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Quantum superarrivals and information transfer through a time-varying boundary

A S MAJUMDAR and DIPANKAR HOME*
S.N. Bose National Centre for Basic Sciences, Block JD, Sector III, Salt Lake, Kolkata 700 098, India
*Department of Physics, Bose Institute, 93/1 A.P.C. Road, Kolkata 700 009, India
Email: archan@boson.bose.res.in; dhom@bosemain.boseinst.ac.in

Abstract. The time-dependent Schrödinger equation is solved numerically for the case of a Gaussian wave packet incident on a time-varying potential barrier. The time evolving reflection and transmission probabilities of the wave packet are computed for several different time-dependent boundary conditions obtained by reducing or increasing the height of the potential barrier. We show that in the case when the barrier height is reduced to zero, a time interval is found during which the reflection probability is larger (superarrivals) compared to the unperturbed case. We further show that the transmission probability exhibits superarrivals when the barrier is raised from zero to a finite value of its height. Superarrivals could be understood by ascribing the features of a real physical field to the Schrödinger wave function which acts as a carrier through which a disturbance, resulting from the boundary condition being perturbed, propagates from the barrier to the detectors measuring reflected and transmitted probabilities. The speed of propagation of this effect depends upon the rate of reducing or raising the barrier height, thus suggesting an application for secure information transfer using superarrivals.

Keywords. Wave packet; potential barrier; information transfer.

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1. Introduction

The physics of wave packet dynamics has revealed several interesting features in recent years [1]. Generally, the reflection/transmission probabilities for the scattering of wave packets by potential barriers are calculated after a complete time-evolution when asymptotic values have been attained. Here we study a phenomenon that occurs during the time evolution. We demonstrate a novel quantum effect which occurs as a result of time-varying boundary conditions. We consider Gaussian wave packets which are incident on time-dependent potential barriers. We are interested in computing the reflection and transmission probabilities for the scattered wave packets. In general, various kinds of perturbations of the potential barrier may be used to alter the time-dependent scattered probabilities. These include increase or decrease or oscillations of the width of the barrier.

In the present paper we choose a simple kind of barrier perturbation. The height of the barrier is reduced to zero linearly with time during an interval of time which is short compared to the time taken by the scattered wave packet to propagate from the barrier to the detectors measuring reflected or transmitted probabilities. We show that this effect of perturbing the boundary leads to superarrivals in the reflected probability. As we shall see below, superarrivals are prominent only when barrier reduction takes place while there is sufficient overlap of the wave packet with the barrier. The time interval during which superarrivals occur and the magnitude of superarrivals depend on the rate at which the barrier height is made zero. We showed in [2] that this essentially nonclassical phenomenon can be explained in terms of the dynamical kick imparted on the wave packet by the time-evolving barrier. Information about barrier perturbation reaches the detector with a finite speed (signal velocity, v_e) which is also proportional to the rate at which the barrier is reduced. This feature is used to suggest a scheme for secure information transfer using superarrivals. In order to reveal superarrivals in the transmitted probability, we start with a zero barrier, which is then raised to a certain height linearly in time while there is sufficient overlap with the incident wave packet.

The plan of the paper is as follows. In §2 we define the reflected probability of a Gaussian wave packet and introduce the concept of superarrivals by considering the case when the barrier height is lowered to zero [2]. In §3 we study the complementary phenomenon of superarrivals occurring in the transmission probability by raising the barrier from zero to some finite value of its height. We will see how the magnitude of superarrivals depends on the various parameters of the problem. Using the phenomenon of superarrivals we explore the possibility of achieving information transfer in a secure way in §4. Section 5 is reserved for some concluding remarks.

2. Superarrivals from a reflecting barrier

In this section we consider the dynamics of a wave packet scattered from a barrier while its height is reduced to zero before the asymptotic value of reflection probability is reached. If an initially localized wave packet $\psi(x,t=0)$ moves to the right and is scattered from a rectangular potential barrier of finite height and width, the time-evolving reflection probability obtained at the left of a point x' is given by

$$|R(t)|^2 = \int_{-\infty}^{x'} |\psi(x,t)|^2 \, \mathrm{d}x. \tag{1}$$

Note that x' lies at the left of the initial profile of the wave packet such that $\int_{-\infty}^{x'} |\psi(x,t=0)|^2 \, \mathrm{d}x$ is negligible. Let us now consider the case if during the time evolution of this wave packet the barrier is perturbed by reducing its height to zero. Let this height reduction take place within a small interval of time compared to the time taken by the reflection probability to attain its asymptotic value $\left|R_0\right|^2$. We compute effects of this perturbation on $|R(t)|^2$. We find that there is a time interval during which $|R(t)|^2$ is remarkably larger in the perturbed case even though the barrier height is reduced. We call this effect 'superarrivals' [2].

Let us consider an initial wave packet given by

$$\psi(x,t=0) = \frac{1}{\left[2\pi\sigma_0^2\right]^{1/4}} \exp\left[-\frac{\left(x-x_0\right)^2}{4\sigma_0^2} + ip_0 x\right], \tag{2}$$

with width σ_0 centered around $x = x_0$ and its peak moving with a group velocity $v_g = \langle p \rangle / m$ towards a rectangular potential barrier. The point x_0 is chosen such that $\psi(x, t = 0)$ has a negligible overlap with the barrier.

For computing $|R(t)|^2$ given by eq. (1) the time-dependent Schrödinger equation is solved numerically. Height of the barrier (V) before perturbation is so chosen that the asymptotic value of the reflection probability is close to 1 for the static case. $|R(t)|^2$ is computed according to eq. (1) by taking various values of x' satisfying the condition $x' \leq x_0 - 3\sigma_0/\sqrt{2}$. The computed evolution of $|R(t)|^2$ corresponds to the building up of reflected particles with time. It means that a detector located within the region $-\infty < x < x'$ measures $|R(t)|^2$ by registering the reflected particles arriving in that region up to various instants. First, we compute $|R_s(t)|^2$ for the wave packet scattered from a static barrier. Next we compute the time evolution of $\left|R_p(t)\right|^2$ in the perturbed case by choosing different rates of barrier reduction. In all cases the barrier reduction starts at a time t_p and the barrier height is reduced to zero linearly in time. t_p is chosen such that at that instant the overlap of the wave packet with the barrier is significant. The time interval during which the barrier is perturbed is denoted by ε. We choose values for which $\varepsilon \ll t_0$, t_0 being the time required for $|R(t)|^2$ to attain the asymptotic value $\left|R_0\right|^2$.

Figure 1 shows the evolution of $|R(t)|^2$ for various values of ε. One sees that during the time interval $t_d < t < t_c$,

$$\left|R_p(t)\right|^2 > |R_s(t)|^2, \tag{3}$$

t_c the instant when the two curves cross each other, and t_d the time from which the curve corresponding to the perturbed case starts deviating from that in the unperturbed case. A detector placed in the region $x < x'$ would therefore register more counts during this time interval $\Delta t = t_c - t_d$ even though the barrier height had been reduced to zero prior to that.

3. Superarrivals in transmission probability from a rising barrier

In this section we will start by considering an initial configuration when there is a right-moving wave packet but no potential barrier. A detector is placed at a point x' far away to the right in this case and it records the incoming particles constituting the wave packet. We denote by $T_s(t)$ the time dependent transmission probability measured at the detector. Now, if a barrier is raised in the path of the wave packet, one expects a part of the incoming wave to be reflected back, and the transmitted probability, $T_p(t)$ to get reduced. However, as in the complementary case of reflection by a reducing barrier, here also we find a time interval during which $T_p(t)$ exceeds $T_s(t)$, thus exhibiting superarrivals in the transmitted wave packet. The barrier is raised from zero to a value V_{B} linearly in time ε. We find that superarrivals persist for a range of detector positions x'.

In order to obtain a quantitative estimate of superarrivals, we define a quantity η, given by

$$\eta = \frac{I_p - I_s}{I_s}, \tag{4}$$

where the quantities I_p and I_s are given by

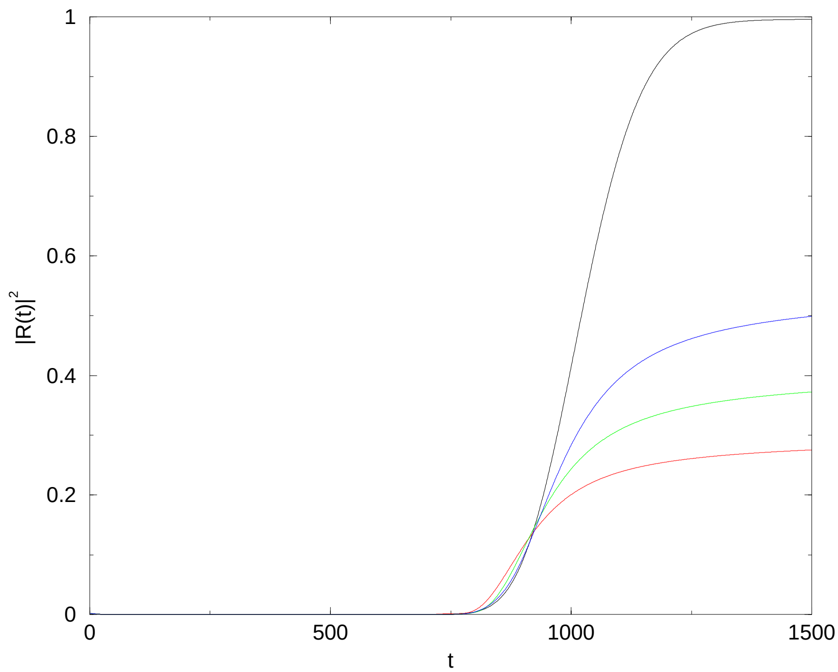

Figure 1. The time evolution of the reflected probability $|R(t)|^2$ is shown for various values of the perturbation duration, ε. The top curve corresponds to the static case. The other curves correspond to the perturbed cases with different rates of barrier reduction. As one increases ε, the magnitude of superarrivals decreases.

$$I_p = \int_{\Delta t} \left|T_p(t)\right|^2 \mathrm{d}t, \tag{5}$$

$$I_s = \int_{\Delta t} |T_s(t)|^2 \mathrm{d}t. \tag{6}$$

The magnitude η of superarrivals is plotted versus the time ε taken for barrier raising in figure 2 for various detector positions. We find that η decreases with increasing ε, or decreasing rate of barrier perturbation in a manner similar to the case of the reflected probability when the barrier height is reduced.

That superarrivals are essentially wave mechanical in origin is demonstrated in the following way. One could consider a probabilistic distribution of particles given by the initial wave packet in terms of the spreads in both position and momentum. Solving the classical Liouville equation for the same time-varying potential and obtaining $|R_s(t)|^2$ and $|R_p(t)|^2$ for the classical case show that there are no superarrivals. This was shown for the case of reflection from a reducing barrier in [2], and the same result holds true for the case of transmission as well.

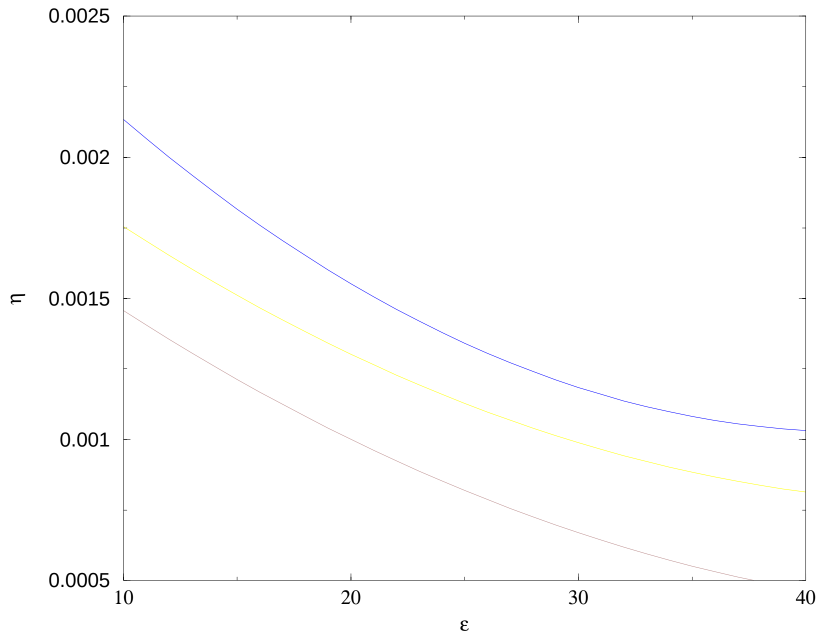

Figure 2. Superarivals in the transmitted wave packet are shown to decrease with increase in ε, the time taken for barrier raising. The three different curves correspond to three different values of the detector position x'.

4. Information transfer using the wave function

In the previous sections we have seen that perturbation of the boundary conditions, represented by the barrier, affects the wave packet by altering its probability amplitude. Superarrivals become more pronounced for larger rate of perturbation. It appears as if the effect of reducing or raising the barrier imparts a dynamical 'kick' to the wave packet, which is then propagated to the detector. In order to investigate the nature of this dynamical effect we consider the question as to how fast the influence of barrier perturbation travels across the wave packet. Note that the information content of a wave packet does not always propagate with the group velocity v_{g} of a wave packet which is usually identified with the velocity of the peak of a wave packet. In situations such as the present one where the incident packet gets distorted on striking the barrier, the concept of signal velocity needs to be defined carefully [3]. A local change in potential affects a wave packet globally, the global effect being manifested through the time evolution of the packet. The action due to a local perturbation (barrier height reduction or raising) propagates across the wave packet at a finite speed, say v_e, affecting the time evolving reflection (or transmission) probability. Thus a distant observer who records the growth of reflection probability becomes aware of perturbation of the barrier (starting at an instant t_p) from the instant t_d when the time-varying reflection (or transmission) probability starts deviating from that measured in the unperturbed case. Thus v_e is given by

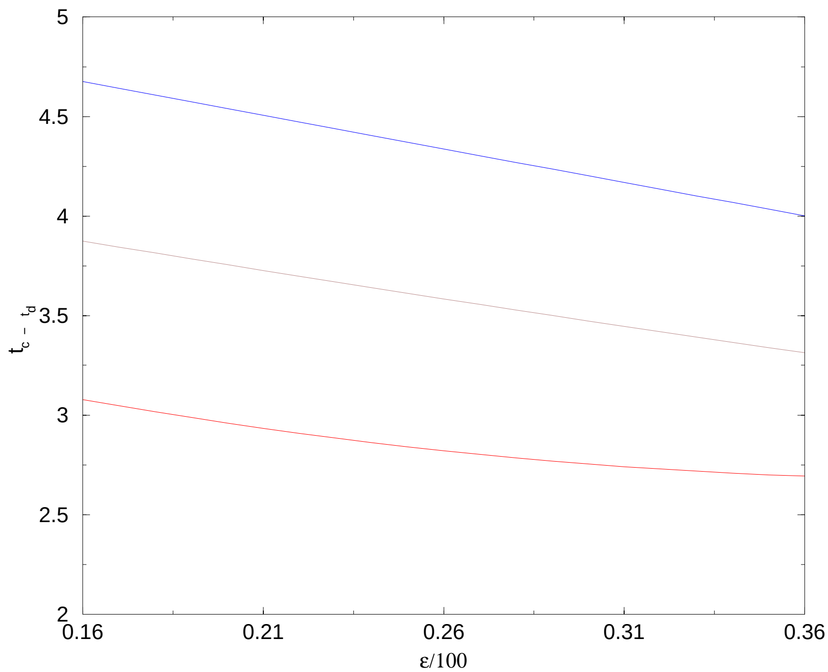

Figure 3. The duration of superarrivals Δt is plotted versus ε. The three different curves denote three different values of the detector position x'.

$$v_e = \frac{D}{t_d - t_p}. \tag{7}$$

It has been shown [2] that the duration of superarrivals Δt, the magnitude of superarrivals η, and the signal velocity v_e all decrease monotonically with increasing ε (or decreasing rate of barrier reduction or raising). The reducing (or rising) barrier imparts a kick (the magnitude of which is proportional to the rate of reduction) on the wave packet. This disturbance is propagated across the wave packet to reach the detector. We see that information about barrier perturbation taking place at location x_c and starting at time t_p propagates through the wave packet and reaches the detector located at x' at time t_d with a finite velocity v_e.

A lot of interest is currently being devoted to study and develop new schemes of quantum information transfer (see, for instance, Alber *et al* [4]), and much work is going on to optimize the capacity of classical and quantum channels. Let us see how the present scheme of superarrivals could, in principle, be used for information transfer through the wave function. In order to do so, it is important to focus on the variation of $\Delta t \equiv t_c - t_d$ (the duration of superarrivals) as a function of ε (the time taken for barrier perturbation). This is plotted in figure 3 for three different values of the detector position x'. Note that Δt decreases monotonically with increasing ε for a wide range of values of ε.

Now suppose a particular curve in figure 3 (functional relation between Δt and ε for a fixed value of detector position x') is chosen as a key which is shared by two persons Alice and Bob who want to exchange information. Alice is at the barrier and receives a continuous inflow of particles whose wave function is given by the initial Gaussian. Alice has the choice of reducing (or raising) the barrier height at completely random different rates. She chooses one particular value of ε for a single run of the experiment, and she wants Bob who is at the detector to be able to decipher this value of ε. Bob monitors the time evolution of $|R_p(t)|^2$ (or $|T_p(t)|^2$) through the detector counts and is able to decipher t_c, t_d, and hence Δt by comparison with the curve $|R_s(t)|^2$ (or $|T_s(t)|^2$) for the static case. He then uses his key to infer the exact value of ε corresponding to the particular value of Δt he has measured. The whole procedure can be repeated with different rates of barrier reduction (or raising) as many times as required by Alice and Bob. In this way an exchange of information takes place between Alice and Bob. This exchange is secure because it would not be possible for any eavesdropper to decipher Alice's chosen value of ε without having access to the key. It is important to note that information transfer takes place in this scheme without any shared entanglement between the two players Alice and Bob. Also, the variable ε can vary continuously in the allowed parameter range.

5. Conclusions

In this paper we have discussed a new quantum mechanical effect which occurs in the time dependent reflection and transmission probabilities of Gaussian wave packets scattered from time varying potential barriers. We have shown that the enhancement of probabilities takes place in both the cases as a result of barrier perturbation. The effect of barrier perturbation, or change in the boundary conditions, propagates across the wave function at a velocity proportional to the rate of barrier perturbation, and manifests itself as a proportional magnitude of superarrivals at the detector. The wave function plays the role of a field or carrier through which information is transmitted. Exploiting this feature of superarrivals, we have suggested a scheme of secure transfer of continuous information. Of course, more detailed calculations are needed in different directions, viz. robustness of the keys used, capacity of the channel etc., in order to establish the viability of this scheme. Nevertheless, what is novel about this kind of approach for information transfer is that it is based on using a dynamical effect of perturbation of the boundary conditions, and it uses purely the wave function for communication. Further investigations with more varied sets of parameters and different types of perturbation in boundary conditions are necessary to test the feasibility of single particle experiments [5] for realizing such a scheme.

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