PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 56, No. 6
June 2001
pp. 755–766

Relativistic nonlinearity and wave-guide propagation of rippled laser beam in plasma

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MS received 20 September 1999; revised 11 January 2001

Abstract. In the present paper we have investigated the self-focusing behaviour of radially symmetrical rippled Gaussian laser beam propagating in a plasma. Considering the nonlinearity to arise from relativistic phenomena and following the approach of Akhmanov *et al*, which is based on the WKB and paraxial-ray approximation, the self-focusing behaviour has been investigated in some detail. The effect of the position and width of the ripple on the self-focusing of laser beam has been studied for arbitrary large magnitude of nonlinearity. Results indicate that the medium behaves as an oscillatory wave-guide. The self-focusing is found to depend on the position parameter of ripple as well as on the beam width. Values of critical power has been calculated for different values of the position parameter of ripple. Effects of axially and radially inhomogeneous plasma on self-focusing behaviour have been investigated and presented here.

Keywords. Laser-matter interaction; nonlinear phenomena; relativistic effect; self-focusing.

PACS Nos 42.65.Jx; 52.60+h; 52.40.Nk

1. Introduction

The propagation of a high irradiance laser beam in a plasma whose optical index depends non-linearly on the light intensity had been investigated by various workers through both theoretical and numerical analysis [1–3]. For intense laser pulses of nearly 10^{18} W/cm^2 and of short duration (pico second or less), the drift velocity of electrons in the plasma can be comparable to the velocity of the light, causing significant increase in the mass of the electron. Consequently it causes the increase in the dielectric constant of the plasma, which leads to self-focusing of the laser beam [4].

The fact that the relativistic mechanism is the only mechanism of self-focusing, which can manifest itself for intense pico-second laser pulse, makes the study very important [5,6]. It could produce ultra high laser irradiance exceeding 10^{19}–10^{20} W/cm^2 over distance much greater than the Rayleigh length determined by natural diffraction. As a result, it would favour the observation of new plasma effects such as high-order harmonic generation by relativistic electrons [7], intense magnetic fields generated by circularly polarized light [8,9], frequency up-shifting [10] and high energy electron acceleration [11].

Most of the theoretical investigation of the self-focusing of laser beams in nonlinear media have been confined to cylindrical beams with a smooth (mostly Gaussian) intensity profile. However, direct and indirect experimental evidence reveals that an apparently smooth looking laser beam are not smooth but has superimposed intensity spikes. Superimposed waves may lead to distortion of self-focusing in nonlinear media [12]. Sodha *et al* [13,14] have investigated self-focusing as well as growth and decay of a radially symmetric ripple, superimposed on a Gaussian laser beam in the plasma. They observed that growth or decay rate depends on the intensity of the main beam, the phase angle Φ_{p} between the electric vector of the main beam and the ripple, the frequency of the laser beam, the electron density in the plasma and the width and position of the ripple. Recently the effect of a magnetic field on the growth of a ring ripple superimposed on a Gaussian electromagnetic wave has been analysed by Singh and Singh [15].

In the present paper we studied the self-focusing of a Gaussian beam with a ring ripple superimposed on it, in a homogeneous as well as inhomogeneous plasma for arbitrary large magnitude of relativistic nonlinearity. The steady state paraxial theory developed by Akhmanov *et al* [16] and presented elsewhere [17], has been employed. The effect of position and the width of the ripple on its growth has been studied in some detail. We have also studied the effect on self-focusing through the ratio of initial width of main beam and ripple.

Section 2 deals with the equation of rippled Gaussian laser beam and nonlinear dielectric constant of plasma considering relativistic effect. In §3, self-focusing theory used in the present analysis has been discussed in some length. Methods used for numerical calculation of critical power has also been discussed in this section. Results of the analysis of self-focusing are presented in §4. Results are compared with the available data and discussed.

2. Equation of rippled Gaussian laser beam and nonlinear dielectric constant

Consider the propagation of a linearly polarized ring rippled superimposed Gaussian laser beam with its electric vector polarized along the y-axis, in a homogeneous plasma along the z-axis.

The electric field of the main beam can be represented by

$$E_0|_{z=0} = E_{00} \exp\left(-\frac{r^2}{2r_0^2}\right) \exp(i\omega t), \tag{1}$$

where ω is the angular frequency of the laser beam, r is the radial co-ordinate of the cylindrical co-ordinate system and r_0 is the initial width of the main beam. The electric field of the ring superimposed on the main beam may be expressed as

$$E_1|_{z=0} = E_{10} \left(\frac{r}{r_{10}}\right)^n \exp\left(-\frac{r^2}{2r_{10}^2}\right) \exp(i\omega t), \tag{2}$$

where phase difference between the main beam and the ring is zero, r_{10} indicates the width of the ring, n is the positive number. As n increases the maximum $(r_{10\,\max} = r_{10}\sqrt{n})$ of the ripple shifts away from the axis. The total electric vector of the beam can thus be written as

$$E = E_0 + E_1$$

or

$$E = E_{00} \exp\left(-\frac{r^2}{2r_0^2}\right) \exp(i\omega t) + E_{10}\left(\frac{r}{r_{10}}\right)^n \times \exp\left(-\frac{r^2}{2r_{10}^2}\right) \exp(i\omega t). \tag{3a}$$

The intensity distribution of the beam is given by

$$\begin{aligned} EE^*|_{z=0} = E_{00}^2 \exp\left(-\frac{r^2}{r_0^2}\right) &\left\{1 + \frac{2E_{10}}{E_{00}}\left(\frac{r}{r_{10}}\right)^n \exp\left[\frac{r^2}{2r_0^2}\left(1 - \frac{r_0^2}{r_{10}^2}\right)\right]\right. \\ &\left. + \frac{E_{10}^2}{E_{00}^2}\left(\frac{r}{r_{10}}\right)^{2n} \exp\left[\frac{r^2}{r_0^2}\left(1 - \frac{r_0^2}{r_{10}^2}\right)\right]\right\}. \end{aligned} \tag{3b}$$

The dielectric constant of the plasma due to relativistic effect of propagation of intense laser beam can be given as [17]

$$\epsilon = \tilde{n}^2 = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}\left[1 + \frac{e^2 E_0 E_0^*}{2m^2\omega^2 c^2}\right]^{-1/2}. \tag{4}$$

Following Sodha *et al* [3], the dielectric constant of the medium can be written as

$$\epsilon(\langle EE^* \rangle) = \epsilon_0 + \Phi(\langle EE^* \rangle),$$

where the linear part of the dielectric constant of the plasma is

$$\epsilon_0 = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2} \tag{5}$$

and nonlinear part of the dielectric constant of the plasma due to relativistic effect

$$s\phi(\langle EE^* \rangle) = \frac{\omega_{\mathrm{p}}^2}{\omega^2}\left\{1 - \left(1 + \frac{\alpha EE^*}{2}\right)^{-1/2}\right\}. \tag{6}$$

Here $\alpha = e^2/(m^2\omega^2 c^2)$, is a term which depends upon the laser frequency (ω).

3. Self-focusing equations

The electric vector E satisfies the wave equation

$$\nabla^2 E - \nabla(\nabla \cdot E) + \frac{\omega^2}{c^2}\epsilon E = 0. \tag{7}$$

In the WKB approximation the second term on the left hand side of the above equation can be neglected, thus

$$\nabla^2 E + \frac{\omega^2}{c^2}\epsilon E = 0. \tag{8}$$

It may be mentioned that neglect of the term $\nabla(\nabla \cdot E)$ in (7) is justified when

$$\frac{c^2}{\omega^2}\left|\frac{1}{c}\nabla^2 \ln \epsilon\right| \ll 1.$$

For the set of parameters considered in the present paper this inequality is valid.

Following Sodha *et al* [18], one can write the effective dielectric constant for arbitrary large magnitude of nonlinearity around $r \approx 0$ (paraxial-ray approximation) where

$$\epsilon = \epsilon_0'(f) + \psi(f)$$

and

$$\epsilon_0'(f) = \epsilon_0 + \Phi\left(\left\langle \frac{k(0)E_{00}^2}{k(f)2f^2}\right\rangle\right)$$

$$\psi(f) = \Phi(\langle EE^*\rangle) - \Phi\left(\left\langle \frac{k(0)E_{00}^2}{k(f)2f^2}\right\rangle\right) \ll \epsilon_0'(f). \tag{9}$$

Here f is the dimensionless beam-width parameter and k is the propagation constant defined below in (10). Again using the WKB approximation and following Akhmanov *et al* [16] and Sodha *et al* [3] we can write

$$E(r,z) = A(r,z)\left[\frac{k(0)}{k(f)}\right]^{1/2} \exp[-ik(f)z], \tag{10}$$

where

$$k(f) = \frac{\omega}{c}[\epsilon_0'(f)]^{1/2}, \quad k(0) = \frac{\omega}{c}[\epsilon_0'(f=1)]^{1/2}.$$

Substituting for E and ϵ in (8) one obtains parabolic equations

$$-2ik(f)\frac{\partial A}{\mathrm{d}z} + \nabla_1^2 A + \frac{\omega^2}{c^2}\psi(f)A = 0. \tag{11}$$

Putting $A(r,z) = A_0(r,z)\exp[-ik(f)S]$ in the above equation and separating real and imaginary parts on both sides, one obtains (here S is eikonal)

$$\frac{\partial S}{\partial z} + \left[\frac{\partial S}{\partial r}\right]^2 = \frac{1}{k^2(f)A_0}\left[\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right] + \frac{\omega^2}{k^2(f)c^2}\Psi(r,f), \tag{12a}$$

$$\frac{\mathrm{d}A_0^2}{\mathrm{d}z} + \frac{\mathrm{d}S}{\mathrm{d}r}\frac{\mathrm{d}A_0^2}{\mathrm{d}r} + A_0^2\left(\frac{\mathrm{d}^2 S}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}S}{\mathrm{d}r}\right) = 0. \tag{12b}$$

Using the identity

$$\frac{\partial A_0^2}{\partial r} = 2A_0\frac{\partial A_0}{\partial r}.$$

Equation (12a) can be written as

$$2\frac{\mathrm{d}S}{\mathrm{d}z}+\left(\frac{\mathrm{d}S}{\mathrm{d}r}\right)^2 = \frac{1}{2k^2(f)A_0^2}\left[\frac{\mathrm{d}^2A_0^2}{\mathrm{d}r^2}-\frac{1}{2A_0^2}\left(\frac{\mathrm{d}A_0^2}{\mathrm{d}r}\right)^2+\frac{1}{r}\frac{\mathrm{d}A_0^2}{\mathrm{d}r}\right] + \frac{\omega^2}{k^2(f)c^2}\psi(f). \tag{12c}$$

Adopting an approach similar to that of Akhmanov *et al* [16] the solution of (12a) and (12b) can be written in terms of inverse radius of curvature of the wavefront (β) and width of the main beam $(r_0 f)$. Substituting the values of A_0^2, S and β from the solutions of (12a) and (12b) and E from (3a) in (6), Φ can be expanded following Sodha *et al* [19] as

$$\Phi(A_0^2) = \Phi(A_0^2|(r=\sqrt{n}fr_{10}) + r^2\Phi'(A_0^2|(r=r_{10}f\sqrt{n}),$$

where

$$\Phi' = \left.\frac{\mathrm{d}\phi}{\mathrm{d}r^2}\right| r = r_{10}f\sqrt{n}.$$

Then

$$\begin{aligned}\Phi(\langle EE^*\rangle) &= \Phi\left(\frac{k(0)A_0^2}{k(f)2}\right)\frac{\omega_\mathrm{p}^2}{\omega^2}\\ &= \Phi\left(\frac{k(0)E_{00}^2}{k(f)2f^2}X\right) - \frac{r^2}{4r_0^2f^4}\frac{\omega_p^2}{\omega^2}\frac{k(0)}{k(f)}\alpha E_{00}^2 D\\ &\quad\times\left(1+\frac{k(0)\alpha E_{00}^2}{k(f)2f^2}X\right)^{3/2},\end{aligned} \tag{13}$$

where

$$\begin{aligned}D &= \exp\left(\frac{-nr_{10}^2}{r_0^2}\right) + n^{n/2}\frac{E_{10}}{E_{00}}\exp\left[-\frac{n}{2}\left(\frac{r_{10}^2}{r_0^2}+1\right)\right]\\ X &= \exp\left(\frac{-nr_{10}^2}{r_0^2}\right) + \frac{2E_{10}}{E_{00}}n^{n/2}\exp\left[-\frac{n}{2}\left(\frac{r_{10}^2}{r_0^2}+1\right)\right]\\ &\quad + \left(\frac{E_{10}}{E_{00}}\right)^2 n^n\exp(-n)\end{aligned}$$

and

$$\Phi\left(\frac{k(0)E_{00}^2}{k(f)2f^2}\right) = \frac{\omega_\mathrm{p}^2}{\omega^2}\left[1-\left\{1+\frac{k(0)\alpha E_{00}^2}{k(f)2f^2}\right\}^{-1/2}\right].$$

Substituting A_0^2, S and β from Akhmanov *et al* [16] and Φ from (13) in (12a) and using paraxial-ray approximation i.e. $(r/r_0 f)^4 \ll 1$ and equating the coefficient of r^2 on both sides of the resulting equation yields

$$\frac{\mathrm{d}^2f}{\mathrm{d}z^2} = \frac{c_1-c_2+1}{k^2(f)r_0^4f^3} - \frac{\omega_\mathrm{p}^2}{k^2(f)c^2}\frac{k(0)\alpha E_{00}^2 D}{k(f)4r_0^2f^3}\left(1+\frac{k(0)\alpha E_{00}^2}{k(f)2f^2}X\right)^{-3/2}. \tag{14}$$

In the above equation, ω_p^2 will be replaced by $\omega_{\mathrm{p}0}^2(1+z/L)$.

In case of radial inhomogeneous plasma i.e. linearly decreasing electron charge density in the radial direction from the axis, the dielectric constant (linear as well as nonlinear part) of the plasma can be written as $\epsilon(\langle EE^*\rangle) = \epsilon_0 + \Phi(\langle EE^*\rangle) - \epsilon_2 r^2$, here ϵ_2 is the radial inhomogeneity constant. Following the above mentioned procedure the self-focusing equation for the radial inhomogeneous plasma can be written as

$$\begin{aligned}\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} = \frac{c_1 - c_2 + 1}{k^2(f) r_0^4 f^3} - \frac{\omega_\mathrm{p}^2}{k^2(f)c^2}\frac{k(0)\alpha E_{00}^2 D}{k(f)4r_0^2 f^3}\left(1 + \frac{k(0)\alpha E_{00}^2}{k(f)2f^2}X\right)^{-3/2} \\ -\frac{\omega^2\epsilon_r f}{k^2(f)c^2},\end{aligned} \tag{15}$$

where

$$\begin{aligned}c_1 = {} & \frac{E_{10}}{E_{00}}\left(\frac{r_0}{r_{10}}\right)^n \left\{\frac{n^2}{8}R_1^2 - (n+1)R_1R_2 + R_2^2 + 2R_1 - 4\right\} + \left(\frac{E_{10}}{E_{00}}\right)^2 \\ & \times \left(\frac{r_0}{r_{10}}\right)^{2n}\left\{-2(2n+1)R_1\frac{r_0^2}{r_{10}^2} + \frac{2r_0^4}{r_{10}^4} + 4(n+1)R_1R_2 - 2R_2^2 + 2R_1 - 2\right\} \\ & + \left(\frac{E_{10}}{E_{00}}\right)^3\left(\frac{r_0}{r_{10}}\right)^{3n}\left\{-\frac{45}{8}n^2R_1^2 + 6(2n+1)R_1\frac{r_0^2}{r_{10}^2}\right. \\ & \left. -4\frac{r_0^4}{r_{10}^4} + 3(n+1)R_1R_2 - R_2^2\right\} \\ & + \left(\frac{E_{10}}{E_{00}}\right)^4\left(\frac{r_0}{r_{10}}\right)^{4n}\left\{-4n^2R_1^2 + 4(2n+1)R_1\frac{r_0^2}{r_{10}^2} - 2\frac{r_0^4}{r_{10}^4}\right\}, \\ c_2 = {} & \frac{E_{10}}{E_{00}}\left(\frac{r_0}{r_{10}}\right)^n\{-nR_1 + 2R_2 - 4\} + \left(\frac{E_{10}}{E_{00}}\right)^2\left(\frac{r_0}{r_{10}}\right)^{2n} \\ & \left\{+\frac{n^2}{2}R_1^2 - 2nR_1R_2 + R_2^2 + 6nR_1 + 2\frac{r_0^2}{r_{10}^2} - 8R_2 - 6\right\} \\ & + \left(\frac{E_{10}}{E_{00}}\right)^3\left(\frac{r_0}{r_{10}}\right)^{3n}\left\{-\frac{9}{4}n^2R_1^2 + 9nR_1R_2 - 3nR_1\frac{r_0^2}{r_{10}^2} + 2R_2\frac{r_0^2}{r_{10}^2}\right. \\ & \left. +30nR_1 - 12R_2 - 4R_2^2 - 8\frac{r_0^4}{r_{10}^4} - 4\right\} + \left(\frac{E_{10}}{E_{00}}\right)^4\left(\frac{r_0}{r_{10}}\right)^{4n} \\ & \times\left\{-26n^2R_1^2 + 12nR_1\frac{r_0^2}{r_{10}^2} + \frac{r_0^4}{r_{10}^4} + 40nR_1R_2 - 6R_2^2\right. \\ & \left. +40nR_1 - 12\frac{r_0^2}{r_{10}^2} - 8R_2\frac{r_0^2}{r_{10}^2} - 8R_2 - 1\right\}.\end{aligned}$$

Here $R_1 = 1 - (r_0^2/r_{10}^2)$ and $R_2 = 1 + (r_0^2/r_{10}^2)$.

For an initial plane wavefront of the beam, the initial conditions on f are $f(z=0) = 1$ and $\mathrm{d}f/\mathrm{d}z/_{z=0} = 0$. When the two terms on the right side of (14) and (15) cancel each other at $z = 0$, $\mathrm{d}^2 f/\mathrm{d}z^2 = 0$. If one further consider a parallel beam at $z = 0$, then $\mathrm{d}f/\mathrm{d}z = 0$ at $z = 0$, thus, if $f = 1$ at $z = 0$; it remains so for all values of z (in other words, the beam propagates without convergence or divergence). The critical power of self-trapping in homogeneous and axial inhomogeneous plasma is therefore

$$(c_1 - c_2 + 1) \Big/ \left(\frac{\omega_\mathrm{p} r_0}{c}\right)^2 = \frac{D\alpha E_{00\mathrm{cr}}^2}{4}\left(1 + \frac{\alpha E_{00\mathrm{cr}}^2 X}{2}\right)^{-3/2}. \tag{16}$$

The corresponding critical power is

$$\begin{aligned} P_\mathrm{cr} &= \frac{c}{8\pi}\int_0^\infty \epsilon^{1/2} E^2 2\pi r \mathrm{d}r \\ &= \frac{1}{8} c r_0^2 E_{00\mathrm{cr}}^2 [\epsilon_0'(f=1)]^{1/2} \\ &\times \left[1 + \frac{2^{\frac{n+4}{2}}\left(\frac{E_{10}}{E_{00}}\right)\left(\frac{r_0}{r_{10}}\right)^n \Gamma\left(\frac{n+2}{2}\right)}{\left(1+\frac{r_0^2}{r_{10}^2}\right)^{\frac{n+2}{2}}} + \frac{E_{10}^2}{E_{00}^2}\left(\frac{r_0}{r_{10}}\right)^2 \Gamma(n+1)\right]. \end{aligned} \tag{17}$$

Here Γ is gamma function.

4. Results and discussion

To study the behaviour of the focusing parameter f with propagating distance z, eq. (14) has been solved numerically. For computation, a typical case $\omega = 1.7 \times 10^{14}\ \mathrm{s}^{-1}$, $\omega_\mathrm{p}/\omega = 0.5$, $r_0 = 30\ \mu$m, $r_{10}/r_0 = 0.7$, $E_{10}/E_{00} = 0.4$ at power $p = 6 \times 10^{12}$ W has been chosen.

Figure 1 represents the variation of dimensionless beam width parameter f with the distance of propagation in a plasma for different values of n (the position of maxima of ripple with respect to maxima of main beam. As n increases the maxima of ripple shift away from the axis where the maxima of the main beam lies). When the ripple is superimposed such that the maxima of the ripple coincides with the maxima of main beam, the ripple has negligible influence on the redistribution of the carriers and its growth is predominantly determined by the main beam. However, if the maxima does not coincide, the intensity of the ripple is crucial for the redistribution of the carriers because the intensity of the main beam decreases away from the axis and may be comparable to the intensity of the ripple around its maximum. Thus it is expected that the growth of the ripple must be significantly affected by the position and width of ripple. Results in figure 1 also indicate stronger self-focusing as n increases i.e. the maxima of the ripple and the main beam get further apart. The focusing of rippled Gaussian laser beam for a given initial beam width is determined by the relative magnitudes of the diffraction and nonlinear terms in eq. (15). The nonlinear term is comprised of two parts, namely (1) the gradient of the nonlinear part of the effective intensity and (2) the gradient of effective intensity with radial distance evaluated at the point where the intensity maximum of the ripple occurs.

As n increases, the radial gradient of the effective intensity at the point under consideration in the plasma increases and becomes dominant over the second above mentioned gradient. Hence the stronger focusing is expected for higher values of n, as observed [19].

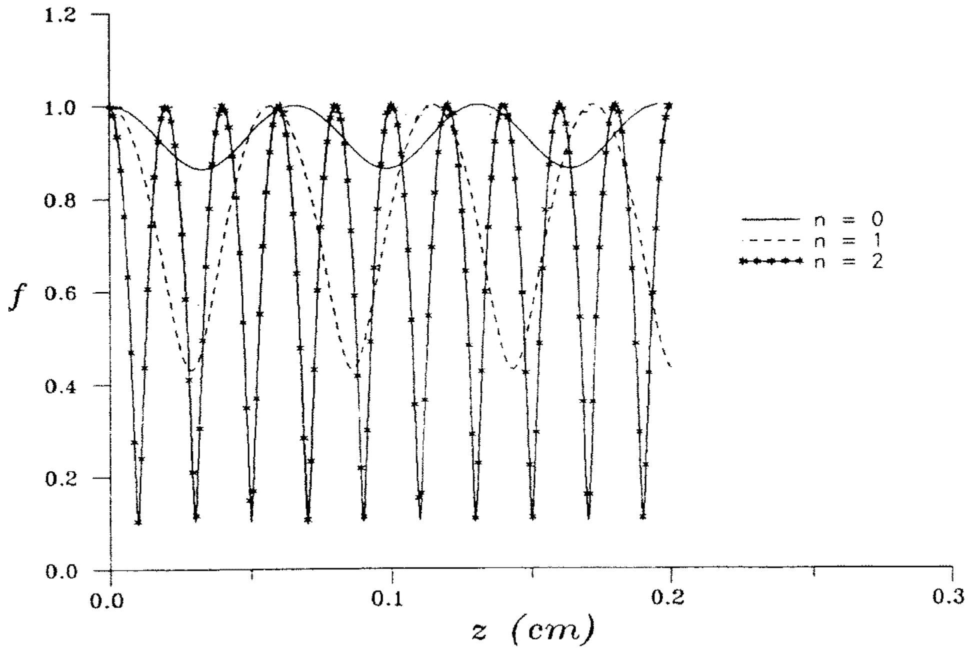

Figure 1. Variation focusing parameter (f) vs propagation distance (z) for relativistic nonlinearity with different values of position parameters (n). Here $r_0 = 0.003$ cm, $E_{10}/E_{00} = 0.4$, $r_{10}/r_0 = 0.7$ and $P = 6 \times 10^{12}$ W.

The observed oscillatory behaviour of f with the distance of propagation (see figure 1) for the given value of E_{10}/E_{00}, $\omega_{\rm p}$, ω, r_0/r_{10} and r_0 for different values of n can be explained as follows:

The second term on the right hand side of (14) dominates the first term at $z = 0$ and $\mathrm{d}^2 f/\mathrm{d}z^2$ is negative, consequently f decreases with the distance of propagation. When the two terms on the right hand side of (14) balance each other $\mathrm{d}^2 f/\mathrm{d}z^2$ becomes zero, and for larger values of z, this expression becomes positive. However, the ripple still continues to converge on account of the curvature it has already gained. With the increasing value of z, $(\mathrm{d}f/\mathrm{d}z)$ becomes less and less negative. Finally at $z = z_f$, $(\mathrm{d}f/\mathrm{d}z) = 0$ and $f = f_{\min}$. Beyond this point f increases with z and attains a maximum value for $f = 1$ at $z = 2z_f$ and then repeats its behaviour. Thus the ripple propagates in a kind of self-made oscillatory wave guide.

Variation of dimensionless $\alpha E_0 E_0^*$ (which is proportional to beam intensity of rippled Gaussian beam within plasma) with respect to distance of propagation z at $r = r_{10} f\sqrt{n}$ for different values of n, indicate that intensity peak increases as n increases. This is expected because stronger focusing will lead to more intense beam as there are no losses in the medium.

The value of critical power also depend on the position of the ripple beam on the main beam (n). It increases with increasing the value of n as noticed from the results presented in table 1.

Table 1. Value of critical power for different position of the ripple beam parameter (n).

n	Critical power in watt
0	2.64×10^{11}
1	3.96×10^{11}
2	9.09×10^{11}
3	5.86×10^{12}

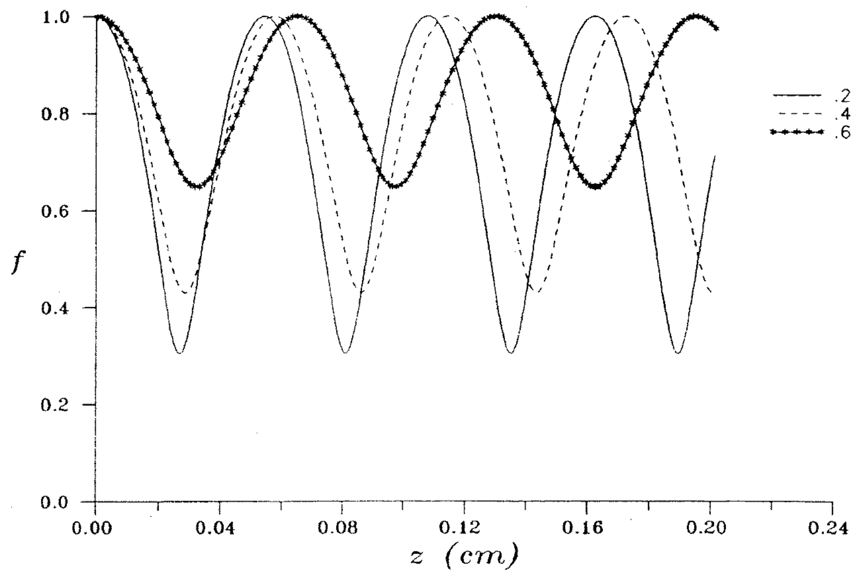

Figure 2. Plots of focusing parameters (f) with propagation distance (z) for different values of amplitude ratio (E_{10}/E_{00}) and relativistic nonlinearity. Here $r_{10}/r_0 = 0.7$, $n = 1$, $r_0 = 0.003$ cm and $P = 6 \times 10^{12}$ W.

Keeping the value of n constant $(n = 1)$ the variation of f with z for different values of r_{10}/r_0 are obtained. It is observed that for small values of r_{10}/r_0, self-focusing effect is more prominent as compared to higher values of r_{10}/r_0.

These results indicate that the effect of changing the ripple width i.e. ratio r_{10}/r_0 on its growth is more important than the position of the ripple (n). This is expected because the change in width of the ripple (r_{10}) would affect the diffraction of the ripple in addition to the nonlinear coupling between main beam and the ripple.

Figure 2 shows the variation of focusing parameter (f) with distance of propagation (z) for particular value of power $P = 6 \times 10^{12}$ W and beam width $r_0 = 0.003$ cm, position parameter $n = 1$, ratio of width of the main beam and ripple $r_{10}/r_0 = 0.7$, for the different values of amplitude ratio of ripple to main beam (E_{10}/E_{00}). For the lower value of amplitude ratio of ripple to main beam E_{10}/E_{00}, value of $f_{\min}$ is found to be low, i.e. for a fixed amplitude of the main beam (E_{00}), if amplitude of ripple decreases, stronger self-focusing

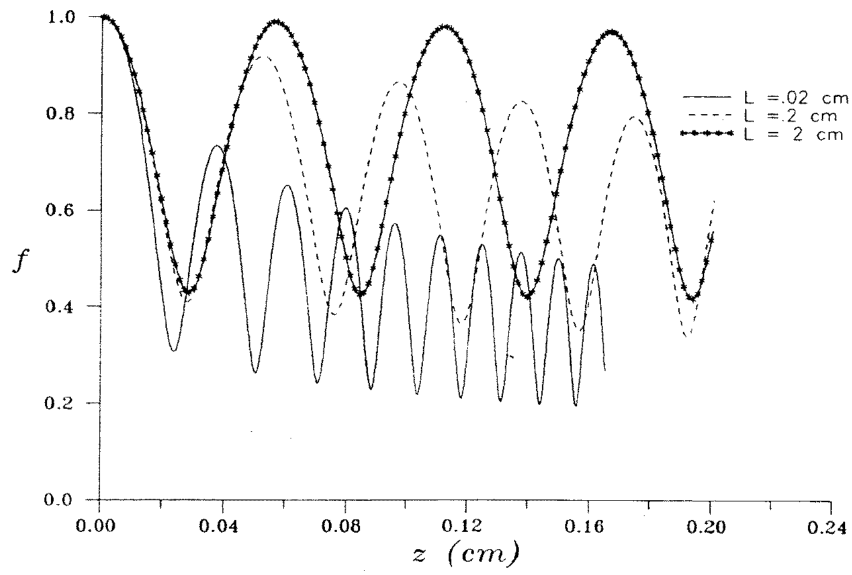

Figure 3. Plots of f vs z for linearly increasing axially inhomogeneous plasma with different values of characteristic scale length (L) and relativistic nonlinearity. Here $n = 1$, $r_{10}/r_0 = 0.7$, $E_{10}/E_{00} = 0.4$, $r_0 = 0.003$ cm.

is observed. At lower value of E_{10}/E_{00}, focusing is observed earlier as compared to higher value of (E_{10}/E_{00}) i.e. $Z_{\min 1}$ is less for lower value of (E_{10}/E_{00}).

Figure 3 gives the plot of f vs z for the linearly increasing axial inhomogeneous plasma of different value of characteristic scale length (L). The alternative beam convergence and divergence has been observed when it propagates in the plasma i.e. oscillatory behaviour is observed. In inhomogeneous plasma the focusing is stronger as compared to homogeneous plasma and as L decreases, this effect becomes more prominent. For lower values of L, focusing length $(Z_{\min})$ decreases i.e. focusing is observed at nearer points. In linearly increasing axial inhomogeneous plasma as z increases the density increases, due to this plasma frequency also increases, which contribute in nonlinear part of self-focusing equation, hence stronger focusing is observed.

To study the self-focusing of laser beam in radial inhomogeneous plasma, eq. (15) has been solved numerically. A set of graphs between f and z for the different values of ϵ_r, representing variation of laser beam aperture as a function of propagating distance in radial inhomogeneous plasma are drawn which predicts a very interesting oscillatory behaviour (figure 4).

Results indicates slow decrease in $f_{\min}$ value with the increase in the values of ϵ_r and focusing is achieved at lower values of z. This can be explained, as the value ϵ_r increases the contribution of self-focusing term also increases so the value of $f_{\min}$ decreases. Linearly decreasing radial inhomogeneity in plasma, simply act as prefixed lens to decrease the self-focusing focal length. Higher the value of ϵ_r stronger the lens effect and earlier the self-focusing.

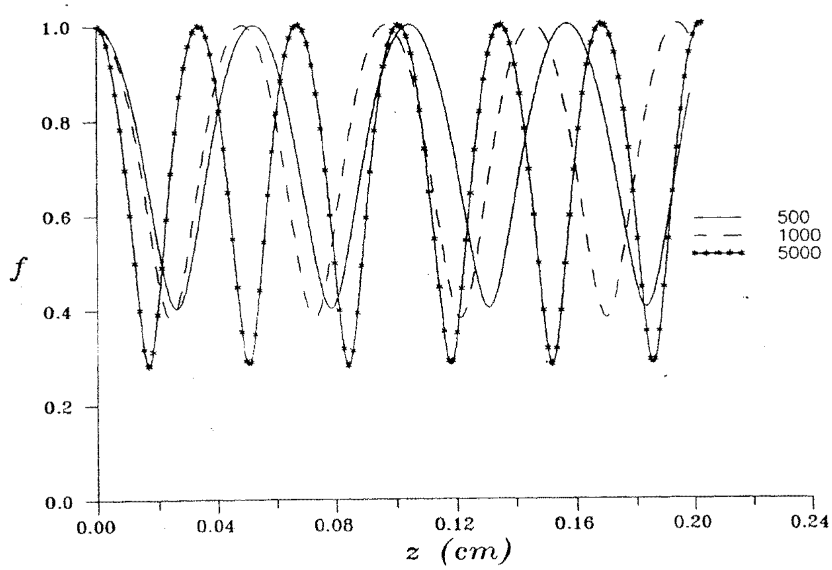

Figure 4. Plots of f vs z in radially inhomogeneous plasma with different values of radial inhomogeneity constant ϵ_r and relativistic nonlinearity. Here $n = 1$, $r_{10}/r_0 = 0.7$, $E_{10}/E_{00} = 0.4$, $r_0 = 0.003$ cm.

Acknowledgement

Part of the study is supported by the Department of Science and Technology, New Delhi. Authors are thankful to Prof. K P Maheshwari, Devi Ahilya Bai University, Indore for useful discussions.

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