent in the PEQ and EQ emission processes. To study this we have defined the PEQ and EQ neutron multiplicities as,

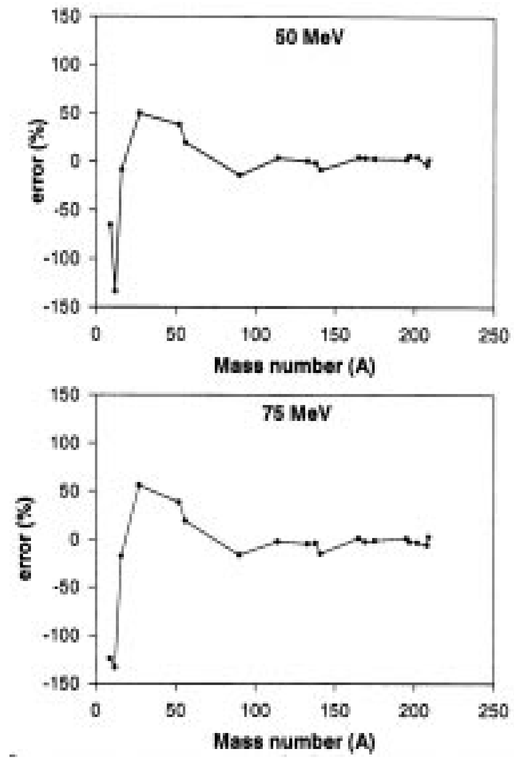

Figure 2. Percentage error of the empirical calculation compared to the cross sections calculated by ALICE for proton induced reactions on different target elements at 50 MeV and 75 MeV projectile energy.

$$N_{\mathrm{PEQ}} = \frac{\sigma_{\mathrm{PEQ}}}{\sigma_{\mathrm{abs}}}$$

$$N_{\mathrm{EQ}} = \frac{\sigma_{\mathrm{EQ}}}{\sigma_{\mathrm{abs}}} \tag{4}$$

The PEQ and EQ neutron multiplicities are plotted against mass number A in figures 3 and 4 respectively along with the regression curves fitted through the data. It is observed that PEQ neutron multiplicity is much less compared to the EQ neutron multiplicity. In figure 3 we find that the PEQ neutron multiplicity is increasing more or less linearly with increasing mass number at all projectile energies, though from the regression plots such linearity is not well established statistically. The slope of the regression line remains more or less the same except at 105 MeV where it decreases slightly. To investigate this trend in the observed PEQ multiplicity we have also analyzed separately the behavior of the emission probability, the number of excited neutrons and the level density ratio. As a result of the above study we attempt to explain the trend as follows: The total number of PEQ neutrons emitted is obtained by summing over all emission energies while that at a particular ejectile energy ε is given by the contribution from all exciton numbers from an initial n_0 to a maximum n_{max}. In the code ALICE-91 n_{max} is fixed in the following way:

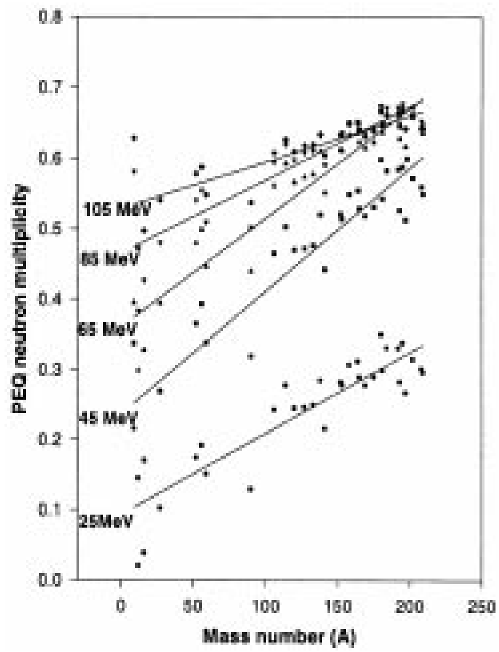

Figure 3. Plot of the PEQ neutron multiplicity for reactions as in figure 1.

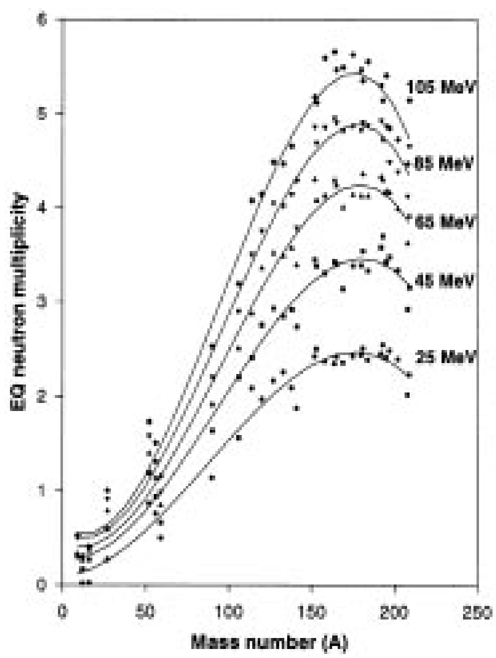

Figure 4. Plot of the EQ neutron multiplicity for reactions as in figure 1.

$$n_{\max} = \overline{n}, \qquad \text{if } \overline{n} \le n_0 + 18$$

$$n_{\max} = n_0 + 18, \quad \text{if } \overline{n} > n_0 + 18 \tag{5}$$

$\overline{n} = (2gE_{\rm c})^{1/2}$ where $E_{\rm c}$ is the excitation energy of the composite nucleus. The single particle level density g is calculated as a linear function of the mass number A ($g = CA/\pi^2$,where C is a constant). So at a given projectile energy $E_{\rm p}$, $\overline{n}$ increases with mass number A. As a result, $n_{\max}$ in ALICE-91 follows the same trend till it reaches the value $n_0 + 18$ in our calculation. Thus for a given $E_{\rm p}$, as the mass number of the target element increases, larger number of exciton states contribute to PEQ emission. Furthermore, in proton induced nuclear reaction $n_0 = 3$. As mentioned earlier, since the p–n interaction is considered three times stronger than the p–p or n–n interaction, the number of excited neutrons and protons at this stage are calculated by ${}_{n_0}X_{\rm n} = 3N/(3N+Z)$ and ${}_{n_0}X_{\rm p} = [1+Z/(3N+Z)]$, respectively, where Z is the number of protons and N is the number of neutrons in the target. In successive stages these numbers are increased by 0.5 at each stage till $n_{\max}$ is reached. We see that for the neutron-rich heavy elements ${}_{n_0}X_{\rm n}$ is larger than that for light elements and subsequently the number of excited neutrons in each exciton state has a larger value. The emission probability and the probability of having one excited neutron with energy ε do not show any definite trend with increasing mass number at a given projectile energy but have a variation of 10% within the mass range considered. But the other two factors mentioned earlier cause the PEQ neutron multiplicity to increase almost linearly with mass number.

It is also observed in figure 3 that at higher projectile energies the PEQ neutron multiplicity has larger values for any target element. The reason may be as follows: For a given target element $\overline{n}$ increases as the projectile energy increases. Thus for a given target element as the incident energy is increased a larger number of exciton states contribute to the PEQ process. Also, since more energy is available for excitation, energy per particle is larger during the initial stages of relaxation process and high energy PEQ emissions are facilitated. As a result, the total emission probability increases since it is integrated over a larger number of energy bins. Consequently, the PEQ neutron multiplicity increases as the projectile energy increases.

Figure 4 shows that the EQ multiplicity increases with increasing A over a large mass range after which it slightly decreases. The EQ emission does not exhibit a linear relationship with the target mass number A. EQ emission cross section $\sigma_{\rm EQ} \sim [e^{2(aU_{\rm r})^{1/2}}]/U_{\rm r}$ where $U_{\rm r}$ is the excitation energy of the residual nucleus and a increases linearly with A. Hence the EQ emission multiplicity should increase monotonically with mass number. For elements with higher mass number the available excitation energy after PEQ emissions may decrease to some extent since a large part of the total energy is carried away by the PEQ ejectiles. This explains the small decrease in the multiplicity at the higher end of the mass range. This trend is strongly reflected in the total neutron multiplicity and also to some extent in the total neutron emission cross section, since EQ emissions are much larger than PEQ emissions. The excitation energy and therefore nuclear temperature available with the residual nuclei after PEQ emissions and the subsequent emission from the equilibrium phase is anticipated to be an interesting subject of study as EQ emissions predominate at all energies even in the high energy spallation region.

3. Conclusion

We have given an empirical relation to calculate quickly and effectively neutron emission cross sections from proton induced reactions in the energy range 25–105 MeV for different target elements. The trend of neutron emission with target mass number as given by the empirical relation can be explained satisfactorily within the conceptual framework of the nuclear reaction model used for the calculation. This simple formula is expected to be useful for large scale computations related to radiation damage studies, proton or neutron therapy, radiological protection in particle accelerators, protection of equipments in accelerators and space travel where high premium is put on easy and quick computational techniques.

Acknowledgement

This work has been carried out as part of a project (No. AERB/20/04) sponsored by the Atomic Energy Regulatory Board, India.

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