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Estimation of dynamic properties of attractors observed in hollow copper electrode atmospheric pressure arc plasma system

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Abstract. Understanding of the basic nature of arc root fluctuation is still one of the unsolved problems in thermal arc plasma physics. It has direct impact on myriads of thermal plasma applications being implemented at present. Recently, chaotic nature of arc root behavior has been reported through the analysis of voltages, acoustic and optical signals which are generated from a hollow copper electrode arc plasma torch. In this paper we present details of computations involved in the estimation process of various dynamic properties and show how they reflect chaotic behavior of arc root in the system.

Keywords. Arc plasma; chaos; Lyapunov exponent; dimension; attractor; fractal.

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1. Introduction

Recently, dynamic characteristics of movement of arc root inside a hollow copper electrode plasma torch have been investigated using voltage, optical and acoustic signals generated from the device and evidence for the existence of chaotic regime and possibility of its online control have been reported based on time series, power spectrum, Lyapunov exponent, dimension, attractor structure of experimental signals and system dynamics [1,2]. The logical evolution of the estimation procedure for identification and characterization of chaos in such experimental systems are scarce and are of much value to future workers in this area. The present paper addresses complexity and benchmarking of the estimation procedure and test results.

Time series of an experiment signal cannot distinguish whether a signal belongs to periodic, chaotic or random system. Part of time series of a chaotic system may look like a periodic signal and a random looking signal may well be a chaotic signal. Phase portrait of a system is an important tool for looking into dynamical details of a system as well as to distinguish periodic and aperiodic systems. However, it fails to distinguish aperiodic and chaotic systems. For aperiodic systems, some times strange looking structures having fractal dimension (termed as strange attractors) appear in state space. Although, dissipative chaotic systems also exhibit strange attractors, it is not a definitive test for evidence of chaos. A number of strange attractors are reported that do not satisfy positivity of Lyapunov exponent, a necessary criteria for a system to be chaotic [3].

Lyapunov exponent of a system is a measure for the determination of rate of expansion of nearby trajectories in phase space. A deterministic system posses as many Lyapunov exponents as dimension of the corresponding phase space and the system is declared as chaotic if at least one of these exponents is positive. There are a number of disputes [4] on this computation and recently it has been reported that random noise also exhibits finite positive value of Lyapunov exponent [5]. Noise being an infinite dimensional process, an infinite dimensional reconstruction phase space is required to achieve the exact theoretical value (zero) of Lyapunov exponent. The requirement not being physical, never comes out in practice. Moreover, every algorithm for computation of Lyapunov exponent depends on a number of parameters of the experimental data. Dependence of Lyapunov exponent on all of these parameters must be studied in detail, for any conclusive statement on the positivity of the exponent. To remove ambiguity, a comparison of results with similar analysis for a set of random data is also essential.

The most widely used algorithm for computation of dimension is due to Grassberger and Procaccia [6]. The approach relies on the assumption that only deterministic dynamics yield convergence of the slope of the correlation integral at a finite and small value, indicating a low dimensional attractor. Unfortunately, this assumption is not always true. An insufficient number of data points, linear correlation in the data, or other causes can lead to false convergence which creates illusion of proof of chaos where there is none [5].

2. Analysis of signals

A copper electrode hollow cathode plasma torch is used for the generation of plasma. Details of system dynamics, typical fluctuating behavior of the signals and structural details of the torch are available elsewhere [1,2]. Three signals representing the dynamic voltage, acoustics and optical outputs are generated from the plasma. Attractors corresponding to different signals from the system have been constructed using standard delay technique [2]. To compare results with periodic, noisy and periodic signals mixed with some percentage of noisy signals, we have considered a periodic signal (figure 1a) and a white noise signal (figure 1b) generated using runtime subroutine RANDOM, available in Microsoft Fortran Version 1.0 (1993). Figure 1c shows expanded view of part of this white noise signal. Figures 2a–b show phase portraits due to the periodic signal, the periodic signal mixed with the white noise signal and the white noise signal respectively. In the first case (figure 2a), only a single orbit is obtained. In the second case (figure 2b), the orbit looks like a toroid due to the presence of noise, while in the third case (figure 2c), the whole space is covered by data points without showing any structure. It can be noted that the phase portraits of experimental signal [2] are completely different in nature from any of these attractors. Phase portraits look neither like random, nor like periodic or quasi-periodic, but exhibit some definite structures in phase space, similar to chaotic systems.

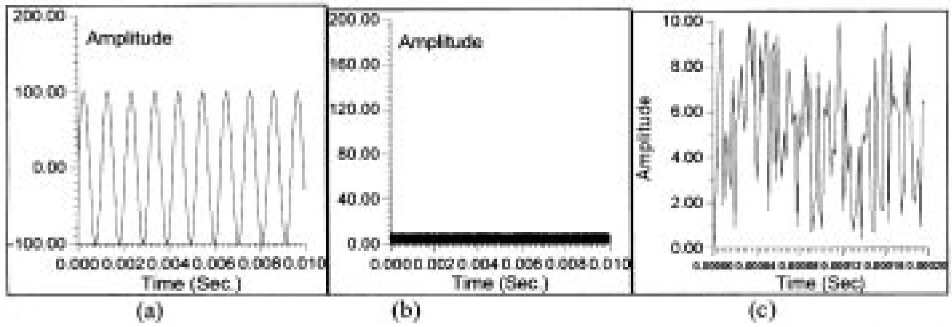

Figure 1. (**a**) Periodic signal, (**b**) white noise signal and (**c**) expanded view of white noise signal.

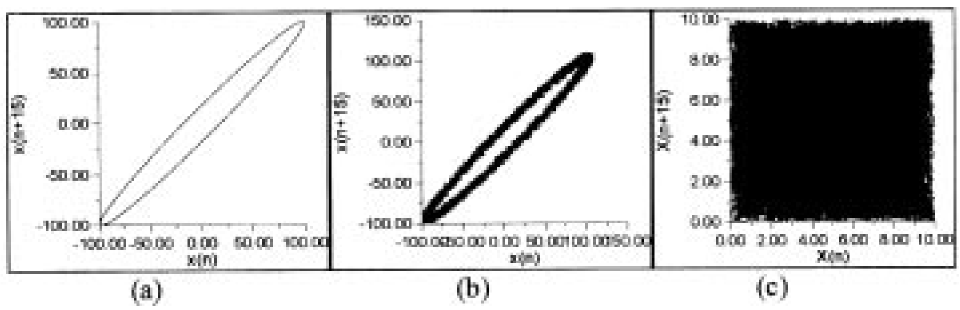

Figure 2. Phase portraits (**a**) periodic signal, (**b**) periodic signal mixed with white noise and (**c**) white noise.

Although, various signals in the system are the macroscopic consequence of the same chaotic discharge, corresponding attractors look different [2]. This is because the phase portrait displays information contained in time series and time series for various signals are different due to their different intrinsic dependence in terms of micro–macro transitions. However, all signals carry identical fractal dimension around 2.3 which is in agreement with the behavior of standard chaotic systems like Henon map: $X_{n+1} = 1 - aX_n^2 + bX_{n-1} (a = 1.4, b = 0.3)$ which exhibits different looking attractors (figures 3a and b) for different embedding delays but shows identical dimension of 1.26. Observed stationarity of computed fractal dimension over a wide range of embedding dimensions [2] proves that the number of experimental data points used in the computation are sufficient. Benchmarking of the code used for dimension computation is available in [7]. Whether these strange attractors are chaotic or not are examined through the estimation of Lyapunov exponent.

Largest Lyapunov exponent from experimental time series is determined using fixed evolution time algorithm proposed by Wolf *et al* [8]. This algorithm is widely used, robust, completely general, capable of computing all the exponents and well suited for a variety of dynamical systems. Details of instruction for computation of this exponent and listing of the code are available in [8]. To avoid ambiguity, names of parameters are chosen identical with that used in [8].

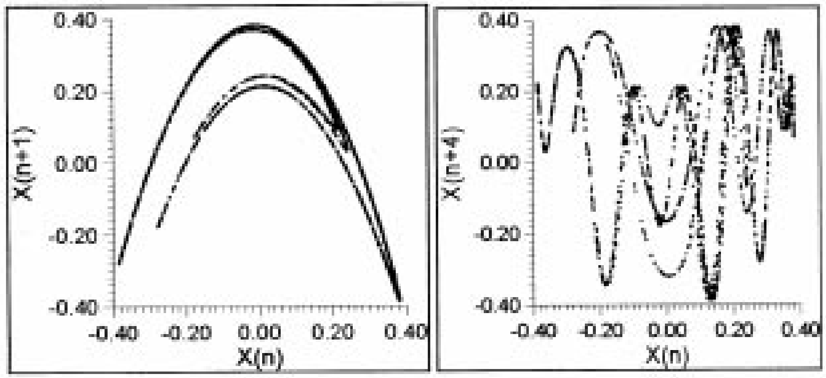

Figure 3. Henon attractor constructed with different delays.

Experimentally, a single observable (voltage, optical or acoustic signal) is sampled at regular interval (DT) to construct the time series of length (NPT) for that particular observable. Construction of phase space with delay coordinates [8] allows one to obtain an attractor from such a time series, whose Lyapunov exponent is identical to that of the original attractor. To estimate λ_1, long term evolution of a single pair of nearby orbits are observed in phase space. Let $X_1 \ldots X_n$ are the elements of a time series $X(t)$, sampled for a particular signal, scaled in the interval [0,1] for convenience. An m-dimensional (DIM) phase portrait is constructed out of it with delay coordinate such that a point in the attractor is denoted by $\{X(t), X(t+\tau) \ldots X(t+[m-1]\tau)\}$, where τ is the delay time (TAU) to be chosen appropriately. A minimum length scale (SCALMN) is defined as the minimum length to be handled in the computation. Any length below this scale will not be entertained. As the computation is based on infinitesimal length scales, a maximum length scale (SCALMX) is also defined (0<SCALMX≤1), which gives the maximum distance between two points for which they will still be considered to be in a common infinitesimal neighborhood. To start the computation $(t = t_0)$, an initial point (fiducial point) is chosen arbitrarily in phase space and distance of its nearest neighbor $L_{\mathrm{i}}(t_0)$ is determined. The system is then allowed to evolve for a certain time (EVOLV), within which the initial separation element $L_{\mathrm{i}}(t_0)$ is evolved to $L_{\mathrm{f}}(t_1)$. Then $(t = t_1)$, a new nearest neighbor is searched out for the fiducial point such that the separation between the two points is $L_{\mathrm{i}}(t_1)$ is a minimum one as well as the angular separation between the evolved and replacement elements $[L_{\mathrm{f}}(t_1)$ and $L_{\mathrm{i}}(t_1)]$ remain within some maximum allowable angular separation ANGLMX. If an adequate replacement point is not available, the points that were being used are retained. The procedure is repeated until the fiducial trajectory has traversed the entire data file of time series and after it the value of λ_1 is estimated as

$$\lambda_1 = \frac{1}{t_M - t_0} \sum_{k=1}^{M} \log_2 \frac{L_{\mathrm{f}}(t_k)}{L_{\mathrm{i}}(t_{k-1})}. \tag{1}$$

In the program $(t_k - t_{k-1})$ is EVOLV, which remains constant throughout the computation and M is the total number of replacement steps.

As it appears, the estimation process depends on a number of parameters like DT, DIM, TAU, EVOLV, SCALMX, SCALMN, and ANGLMX. It is significant that the values of all of these parameters are to be chosen properly for correct computation of the exponent value. Dependence of λ_1 has to be studied against variation of all of these parameters except NPT, DT and ANGLMX. The value of NPT is fixed for all signals and is set to a sufficiently large value 20000. DT is the sampling rate of experimental data and is fixed according to Nyquist criteria. In the present case, ANGLMX is set to 0.3 radian and sampling interval remains within a few microsecond.

Choice of DIM is not a very strict one. Ideally it should be more than twice the dimension of the underlying attractor. However, DIM lower than this also works well [8] in practice. For higher values of DIM computed value of λ_1 remains unaltered and for too low values of it proper unfolding of the attractor does not take place. However, noise being an infinite dimensional process, may corrupt the true data in phase space for very large value of DIM, resulting incorrect computation of λ_1. Therefore, DIM should be kept as minimum as possible but higher than the minimum required one for correct computation. If the dimension of the attractor is not known beforehand, a study of λ_1 against DIM is needed.

Lyapunov exponents are defined by the long-term evolution of the axes of an infinitesimal sphere of states. The use of finite amount of experimental data does not allow one to probe the desired infinitesimal length scales of an attractor. The scales are also inaccessible due to the presence of noise on finite length scales. In the algorithm SCALMN is the length scale on which noise is expected to appear and SCALMX is the length scale over which the local structure of the attractor is no longer being probed. Therefore, for proper choice, a check of stationarity of λ_1 on both of these parameters are needed.

Computation also depends on choice of EVOLV, the allowed evolution time. It should be as maximum as possible to avoid accumulation of orientation error. However, for too large EVOLV, the two trajectories that are being considered for evaluation of λ_1, may pass through a folding region of the attractor and lead to an underestimation of λ_1. Practically it has to be selected considering maximization of evolution time and the tradeoff between replacement vector size and orientation error [8]. The program used for evaluation of λ_1 keeps EVOLV constant throughout the computation. Therefore, for choice of any particular value of EVOLV, a study of dependence of λ_1 on EVOLV is necessary.

Choice of delay time TAU for reconstruction of attractor is also a crucial point in this case. For too less a value of TAU, attractor may not get properly unfolded in phase space, while for too large a value of TAU the consecutive points in phase space may get totally uncorrelated. Therefore, for proper choice of TAU, stationarity of λ_1 against TAU has to be secured.

Figure 4 shows a check of stationarity of λ_1 against TAU for an acoustic signal. TAU has been varied from 2 to 10 and in each case, evolution of λ_1 is studied (figures 4a–i). In all cases, as computation proceeds, λ_1 tries to reach a stationary value. A least square fit of the data in the stationary region (straight line in the figures) gives the value of λ_1 for that particular TAU. Figure 5a shows dependence of λ_1 on a range of TAU. As λ_1 remains almost stationary for TAU within five to eight, TAU=5 (in units of DT) is selected for the present computation. Similar studies are made for all other parameters also. Although, ultimate dependence of λ_1 on these parameters are presented (figure 5), evolution of λ_1 in all these cases are not presented due to lack of space. Figure 5b presents dependence of λ_1 on SCALMX. Horizontal line in the figure presents the level where the value of λ_1 remains

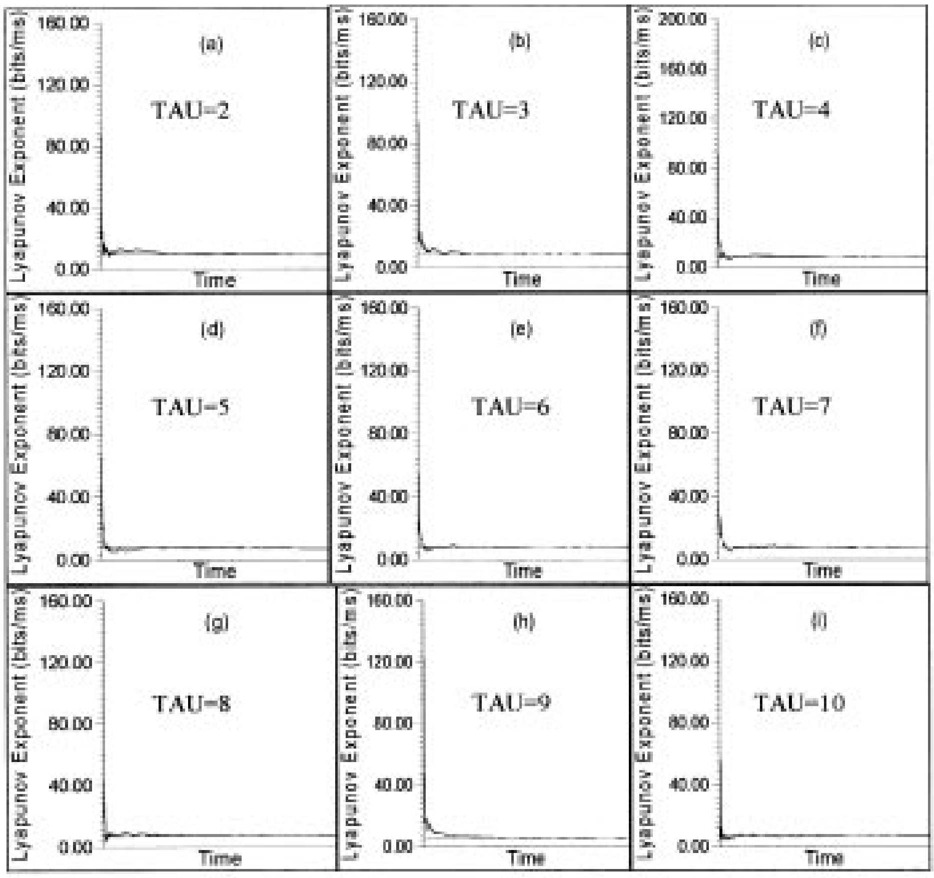

Figure 4. Check of stationarity of λ_1 against TAU. (**a**) $\lambda_1 = 9.502$, (**b**) $\lambda_1 = 8.798$, (**c**) $\lambda_1 = 8.752$, (**d**) $\lambda_1 = 7.5$, (**e**) $\lambda_1 = 8.074$, (**f**) $\lambda_1 = 7.644$, (**g**) $\lambda_1 = 7.652$, (**h**) $\lambda_1 = 6.049$ and (**i**) $\lambda_1 = 7.054$.

most stable over a wide range of SCALMX. It is seen that SCALMX=20% of horizontal extent of the attractor can be taken for a good estimate of λ_1. Lesser the SCALMN, more accurate is the exponent calculation. However, to avoid noisy length scales, there must be a limit to the choice of this minimum. Figure 5c shows the dependence of λ_1 on SCALMN. It is observed that the value of λ_1 remains almost constant on the lower side of the minimum cut off length shown. For correct computation of λ_1, SCALMN is selected as 0.0004% of the horizontal extent of the attractor. One obtains an embedding if DIM is chosen more than twice the dimension of the underlying attractor. However, in practice one obtains a correct evaluation for values of DIM much lower than this [8]. Dependence of λ_1 on DIM is displayed in figure 5d. 'Plus' and 'square' signs correspond to the acoustic and the random signal respectively. It is seen that λ_1 is almost constant for DIM more than 3 for the acoustic signal but varies drastically for the random signal. Since value of DIM past the minimally

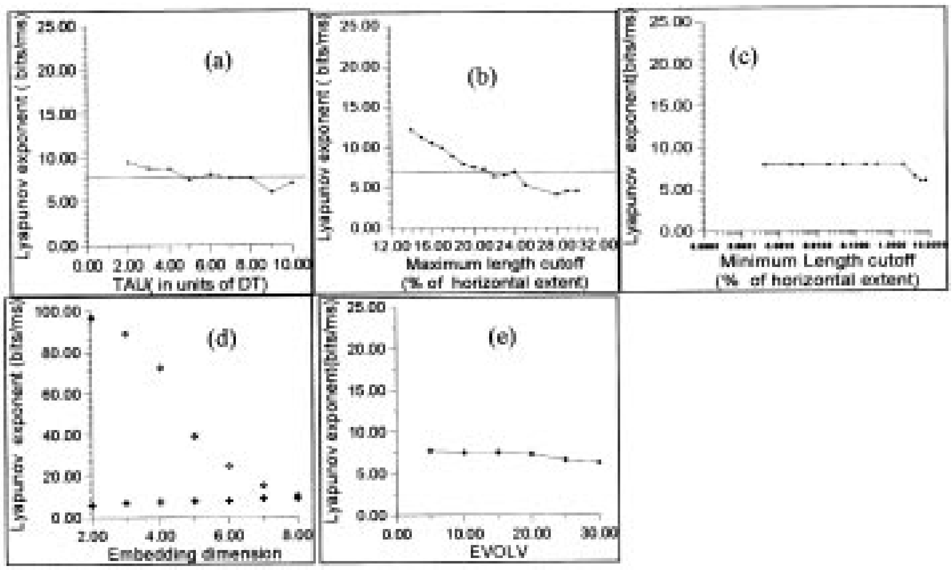

Figure 5. Check of dependence of λ_1 on (**a**) TAU, (**b**) SCALMX, (**c**) SCALMN, (**d**) DIM and (**e**) EVOLV.

required unnecessarily increases the level of contamination due to noise, DIM is chosen safely as 4 for correct computation of λ_1. This behavior of λ_1 with DIM is exactly similar to that of correlation dimension on DIM [2]. Both of these indicate that the value of DIM equal to 4 is optimum for this analysis. EVOLV is the maximum time permitted for free evolution between replacements in the 'Wolf's algorithm'. Larger value of EVOLV reduces the frequency of replacement and reduces the accumulation of orientation error, but allows the volume element to grow overly large and incorporate folding, causing underestimation of λ_1. Therefore, for correct estimation of λ_1, EVOLV should be chosen in a region where local variation of EVOLV does not change the value of λ_1. Figure 5e presents the variation of λ_1 with EVOLV. It is seen that λ_1 is almost stationary for EVOLV within 10 to 20 (in units of DT). Therefore, EVOLV$=$10 is selected for correct computation of λ_1.

We have chosen the optimum algorithmic parameters as per the above discussion and computed the value of λ_1 for the experimental signal (figure 6). The solid horizontal line is the least square fit of the data where value of λ_1 is almost stationary. It provides a value $\lambda_1 = 7.55$ bits/ms.

As white noise also exhibits spurious positive Lyapunov exponent, it can be questioned whether the positive exponent, so obtained, is spurious or not. This test has also been carried out using the same technique and results are already available in [2]. For random noise and periodic signals the exponent tries to reach zero but remains slightly positive, while in the case of experimental signal, the value of λ_1 shows a stationary value, well above the level of zero. The first two cases confirm that the algorithm is correct and the test as a whole reconfirms that the computed exponent is not spurious and definitely positive.

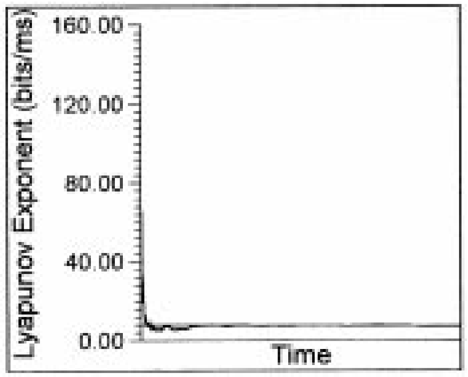

Figure 6. Computation of λ_1 for experimental signal (computation parameters: NPT = 20000, TAU = 5, DT = 0.002, SCALMX = 20% of horizontal extent, SCALMN = 0.004% of horizontal extent, DIM = 4, EVOLV = 10).

3. Summary and conclusion

Proper prediction and correct characterization of hollow cathode arc discharge system require a rigorous estimation procedure to be followed using the standard tools of chaos diagnosis. A number of complex issues and tests are required to be taken into account before any conclusive statement. This necessitates a detailed accounting of the estimation procedure with justification for each step. In this paper we have presented details of such estimation procedure and explained how it confirms presence of chaos exhibited by various signals in the arc discharge system.

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