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Higher dimensional global monopole in Brans–Dicke theory

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Abstract. The gravitational field of a higher dimensional global monopole in the context of Brans–Dicke theory of gravity is investigated. The space time metric and the scalar field generated by a global monopole are obtained using the weak field approximation. Finally, the geodesic of a test particle due to the gravitational field of the monopole is studied.

Keywords. Global monopole; Brans–Dicke theory; higher dimension.

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1. Introduction

The idea of higher dimensional theory was originated in super string and super gravity theories to unify gravity with other fundamental forces in nature. Solutions of Einstein field equations in higher dimensional space times are believed to be of physical relevance possibly at the extremely early times before the universe underwent compactification transitions [1].

Phase transitions in the early universe can give rise to topological defects of various kinds. Topological objects such as monopoles, strings and domain walls have an important role in the formation of our universe. Monopoles are point like topological objects that may arise during phase transitions in the early universe. The gravitational field of a monopole exhibits some interesting properties, particularly those concerning the appearance of non-trivial space time topologies [2]. In 1989, Barriola and Vilenkin (BV) [2] described a monopole solution resulting from a breaking of the global SO(3) of a triplet scalar field in a Schwarzchild background. In 1996, Banerjee *et al* [3] extended the work of BV in a five dimensional space time. In 1997, Barros and Romero [4] studied gravitational field of global monopole in Brans–Dicke theory of gravity by taking the same energy momentum tensors as BV have taken.

In this work, we have studied the higher dimensional global monopole in Brans–Dicke theory in the weak field approximation by considering the same energy momentum tensor components as that of Banerjee *et al* [3].

2. Basic equations

Let us consider Brans–Dicke field equation in the form [4]

$$G_{ab} = \frac{8\pi}{\phi} T_{ab} + \frac{\omega}{\phi^2}\phi_{,a}\phi_{,b} + \frac{1}{\phi}(\phi_{;a;b} - g_{ab}\Box\phi), \tag{1}$$

$$\Box\phi = \frac{8\pi T}{2\omega + 3}, \tag{2}$$

where ϕ is the scalar field, ω is a dimensionless coupling constant, and T denotes the trace of T_a^b, the energy momentum tensor of the matter field.

The energy momentum tensor of a static global monopole can be approximated (outside the core) as [3]

$$T_a^b = \left(\frac{\eta^2}{r^2},\ \frac{\eta^2}{r^2},\ 0,\ 0,\ \frac{\eta^2}{r^2}\right), \tag{3}$$

where η is the energy scale of the symmetry breaking.

The general five dimensional static metric admitting spherical symmetry in the usual three-dimensional space is given by

$$\mathrm{d}s^2 = \mathrm{e}^{\gamma}\mathrm{d}t^2 - \mathrm{e}^{\beta}\ \mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\ \mathrm{d}\phi^2\right) - \mathrm{e}^{\mu}\ \mathrm{d}\psi^2, \tag{4}$$

where γ, β, μ and ϕ are all functions of r only.

Substituting this in eqs (1) and (2) and taking into account eq. (3), we obtain the following set of equations:

$$-\mathrm{e}^{-\beta}\left(\frac{\mu''}{2} + \frac{\mu'^2}{4} - \frac{\beta'\mu'}{4} - \frac{\beta'}{r} + \frac{\mu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} = \frac{16\pi\eta^2\omega}{\phi(2\omega+3)r^2} - \frac{\phi'\gamma'\mathrm{e}^{-\beta}}{2\phi}, \tag{5}$$

$$-\mathrm{e}^{-\beta}\left(\frac{1}{r^2} + \frac{\gamma'}{r} + \frac{\mu'}{r} + \frac{\mu'\gamma'}{4}\right) + \frac{1}{r^2} = \frac{16\pi\eta^2\omega}{\phi(2\omega+3)r^2} - \omega\mathrm{e}^{-\beta}\frac{\phi'^2}{\phi^2} - \frac{\mathrm{e}^{-\beta}}{\phi}\left(\phi'' - \frac{\phi'\beta'}{2}\right), \tag{6}$$

$$-\mathrm{e}^{-\beta}\left(\frac{\gamma''}{2} + \frac{\gamma'^2}{4} - \frac{\gamma'\beta'}{4} + \frac{\gamma'}{2r} - \frac{\beta'}{2r} + \frac{\mu'}{2r} + \frac{\mu''}{2} + \frac{\mu'^2}{4} - \frac{\beta'\mu'}{4} + \frac{\gamma'\mu'}{4}\right) = -\frac{\mathrm{e}^{-\beta}\phi'}{r\phi} - \frac{24\pi\eta^2}{\phi(2\omega+3)r^2}, \tag{7}$$

$$-\mathrm{e}^{-\beta}\left(\frac{\gamma''}{2} + \frac{\gamma'^2}{4} - \frac{\gamma'\beta'}{4} + \frac{\gamma'}{r} - \frac{\beta'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} = \frac{16\pi\eta^2\omega}{(2\omega+3)r^2} - \frac{\mu'\phi'\mathrm{e}^{-\beta}}{2\phi}, \tag{8}$$

$$e^{-\beta}\left(\phi''+\frac{\phi'}{2}\left(\gamma'-\beta'+\mu'+\frac{4}{r}\right)\right)=-\frac{24\pi\eta^2}{r^2(2\omega+3)}. \tag{9}$$

Subtracting (8) from (5), we get

$$-e^{-\beta}\left(\frac{\mu''-\gamma''}{2}+\frac{\mu'^2-\gamma'^2}{4}-\frac{\beta'}{4}(\mu'-\gamma')+\frac{\mu'-\gamma'}{r}\right)=\frac{\phi'}{2\phi}(\mu'-\gamma'). \tag{10}$$

So without loss of any generality one can take

$$\mu=\gamma \tag{11}$$

as the solution of (10).

3. Solutions in the weak field approximation

Now we shall solve the field eqs (5)–(9) in the weak field approximation.

So we take the metric coefficients and the scalar field of the form [4]

$$e^{\gamma}=e^{\mu}=1+f(r),\quad e^{\beta}=1+g(r),$$

and

$$\phi(r)=\phi_0+\varepsilon(r), \tag{12}$$

where ϕ_0 is a constant which may be identified with G^{-1} when $\omega\to\infty$ (G = Newtonian constant).

Here the functions $f(r)$, $g(r)$ and $\varepsilon(r)/\phi_0$ are small in the sense $|f(r)|,|g(r)|$ and $|\varepsilon(r)/\phi_0|\ll 1$ and in course of calculations we should retain terms to the first order in η^2/ϕ_0.

With this approximation, we can easily see that [4]

$$\frac{\phi'}{\phi}=\frac{\varepsilon'}{\phi_0(1+\varepsilon/\phi_0)}=\frac{\varepsilon'}{\phi_0},\quad \frac{\phi''}{\phi}=\frac{\varepsilon''}{\phi_0},$$

$$\beta'=\frac{g'}{e^{\beta}}=\frac{g'}{1+g}=g',$$

$$\gamma'=\mu'=f',$$

and so on.

From eq. (9), we have

$$\varepsilon''+\frac{\varepsilon'}{r}=-\frac{24\pi}{(2\omega+3)}\cdot\frac{\eta^2}{r^2}. \tag{13}$$

Solving (13), we get

$$\varepsilon = -\frac{24\pi\eta^2}{(2\omega+3)}\ \ln\frac{r}{r_0} - \frac{A_0}{r}, \tag{14}$$

where r_0, A_0 are integration constants.

Now the field equations reduce to the following equations in the above weak field approximations

$$-\left(\frac{f''}{2}+\frac{f'}{r}-\frac{g'}{r}\right) = \frac{16\pi\eta^2\omega}{\phi_0(2\omega+3)r^2}, \tag{15}$$

$$-\frac{2f'}{r} = \frac{16\pi\eta^2\omega}{(2\omega+3)r^2} - \frac{\varepsilon''}{\phi_0}, \tag{16}$$

$$\left(f''+\frac{f'}{r}-\frac{g'}{2r}\right) = \frac{\varepsilon'}{\phi_0 r} + \frac{24\pi\eta^2}{\phi_0(2\omega+3)r^2}, \tag{17}$$

$$-\left(\frac{f''}{2}+\frac{f'}{r}-\frac{g'}{r}\right) = \frac{16\pi\eta^2\omega}{\phi_0(2\omega+3)r^2}. \tag{18}$$

Putting the value of ε, we solve (16) and get

$$f = \frac{4\pi\eta^2(3-2\omega)}{\phi_0(2\omega+3)}\ \ln\frac{r}{r_0} + \frac{A_0}{\phi_0 r}. \tag{19}$$

Now solving (15), we get

$$g = \frac{6\pi\eta^2(2\omega+1)}{\phi_0(2\omega+3)}\ \ln\frac{r}{r_0}. \tag{20}$$

Thus the line element (4) which represents the space time generated by the higher dimensional monopole can be written as

$$\begin{aligned} \mathrm{d}s^2 = {} & \left(1+\frac{4\pi\eta^2(3-2\omega)}{\phi_0(2\omega+3)}\ \ln\frac{r}{r_0}+\frac{A_0}{\phi_0 r}\right)\mathrm{d}t^2 - \left(1+\frac{6\pi\eta^2(2\omega+1)}{\phi_0(2\omega+3)}\ \ln\frac{r}{r_0}\right)\mathrm{d}r^2 \\ & -r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2) - \left(1+\frac{4\pi\eta^2(3-2\omega)}{\phi_0(2\omega+3)}\ \ln\frac{r}{r_0}+\frac{A_0}{\phi_0 r}\right)\mathrm{d}\psi^2. \end{aligned} \tag{21}$$

We see that this solution does not reduce to Banerjee *et al* solution as $\omega\to\infty$ [3].

4. Geodesics

In this section, we shall discuss the path of a relativistic particle of mass m, moving in the gravitational field of a monopole described by (21) using the formalism of Hamilton and Jacobi (H–J). Accordingly, the H–J equation is [5]

$$\frac{1}{A}\left(\frac{\partial S}{\partial t}\right)^2 - \frac{1}{B}\left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{r^2}\left[\left(\frac{\partial S}{\partial x_1}\right)^2+\left(\frac{\partial S}{\partial x_2}\right)^2\right] - \frac{1}{A}\left(\frac{\partial S}{\partial \psi}\right)^2 + m^2 = 0, \tag{22}$$

where

$$A = 1 + \frac{4\pi\eta^2(3-2\omega)}{\phi_0(2\omega+1)}\ \ln r + \frac{A_0}{\phi_0 r}, \quad B = 1 + \frac{6\pi\eta^2(2\omega+1)}{\phi_0(2\omega+3)}\ln r \tag{23}$$

and x_1, x_2 are the co-ordinates on the surface of the 2-sphere. Take the ansatz [5]

$$S(t,\ r,\ x_1,\ x_2,\ \psi) = -Et + S_1(r) + p_1 x_1 + p_2 x_2 + J\cdot\psi \tag{24}$$

as the solution to the above H–J equation (22). Here constants E, J are identified as the energy and five dimensional velocity and p_1, p_2 are momentum of the particle along different axes on 2-sphere with $p = (p_1^2 + p_2^2)^{1/2}$, as the resulting momentum of the particle.

Now substituting (24) in (22), we get

$$S_1(r) = \varepsilon \int \left[B\left(\frac{E^2-J^2}{A} - \frac{p^2}{r^2} + m^2\right)\right]^{1/2} \mathrm{d}r, \tag{25}$$

where $\varepsilon = \pm 1$.

In H–J formalism, the path of the particle is characterized by [5]. $\partial S/\partial E =$ constant, $\partial S/\partial p_1 =$ constant, $\partial S/\partial p_2 =$ constant and $\partial S/\partial J =$ constant.

Thus we get (taking constants to be zero without any loss of generality),

$$A = \varepsilon \int \frac{EB}{A}\left[B\left(\frac{E^2-J^2}{A} - \frac{p^2}{r^2} + m^2\right)\right]^{-1/2} \mathrm{d}r, \tag{26}$$

$$\chi_i = \varepsilon \int \frac{p_i B}{r^2}\left[B\left(\frac{E^2-J^2}{A} - \frac{p^2}{r^2} + m^2\right)\right]^{-1/2} \mathrm{d}r, \tag{27}$$

$$\psi = \varepsilon \int \frac{JB}{A}\left[B\left(\frac{E^2-J^2}{A} - \frac{p^2}{r^2} + m^2\right)\right]^{-1/2} \mathrm{d}r. \tag{28}$$

From (26), we get the radial velocity as

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \left(\frac{A}{EB}\right)^{1/2}\left[B\left(\frac{E^2-J^2}{A} - \frac{p^2}{r^2} + m^2\right)\right]^{1/2}. \tag{29}$$

Now the turning points of the trajectory are given by $\mathrm{d}r/\mathrm{d}t = 0$, as a consequence the potential curves are

$$\frac{E}{m} = \left[\frac{J^2}{m^2} + \left(\frac{p^2}{r^2 m^2} - 1\right)A\right]^{1/2}. \tag{30}$$

Thus the extremals of the potential curve are the solution of the equation

$$m^2 r^2 A_0 + \frac{8\pi\eta^2 p^2(2\omega-3)}{2\omega+3} r \ln r + \frac{4\pi\eta^2(2\omega-3)m^2 r^3}{(2\omega+3)} = r\left(2p^2\phi_0 + \frac{4(2\omega-3)\pi\eta^2 p^2}{(2\omega+3)}\right). \tag{31}$$

It has real solution provided

$$2p^2\phi_0 + \frac{4(2\omega-3)\pi\eta^2 p^2}{2\omega+3} > 0, \quad \text{i.e.,} \quad \omega > \frac{3-3\phi_0}{2\phi_0+4\pi\eta^2}.$$

Thus the trajectory of the test particle can be trapped by the monopole due to the restriction on the coupling parameter ω.

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