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An empirical approach to the theory of particle and nuclear phenomena: Review and some new ideas

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Abstract. Experimental data on masses and lifetimes of unstable particles falls into a pattern, a brief review of some interesting consequences is presented here. From the experience in semiclassical methods and recent advances in quantum chromodynamics, it is proposed that an appropriate generalization of the Gutzwiller trace formula for field theories may lead to a systematic semiclassical chromodynamics theory. The theory can be developed to get appropriate dynamics leading to an explanation of pattern discovered in the empirical data.

Keywords. Nuclear masses and lifetimes; trace formula; semiclassical chromodynamics; quantum chaos; random matrix theory.

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1. Introduction

An essential problem of particle physics is to explain the relationship between the measured mass of fundamental particles and their lifetimes. This is a formidable problem. It is shown in this paper that the main problem can be broken into various parts and the inter-relationship between nuclear and particle properties show possibilities by which phenomena can be better understood. It is possible to explain missing fundamental particles, their maximum mode of decay, prediction of the half-lives of isospin particles, and so on [1].

Besides, a possible way to understand above mentioned relations will be presented. This is based on the observation of random matrix universality found in the fluctuation measures on energy levels of nuclei, chaotic quantum systems, quantum dots, Riemann zeros etc. Semiclassical trace formula is most useful in understanding the universal features as well as in quantizing chaotic systems. A way to unravel these mysteries would be through a semiclassical route by generalizing the semiclassical trace formula of Gutzwiller [2]. It should be noted that Gutzwiller trace formula was inspired by the Selberg trace formula in number theory. Thus, here we have some deep connections between number theory, semiclassical methods, nuclear physics and chaos. After presenting the empirical approach, some recent developments [3] on how nuclear masses are obtainable from the quantum chromodynamics (QCD) will be presented in §2. In §3, we briefly propose a sketch of what may be called semiclassical chromodynamics. Section 4 contains a brief summary.

2. Masses and lifetimes of nuclei, mesons, etc.

2.1 *Empirical approach*

Employing the measured masses and lifetimes of unstable nuclei and other particles, it was shown that [1,4,5] their product follows a simple relation:

$$\frac{\hbar}{MT} = \frac{n}{2^n}, \tag{1}$$

where M is neutron mass (ergs) in β-emitting nuclei, binding energy of the nuclei in α emitters, and entire mass of the decaying particles in the case of fundamental particles. T is the observed half-life related to the width Γ and n is an integer, presumably containing information about the strength of interaction [5]. The above relation leads to an interesting connection $\Gamma = M.n/2^n$ which means that the Heisenberg energy spread in decay is a fraction $n/2^n$ of the rest mass. It is to be noted that the ratio $n/2^n$ can be interpreted following Cantor in that if n is discrete set of numbers, 2^n is the corresponding continuous set of numbers.

In particular, to appreciate the efficacy of (1), consider first the behaviour of fundamental particles and write $p = -\log_{10}\frac{\hbar}{MT} = -\log_{10}\frac{n}{2^n}$, thus giving n from observed values of M and T. In figure 1, $\log_{10}\frac{\hbar}{MT}$ is plotted against n. As seen here, all known particles lie on a straight line.

There are two important gaps in the line, first being between $n = 30$ and $n = 43$ and again between $n = 63$ and $n = 97$. A curious result is that the values of p take values of numbers very close to primes for values of n = 6, 10, 14, 21, 28, 42, and 49 [5b].

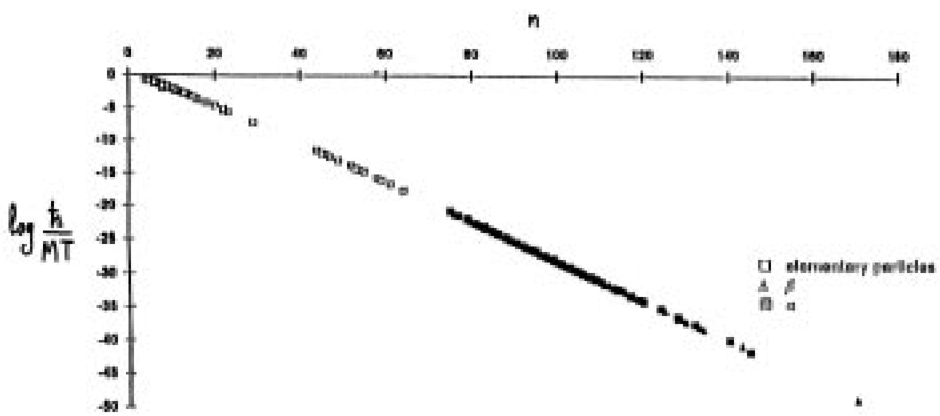

Figure 1. $\log\frac{\hbar}{MT}$ vs n for elementary particles, β- and α-emitters is plotted, a remarkable trend is evident.

It was also shown in [1] that periodicity can be introduced by replacing (1) by

$$\frac{\hbar}{MT} = \frac{f(n)}{2^{f(n)}}, \tag{2}$$

where $f(n) = n\sin(\alpha n + 7)$. The constant α is given values 1, 2, or 3 to fit the observed values of MT. For most of the particles the value is 2, 1 and 3 are kept for the scale.

We now show that (1) by itself is capable of explaining many aspects of particle and nuclear physics.

2.1.1 *Lifetimes of isospin particles*: The classification of mesons in terms of n allows for some interesting regularities. The n-values of isospin singlets and neutral member of the isospin triplets belonging to SU(3) octets are found to be related to each other through some periodicity. The following equations show that (π^0, η) for pseudoscalar mesonic octet, (ρ^0, ω) for vector mesonic octet and (Λ, Σ^0) for $\frac{1}{2}^+$ baryonic octet-pairs of states with different values of isospin (I), but the same value of its third component I_3 show regularities:

$$\begin{aligned} n_{\pi^0} - n_\eta &= 2^2 + 1 \quad \text{for pseudoscalar mesons,} \\ n_{\rho^0} - n_\omega &= 2^2 + 1 \quad \text{for vector mesons,} \\ n_\Lambda - n_{\Sigma^0} &= 2^5 + 1 \quad \text{for baryons.} \end{aligned} \tag{3}$$

On the other hand, singlets belonging to SU(3) octets and their orthogonal singlets with the same value of I and $I_3 (= 0)$ show a slightly different kind of periodicity of n values. Examples of such pair of mesons are (ϕ, ω), (η, η') for which

$$\begin{aligned} n_\phi - n_\omega &= 2^1, \\ n_\eta - n_{\eta'} &= 2^3. \end{aligned} \tag{4}$$

Both the symmetries can be combined to give a relation

$$\mid \Delta n \mid = 2^q + \mid \Delta I \mid, \tag{5}$$

where Δn and ΔI represent differences in their integer n and isospin values respectively. It is to be noted that this relation holds for only integral values of ΔI. These regularities in the n-difference point out to the quantization of the Γ/M ratios for the isospin levels which can be written in a mathematical form as [6]

$$\left[\frac{MT}{\hbar}\right]_{n+\alpha} = k \left[\frac{MT}{\hbar}\right]_n, \tag{6}$$

where subscripts denote the corresponding n-values for $MT/\hbar$. Note that n and α are integers, and α is the index using isospin. This equation can be simplified by using (1) to

$$k = \frac{n}{n+\alpha} 2^\alpha. \tag{7}$$

2.1.2 *Lifetime of proton*: We apply the above considerations to proton by using the corresponding value of n of its doublet member, the neutron. Since $I_3 = 1/2$ $(-1/2)$ for proton (neutron), thus $\Delta I_3 = 1$ as we go from neutron to proton. For the case of neutron where $n = 97$, it is argued in [6] that $\alpha = 2^7 - 97 = 31$. The corresponding case of proton for which $\alpha = 31$, $q = 7 + 1$, thus $n = 2^{7+1} - 31 = 225$. With this value of n for the proton, the lifetime of proton becomes 5.33×10^{33} years which is very close to the present experimental limit: more than $10^{31} - 5 \times 10^{33}$ years. It is really intriguing to note that the lifetime of a particle with known mass can be calculated using that of the other particle which is related to the former through isospin symmetry.

2.1.3 *Excited nuclear levels and elementary particles*: The transition of a ground state nucleus to an excited one of higher energy occurs under an external influences when the required energy is transferred to the nucleus through interaction with an energetic particle. When excited externally, one or many nucleons occupy higher energy levels. Since the nucleon levels are separated by finite energy intervals, the nucleus cannot receive an arbitrary amount of energy but only in certain quanta, precisely corresponding to the energies of nucleon transitions from lower to higher states. In this section, we propose that the energy is absorbed by the nucleus in a discrete way in the form of virtual mesons, mainly the light ones. As a result, the system gets excited to a level whose width and energy depend on the mass and decay time of the meson involved in the process. The type of virtual meson depends on the external energy imparted to the nucleus.

Each excited level of a nucleus is associated with an elementary particle which is identified by comparing the ratio $\log E_r/\Gamma$ [7] for an excited level of energy E_r and width Γ with $\log MT/\hbar$ for an elementary particle of mass M and lifetime T. In figure 2, integer D_n is given by $p = \log(2^{D_n}/D_n)$. The quantity n for a resonance level is calculated from E_r/Γ by using (1) which helps in identifying the particle involved in the excitation process. The elementary particles involved are found to be light mesons. A study on number distribution of various ensuing n-values show that it is the ω meson that is mostly involved in the excitation process.

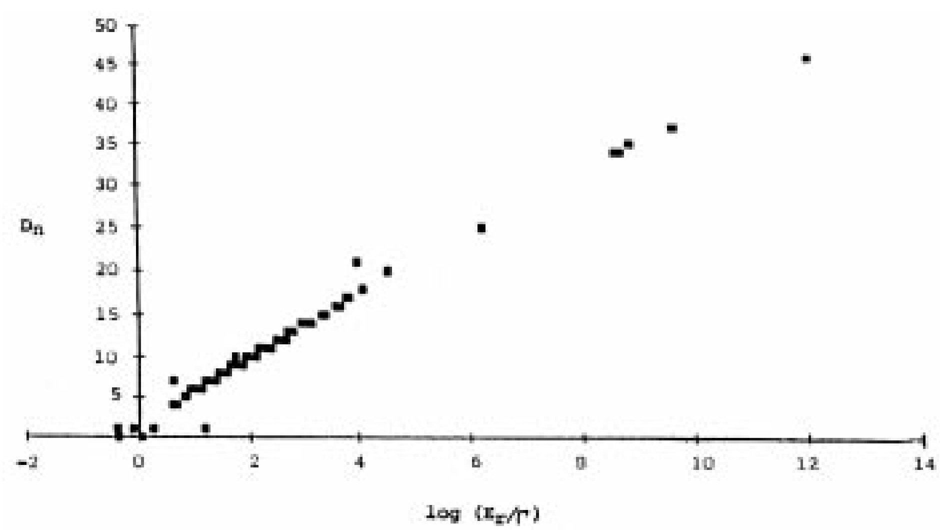

Figure 2. A derived integer, D_n (see text) is plotted against $\log E_r/\Gamma$ for resonances, the trend is clearly brought out.

2.2 *QCD approach*

Quarks and gluons are the ingredients of protons and neutrons (which is 99% of the known mass) and the theory which describes the strong interactions binding the nuclei together is QCD. Since the heavier quarks play a minor role in determining the structure of proton and neutron, it is enough to consider a truncated version of QCD where only up and down quarks are taken into account along with gluons. The theory of colour gluons is derived from the Yang–Mills or a non-abelian gauge symmetry. Gauge invariance also demands that the vector particles like colour gluons have no mass. Based on phenomenological evidence that up and down quark mass terms are small, one may go further and set them to zero, leaving QCD with no mass terms called QCD lite by Wilczek [8]. Surprisingly, even with QCD lite, the mass of protons and neutrons turn out to be accurate to within 10% [9]. Thus, most of the mass of ordinary matter is basically the energy associated with quark motion and colour gluon fields – we make use of this finding in our proposal in §3. And, although the masses seem to come without mass, the mass is acquired in the same way as photons acquire it in a superconductor. This means that there exists a background condensate of Higgs field – a phase-coherent object – giving us all the mass including that of electrons. In QCD lite, there is only one dimensionless parameter, α_s, that governs the strength of the strong interaction. According to QCD, the proton mass in Planck units can be written as [8]

$$m_{\text{proton}} \sim \exp\left(-\frac{k}{\alpha_{\text{unified}}}\right) M_{\text{Planck}},$$

where $M_{\text{Planck}} = \sqrt{(\hbar c/G)}$, $\alpha_{\text{unified}} = 1/25$ is the value where the strong, weak and electromagnetic interactions unify, and $k = 11/2\pi$ is a calculable factor characterizing antiscreening.

Since M_{Planck} is roughly $10^{18} m_{\text{proton}}$, this above formula explains how large ratios originate. Although this formula works remarkably well for ratios, it does not give unique values for physical quantities. Rather it gives several consistent solutions giving many different possible worlds.

3. Towards semiclassical chromodynamics

An important aspect is that the empirical results in §2.1 give us our own world in contrast to several worlds in §2.2. We now propose a program where we plan to begin with QCD, take into account the lessons from §2.2, and use the number-theoretic patterns discovered in [4,5], to semiclassically quantize the field theory. Eventually, one has to argue a way to obtain energy levels of lighter nuclei. Owing to the fact that nuclei belong to the non-perturbative regime of QCD, we begin our journey with a reminder of one of the profound results of mathematical physics of the last century – the Gutzwiller trace formula [2]. We feel that it is important to mention that the logical program remains the same for any other field theory, so the considerations are quite general.

3.1 *Quantizing chaos via Gutzwiller trace formula*

It is an outstanding problem to find analytically exact expressions for quantum states at energies where the underlying classical motion is fully chaotic. The most popular method is semiclassical simply because fully chaotic systems are in a non-perturbative regime and are generally inaccessible to analytic methods. We believe that for the case of nuclear properties in so far as they may be obtained in a non-perturbative manner from QCD, semiclassical theories may be well-suited.

With the exception of a recent work [10], it is possible to quantize chaotic dynamical systems only by the powerful Gutzwiller trace formula. To understand this, consider a one-body system described by a Hamiltonian, $H = \mathbf{p}^2/2 + V(\mathbf{q})$, which supports classically chaotic dynamics; ($\mathbf{q}$, $\mathbf{p}$) are the phase space coordinates. To obtain the energy levels E_i of the system, the density of energy levels is expressible as the trace formula:

$$\sum_i \delta(E - E_i) \sim \sum_{r,\mathrm{po}} \frac{\cos[\frac{r}{\hbar}S_{\mathrm{po}} - \mu_{\mathrm{po}}]}{\sqrt{|\,\mathbf{I} - \mathbf{M}^r_{\mathrm{po}}\,|}} \tag{8}$$

where 'po' and r denote the periodic orbits and their repetitions obtained by solving the classical equations of motion. S_{po} and μ_{po} are respectively the classical actions and Maslov indices corresponding to the periodic orbits. $\mathbf{M}$ is the monodromy matrix giving the stability properties of the periodic orbits. Thus, knowing the periodic orbits and its characteristics, one can quantize any system. The important point is that the above expression is only a dominant term in the semiclassical expansion, corrections can all be found [11]. There are many examples where this formula has given significant results [12]. It should be noted that the semiclassical framework has been developed for certain problems in nuclear physics based on the trace formula [13].

3.2 *Generalizing trace formula for field theories*

Whereas trace formula helps in quantizing linear partial differential equations, nonlinear partial differential equations (PDE) need a generalization. The equations for QCD are highly nonlinear and any success in treating them depend on this development. Of course, there have been analytic solutions known for a long time [14] for the classical SU(2) Yang–Mills equations of motion, they correspond only to the equilibrium points or the saddles in the semiclassical quantization. One needs to go further.

An example has been considered recently [15,16] where the authors considered a nonlinear PDE, the Kuramoto–Sivashinsky equation. This equation arises in describing amplitudes for interfacial instabilities in various contexts [17]. A study of unstable modes of this equation were initiated [18] in order to study the temporally stationary solutions. In [15], it was shown that in the limit of weak turbulence or 'spatiotemporal chaos', cycles of longer and longer periods are determined and used to evaluate global averages.

The first step is to find the classical solutions. For the case of nonlinear PDE or QCD, the solutions will be spacetime solutions, which are patterns; swirls in case of turbulence. These patterns will in general be unstable. Next step is to classify which needs a codification of patterns. In terms of code, one can then find recurrent patterns. These are important as they form the invariant set and would dictate long-time spatially extended dynamics. One can then determine the stability characteristics by finding whether the pattern is stable or unstable along certain eigen-directions. In our context, simplest theory to consider is the QCD lite and develop (what we call as) semiclassical chromodynamics lite (SCD lite). Here, then, the observable will be energy corresponding to quark and gluon motion. Energy corresponding to quantized modes of the color fields would give masses. The stability of the modes would correspond to lifetimes. It is here, as one may anticipate (§2.2), that the empirical relations will be most invaluable.

The problem is to get the correct dynamics, much help is available thanks to so much study existing on QCD, and, on empirical relations.

4. Summary

The good agreement between experimentally measured quantities and those coming from (1) suggests that the expression has deeper meaning. As mentioned in §2.1, the ratio $\frac{n}{2^n}$ or generally $\frac{f(n)}{2^{f(n)}}$ can be interpreted following Cantor in that if n is discrete set of numbers, 2^n is the corresponding continuous set of numbers. In fact, a theorem states: If A contains n elements where n is a positive integer then B which has a continuum of numbers, i.e. is of higher cardinality, contains 2^n elements. If A consists of the set of all integers then B is equivalent to a continuum of all real numbers from 0 to 1.

We have proposed here a possible semiclassical field theory which can handle non-perturbative effects so important in nuclear physics. The proposition here has to take shape in years to come, though.

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