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Masses of *S* and *P* wave mesons and pseudoscalar decay constants using a confinement scheme

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Abstract. In the framework of relativistic harmonic confinement model for quarks and antiquarks, the masses of *S*- and *P*-wave mesons and pseudoscalar decay constants from light flavour to heavy flavour sectors are computed. The residual two-body Coulomb interaction and the spin-dependent interaction of the confined one gluon exchange effects (COGEP) such as spin–spin and spin–orbit interactions are perturbatively incorporated with the confinement energy to get the respective vector-pseudoscalar meson mass differences. Here we employ the same parametrization and model parameters as used in a recent study of low-lying hadron masses and leptonic decay widths. The results are being compared with the values obtained from other theoretical models and the experimental values.

Keywords. Unified confinement scheme; pseudoscalar mesons; decay constants; confined one gluon exchange potential.

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There is a renewed interest in meson spectroscopy due to vast data of light flavour mesons to heavy flavour mesons from many experimental facilities world over [1–5]. Theoretically, our knowledge of hadron physics is mainly based on phenomenological quark confinement models [6–10], as the hadron domain generally falls in the non-perturbative regime of quantum chromodynamics. Though some of the successful models could reproduce the masses of the low-lying hadrons, their predictions for the excited states and decay properties of mesons were not satisfactory with respect to the experimental values [11,12]. The pseudoscalar decay constants of the heavy-light mesons have also been estimated in the context of many QCD-motivated approximations. The predictions for each of these constants cover a wide range of values from one model to another. They also predict different behaviours for the decay constants with increasing heavy quark mass. Phenomenologically, it is important to have reliable estimates of these decay constants as they are useful in many weak processes such as quark mixing, CP violation etc. Due to the availability of experimental data corresponding to $L = 1$ mesonic states (*P*-wave mesons), it is necessary to predict these states from any successful phenomenological model.

Recently, an attempt has been made to provide a unified confinement scheme for the study of hadrons from light flavour to heavy flavours. Based on this scheme, we have been able to predict the masses of open flavour hadrons and the leptonic decay widths of vector mesons [13]. In this paper, we present the computed masses of mesons as well as the pseudoscalar decay constants from light flavour to heavy flavour combinations. The spin–spin interaction and the spin–orbit interaction of the confined one gluon exchange potential have been employed perturbatively to get the vector-pseudoscalar mass differences and to predict the masses of P-wave mesonic states.

The mass of a meson in an energy eigenstate N and spin state J containing different quark–antiquark combinations is computed as [13]

$$M_N^J(q_1,q_2) = \sum_{i=1}^{2} \varepsilon_N(q_i)_{\rm conf} + \varepsilon_N(q_1,q_2)_{\rm coul} + \varepsilon_N^J(q_1,q_2)_{\rm SD}. \tag{1}$$

Here we treat q_1 or q_2 as either quark or antiquark. The first term in eq. (1), corresponds to the total confinement energy (total dynamical inertial mass) of the constituent quarks, which has been computed using a phenomenological model like the RHM [14]. The second term corresponds to the residual Coulomb energy among the confined constituent quarks. The third term corresponds to the spin-dependent interaction energy (spin-hyperfine, spin–orbit etc.) among the confined constituent quarks. For total orbital angular momentum $\ell = 0$ states the spin–orbit and tensor terms do not contribute. The spin-averaged (centre of weight) masses of the mesons are obtained without the spin dependent term of eq. (1).

According to the unified scheme based on RHM, the intrinsic energy of the quark/antiquark in a mesonic system is given by [13]

$$\varepsilon_N(q)_{\rm conf} = \sqrt{(2N+3)\Omega_N(q) + M_q^2 - \frac{3M_q}{\sum_{i=1}^{2} M_{q_i}}\Omega_0(q)}, \tag{2}$$

where M_q is the mass parameter of the quark in this scheme.

The size parameter, $\Omega_N(q)$ of RHM radial wave function is energy dependent and is given by

$$\Omega_N(q) = A(E_N + M_q)^{1/2}. \tag{3}$$

The wave functions for the two quark systems are constructed by retaining the nature of single particle wave function but with a two particle size parameter $\Omega_N(q_i q_j)$ defined as in the recent study of hadronic properties [13].

The residual Coulomb energy in eq. (1) is calculated perturbatively in the confinement basis as

$$\varepsilon_N(q_1,q_2)_{\rm coul} = \langle NS|V_{\rm coul}(r)|NS\rangle, \tag{4}$$

where the residual colour Coulomb potential is given by

$$V_{\rm coul}(r) = \alpha_s^{\rm eff}(\mu)\frac{1}{r}. \tag{5}$$

Here $\alpha_s^{\rm eff}(\mu)$ is the effective strong coupling constant defined as in [13].

The spin-dependent energy containing the hyperfine interaction and spin–orbit interaction is computed as

$$\varepsilon_N^J(q_i,q_j)_{\rm SD} = \langle NJ|V_{\rm SD}|NJ\rangle, \tag{6}$$

where the spin-hyperfine potential and the spin–orbit interaction of the residual (effective) confined one gluon exchange potential (COGEP) are given by [13–18]

$$V_{\sigma_i\cdot\sigma_j} = \frac{\alpha_s N_i^2 N_j^2}{4}\frac{\lambda_i.\lambda_j}{[E_i+m_i][E_j+m_j]}[4\pi\delta^3(r) - C_{\rm CCM}^4 r^2 D_1(r_{ij})]\left(-\frac{2}{3}\sigma_i\cdot\sigma_j\right) \tag{7}$$

and

$$\begin{aligned} V_{q_iq_j}^{LS} = {} & \frac{\alpha_s}{4}\cdot\frac{N_i^2N_j^2}{(E_i+M_i)(E_j+M_j)}\ \frac{\lambda_i\cdot\lambda_j}{2r_{ij}} \\ & \times\{[r\times(\hat{p}_i-\hat{p}_j)\cdot(\vec{\sigma}_i+\vec{\sigma}_j)]\ \cdot\ [D_0'(r_{ij})+2D_1'(r_{ij})] \\ & +\{[r\times(\hat{p}_i+\hat{p}_j)\cdot(\vec{\sigma}_i-\vec{\sigma}_j)]\ \cdot\ [D_0'(r_{ij})+2D_1'(r_{ij})], \end{aligned} \tag{8}$$

where $D_0(ij)$ and $D_1(r_{ij})$ are the confined gluon propagators and $N_{i/j}$'s are the RHM normalization constants [15–17].

In the construction of mesonic states, the spurious motion of centre is accounted by keeping the centre of mass at the ground state. Accordingly, the second term proportional to $(\hat{p}_1+\hat{p}_2)$ in eq. (8) will not contribute to the spin–orbit interaction of the quark–antiquark system.

As in the earlier case, the propagators D_0 and D_1 are fitted to a familiar function as $\sim k_{0,1}((\exp[-C_{\rm CCM}^2 r_{ij}^2/2])/r_{ij})$ [15,16]. The $C_{\rm CCM}$ parameter can be identified as the effective mass of the confined gluons and it is taken as 810 MeV inspired from the low lying glueball mass [19]. It also provides the right energy difference between pseudoscalar-vector meson systems.

Using the two particle size parameter obtained from the RHM scheme for the confined quarks [13], we calculate the matrix elements of the Coulombic term and the matrix elements of the spin-dependent interactions among the confined quarks for the different confined energy states for the mesonic systems. The computed *S*- and *P*-wave masses of the mesons are listed in tables 1 and 2 respectively alongwith other theoretical predictions [20–23].

The pseudoscalar decay constant $(f_{\rm P})$ which is an important input parameter in the study of hadronic decays is also obtained using the Van Royen–Weisskopf formula given by [20]

$$f_{\rm P}^2 = \frac{3}{\pi M_p}|R_{00}|^2, \tag{9}$$

where M_p is the ground state mass of pseudoscalar meson. Using the two particle radial wave function evaluated at the origin (R_{00}) the pseudoscalar decay constants for the ground states are computed and are tabulated in table 3 alongwith the results of other theoretical models [8,22,24–26].

To summarize, we have presented in this paper, the masses of *S*- and *P*-wave mesons as well as the decay constants of pseudoscalar mesons based on a unified confinement scheme described by RHM and CCM [13,14,19]. The masses of the constituent quarks and other model parameters are the same as used in [13]. Using these parameters the spin averaged masses starting from $b\bar{b}$ mesons to $q\bar{q}$ mesons with all open flavour combinations are computed and are listed in table 1.

Table 1. Masses and charge radii of the *S*-wave mesons.

System	Spin-averaged meson mass	Pseudoscalar meson mass (MeV)				Vector meson masses (MeV)				Charge radii (fm)	
		Present	Others		Expt.	Present	Others		Expt.	Present	Others
$b\bar{b}(1S)$	9452	9425	9377[a]	9427[d]	–	9461	9464[a]	9462[d]	9461	0.186	0.230[a]
$b\bar{b}(2S)$	10023	10012	9963[a]	9994[d]	–	10027	10007[a]	10013[d]	10023	0.400	0.500[a]
$b\bar{b}(3S)$	10326	10319	10298[a]	10339[d]	–	10329	10339[a]	10355[d]	10355	0.707	0.750[a]
$b\bar{b}(4S)$	10575	10572	10573[a]		–	10574	10602[a]		10580	1.165	0.950[a]
$b\bar{c}(1S)$	6300	6256	6264[a]		–	6314	6337[a]			0.236	
$b\bar{c}(2S)$	6951	6929	6856[a]		–	6968	6899[a]			0.509	
$b\bar{c}(3S)$	7322	7308	7244[a]		–	7326	7280[a]			0.898	
$b\bar{c}(4S)$	7630	7622	7562[a]		–	7633	7594[a]			1.477	
$b\bar{s}(1S)$	5382	5298	5375[b]		5375	5409	5422[b]			0.256	
$b\bar{s}(2S)$	6290	6264			–	6299				0.549	
$b\bar{s}(3S)$	6774	6764			–	6777				0.969	
$b\bar{u}(1S)$	5222	5094	5091[e]	5342[c]	5279	5265	5182[e]	5347[c]	5352	0.258	
$b\bar{u}(2S)$	6224	6192			–	5094				0.553	
$b\bar{u}(3S)$	6743	6732			–	6747				0.975	
$c\bar{c}(1S)$	3068	2985	2980[a]	2993[c]	2980	3096	3097[a]	3091[c]	3097	0.385	0.420[a]
$c\bar{c}(2S)$	3674	3626	3608[a]	3640[c]	–	3690	3686[a]	3688[c]	3686	0.822	0.850[a]
$c\bar{c}(3S)$	4073	4047			–	4082		4104[c]	4040	1.441	1.200[a]
$c\bar{c}(4S)$	4420	4408			–	4420		4456[c]	4415	2.355	1.480[a]
$c\bar{s}(1S)$	2085	2009	1969[b]	1968[c]	1968	2110	2110[b]	2076[c]	2110	0.491	
$c\bar{s}(2S)$	2805	2778			–	2805				1.039	
$c\bar{s}(3S)$	3274	3264			–	3277				1.810	
$c\bar{q}(1S)$	1912	1815	1850[e]	1897[c]	1869	1909	1956[e]	2004[c]	2007	0.506	
$c\bar{q}(2S)$	2681	2653			–	2690				1.063	
$c\bar{q}(3S)$	3172	3162			–	3175				1.846	
$s\bar{q}(1S)$	817	520	485[e]	495[c]	497	916	909[e]	916[c]	892	0.869	
$s\bar{q}(2S)$	1603	1531	1412[e]		–	1627	1749[e]	1680		1.733	
$s\bar{q}(3S)$	2135	2114	1832[e]		–	2142				2.920	
$q\bar{q}(1S)$	612	140	150[e]	135[c]	139	769	767[e]	812[c]	768	0.958	
$q\bar{q}(2S)$	1441	1345	1293[e]	1439[c]	1300	1473	1369[e]		1450	1.852	
$q\bar{q}(3S)$	1987	1962			–	1995				3.079	

q corresponds to *u* or *d*.
[a]Ref. [20], [b][22], [c][21], [d][23], [e][4].

The hyperfine parameter C_{CCM} in the spin-dependent term of COGEP has been identified as the effective mass of the confined gluon and is taken as 810 MeV, as the di-gluon glueball state is expected in the range 1600–1700 MeV [26]. Using this CCM parameter, the hyperfine contribution to the pseudoscalar mesons and to the vector mesons of all flavour combinations are computed and are also listed in table 1. The masses of the mesons obtained here are fairly in good agreement with other theoretical model predictions as well as with the experimental values (see table 1). The charge radii for the mesons are also computed and are listed in table 1 along with the *S*-wave masses of the mesons. We have also predicted the *P*-wave masses of the mesons from $b\bar{b}$ to $q\bar{q}$ flavour combinations. Though there are no experimental data for the *P*-wave masses of many of the open flavour mesons like $b\bar{s}$, $b\bar{c}$, $c\bar{s}$ etc., our results for *P*-wave $b\bar{b}$ mesons and $c\bar{c}$ mesons are in good agreement with the experimental values. At the lower flavour sector, we may not be able to compare with the experimental states as these states are largely impure with various contributions

Table 2. Masses of the *P*-wave mesons.

	Masses in MeV							
	1*P*				2*P*			
System	Spin-averaged	χ_0	χ_1	χ_2	Spin-averaged	χ_0	χ_1	χ_2
$b\bar{b}$	9907 (9900)	9839 (9860)	9873 (9892)	9941 (9913)	10217 (10260)	10197 (10232)	10207 (10255)	10227 (10268)
$b\bar{c}$	6801	6709	6755	6847	7182	7156	7169	7195
$b\bar{s}$	6091	5927	6009	6173	6599	6561	6580	6618
$b\bar{q}$	6010	5798	5904	6116	6559	6515	6537	6585
$c\bar{c}$	3497 (3525)	3431 (3417)	3464 (3510)	3530 (3556)	3907	3891	3899	3916
$c\bar{s}$	2586	2514	2550	2622	3079	3065	3072	3086
$c\bar{q}$	2449	2385	2417	2481	2969	2959	2964	2974
$s\bar{q}$	1320	1236	1278	1362	1902	1890	1896	1908
$q\bar{q}$	1142	1134	1138	1146	1748	1746	1747	1749

Note: The values shown in the paranthesis indicate experimental results of mesonic masses [4].

Table 3. The pseudoscalar decay constants (in MeV) in comparison with values from other theoretical models as well as experiments.

Meson	Present	Other theoretical models			Experimental
f_{η_b}	711	660[a]	772[b]	812[d]	715 ± 15[f]
$f_{\eta_{bc}}$	607	510[a]	456[b]	500[d]	–
f_{B_s}	600		266 ± 10[c]	235[e]	–
f_B	581			203[e]	–
f_{η_c}	420	547[a]	426[b]	509[d]	410 ± 15[f]
f_{D_s}	387		309 ± 15[c]	227[e]	344 ± 52[g]
f_D	336			185[e]	< 310[g]
f_K	320			160[e]	160 ± 1.4[g]
f_π	239			139[e]	131 ± 0.1[g]

[a,b]Ref. [24], [c][22], [d][8], [e][25], [f][26], [g][4].

from gluonic contents and mixing of other multiquark states [5]. Though there are many models for heavy flavour sector and low flavour sector separately, there are very few models like the present one, A J Sommerer *et al* [21] etc. which could predict the properties of the mesons from light–light flavour to heavy–heavy flavour combinations successfully. The pseudoscalar decay constants of the mesons in table 3 show good agreement with the experimental values of the η_b, η_c and D_s mesons. However, the present study overestimates pseudoscalar decay constants in the low flavour sectors. Though there is no experimental $f_{\rm P}$ values for η_{bc}, B_s and B mesons, our values are found to differ from the values predicted by other theoretical models. The values for decay constants obtained here for f_K to f_{D_s} lie in the same range as predicted by lattice calculations [27]. The overall behaviour of the $f_{\rm P}$ values from light flavour to heavy flavour is as expected from the experimental trend. The deviation for the low flavour mesons suggests the importance of higher order radiative corrections and relativistic corrections to the decay constants for low flavour mesons, while such corrections are not important in the heavy flavour sector. Further studies on various properties of the hadrons and their decays based on the present scheme are under investigation.

Parameters used in the calculations

Confinement mean field parameter, $A = 2166\ ({\rm MeV})^{3/2}$;
$M_u = M_d = 82.8$ MeV; $M_s = 357.5$ MeV; $M_c = 1428$ MeV; $M_b = 4637$ MeV; $k = 5.19427; C_0 = 1.47$.
CCM parameter $C_{\rm CCM} = 810$ MeV.

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