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First experimental test of Bell inequalities performed using a non-maximally entangled state

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Abstract. We describe the realisation of a new test of Bell inequalities using a new scheme obtained by the superposition of type I parametric down conversion produced in two different non-linear crystals pumped by the same laser, but with different polarisations. This experiment is the first test of Bell inequalities using a non-maximally entangled state and thus represents an important step in the direction of eliminating the detection loophole.

Keywords. Bell inequalities; non-locality; EPR.

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The idea that quantum mechanics (QM) could be an incomplete theory, representing a statistical approximation of a complete deterministic theory (where observable values are fixed by some hidden variable) appeared already in 1935 thanks to the celebrated Einstein-Podolsky-Rosen paper [1].

Historically, the quest for hidden variable theories stopped when Von Neumann published a theorem asserting the impossibility of constructing a hidden variable theory reproducing all the results of QM. For a long time, the prestige of Von Neumann led to an acritical acceptation of this theorem, but it was then discovered that one of his hypotheses was too restrictive so that the program of a hidden variable theory was still possible.

The next fundamental progress in discussing possible extensions of QM was the discovery of Bell [2] that any realistic local hidden variable (LHV) theory must satisfy certain inequalities which can be violated in QM leading in principle to a possible experimental test of the validity of QM as compared to LHV.

Since then, many interesting experiments have been devoted to a test of Bell inequalities [3–7], leading to a substantial agreement with quantum mechanics and disfavouring LHV theories, but, so far, no experiment has yet been able to exclude definitively such theories. In fact, so far, one has always been forced to introduce a further additional hypothesis [9], due to the low total detection efficiency, stating that the observed sample of particle pairs is a faithful subsample of the whole. This problem is known as *detection or efficiency loophole*. The research for new experimental configurations able to overcome the detection loophole is of course the greatest interest.

In the 90's big progresses in this direction have been obtained by using parametric down conversion (PDC) processes.

This technique [3] has been largely employed for generating 'entangled' photon pairs, i.e. pairs of photons described by a common wave function which cannot be factorized into the product of two distinct wave functions pertaining to separated photons.

The generation of entangled states by parametric down conversion (PDC) has replaced other techniques, such as the radiative decay of excited atomic states, as it was in the celebrated experiment of Aspect *et al* [4], for it overcomes some former limitations. In particular, having angular correlations better than 1 mrad, it overcomes the poor angular correlation of atomic cascade photons, that was at the origin of the small total efficiency of this type of experiments in which one is forced to select a small subsample of the produced photons, leading inevitably to the detection loophole.

The first experiments using this technique, were performed with Type I PDC, which gives phase and momentum entanglement and can be used for a test of Bell inequalities using two spatially separated interferometers [5], as realised by [6]. The use of beam splitters, however, strongly reduces the total quantum efficiency.

In alternative, one can generate a polarisation entangled state [8]. It appears, however, that the creation of couples of photons entangled from the point of view of polarisation, which is by far the most diffuse case due to the easy experimental implementation, still suffers severe limitations, as it was pointed out recently in the literature [9]. The essence of the problem is that in generating this state, half of the initial photon flux is lost (in most of the used configurations), and one is, of necessity, led to assume that the photon's population actually involved in the experiment is a faithful sample of the original one, without eliminating the efficiency loophole.

Recently, an experiment where a polarisation entangled state is directly generated, has been realised using Type II PDC [7]. This scheme has permitted, at the price of delicate compensations for having identical arrival time of the ordinary and extraordinary photon, a much higher total efficiency than the previous ones, which is, however, still far from the required value of 0.81. Also, some recent experiments studying equalities among correlation functions rather than Bell inequalities [11] are far from solving these problems [12]. A large interest remains therefore for new experiments increasing total quantum efficiency in order to reduce and finally overcome the efficiency loophole.

Some years ago, a very important theoretical step in this direction has been performed recognising that, while for maximally entangled pairs a total efficiency larger than to 0.81 is required to obtain an efficiency-loophole free experiment, for non maximally entangled pairs this limit is reduced to 0.67 [10] (in the case of no background). However, it must be noticed that, for non-maximally entangled states, the largest discrepancy between quantum mechanics and local hidden variable theories is reduced: thus a compromise between a lower total efficiency and a still sufficiently large value of this difference will be necessary when realising of an experiment addressed to overcome the detection loophole.

Considering a polarization entangled state of photons of the form

$$|\psi\rangle = \frac{|H\rangle|H\rangle + f|V\rangle|V\rangle}{\sqrt{(1+|f|^2)}}, \tag{1}$$

where H and V indicate horizontal and vertical polarisations respectively, the parameter f describes how much the state 1 differs from a maximally entangled one.

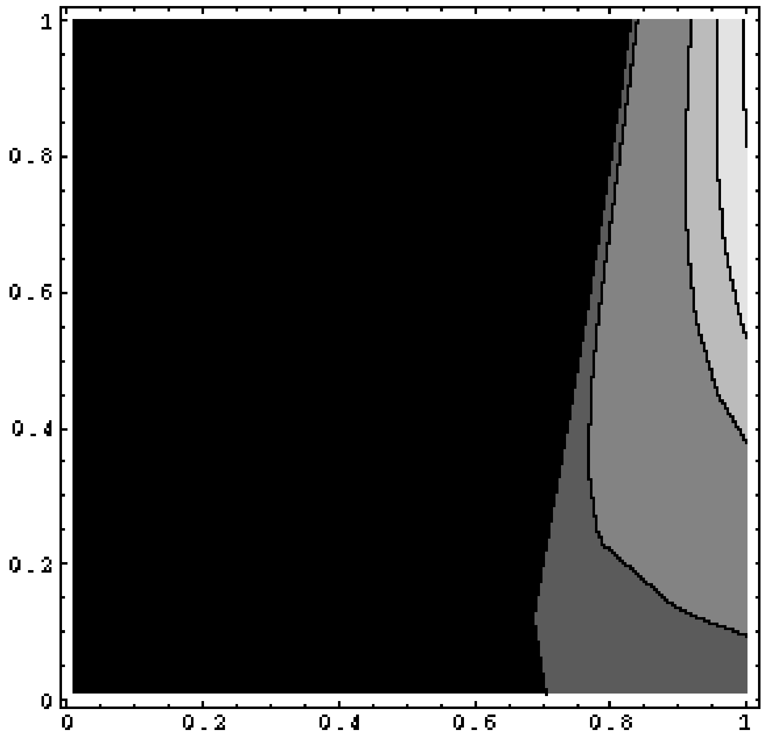

Figure 1. Contour plot of the quantity CH/N (see eq. (2), N is the total number of detections) in the plane with f (non maximally entanglement parameter, see the text for the definition) as y-axis and η (total detection efficiency) as x-axis. The leftmost region corresponds to the region where no detection loophole free test of Bell inequalities can be performed. The contour lines are at 0, 0.1, 0.15, 0.2.

The region corresponding to the one where the detection loophole is eliminated in the plane f and η (total detection efficiency) is shown in figure 1.

Let us now describe more in detail our experimental set-up. It derives from developing [13] a proposal made by Hardy [14] and is based on the creation of a polarisation (non maximally-) entangled states of the form 1 via the superposition of the spontaneous fluorescence emitted by two non-linear crystals driven by the same pumping laser. The crystals are put in cascade along the propagation direction of the pumping laser and the superposition is obtained by using an appropriate optics.

More in detail (see figure 2), two crystals of $LiIO_3$ (10×10×10 mm) are placed along the pump laser propagation, 250 mm apart, a distance smaller than the coherence length of the pumping laser. This guarantees indistinguishability in the creation of a couple of photons in the first or in the second crystal. A couple of planoconvex lenses of 120 mm focal length centered in between, focalises the spontaneous emission from the first crystal into the second one maintaining exactly, in principle, the angular spread. A hole of 4 mm diameter is drilled into the centre of the lenses in order to allow transmission of the pump radiation without absorption and, even more important, without adding stray-light,

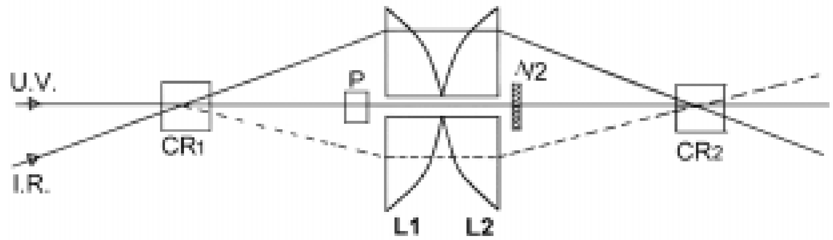

Figure 2. Sketch of the source of polarisation entangled photons. CR1 and CR2 are two $LiIO_3$ crystals cut at the phase-matching angle of 51°. L1 and L2 are two identical plano-convex lenses with a hole of 4 mm in the centre. P is a $5 \times 5 \times 5$ mm quartz plate for birefringence compensation and $\Lambda/2$ is a first order half wave-length plate at 351 nm. U.V. identifies the pumping radiation at 351 nm. The infrared beam (I.R.) at 789 nm is generated by a diode laser and is used for system alignment only. The parametric amplifier scheme, described in the text, is shown as well. The dashed line identifies the idler radiation at 633 nm. A second half-wave plate on the I.R. beam (not shown in the figure) allows amplified idler emission from the second crystals too. The figure is not in scale.

because of fluorescence and diffusion of the UV radiation. This configuration, which realises what is known as an 'optical condenser', was chosen among others, using an optical simulation program, as a compromise between minimisation of aberrations (mainly spherical and chromatic) and losses due to the number of optical components. The pumping beam at the exit of the first crystal is displaced from its input direction by birefringence: the small quartz plate (5×5×5 mm) in front of the first lens of the condensers compensates this displacement, so that the input conditions are prepared to be the same for the two crystals, apart from alignment errors. Finally, a half-wavelength plate immediately after the condenser rotates the polarisation of the argon beam and excites in the second crystal a spontaneous emission cross-polarised with respect to the first one. With a phase matching angle of 51°, the spontaneous emissions at 633 and 789 nm (which are the wave-lengths to be used for the test) are emitted at 3.5° and 4° respectively. The dimensions and positions of both plates are carefully chosen in order that they do not intersect this two conjugated emissions.

We have used as photo-detectors two avalanche photodiodes with active quenching (EG&G SPCM-AQ) with a sensitive area of 0.025 mm^2 and dark count below 50 counts/s. The PDC signal was coupled to an optical fiber (carrying the light to the detectors) by means of a microscope objective with magnification 20. The quantum efficiency, including the fiber coupling, has been measured to be 0.535 ± 0.008 at 633 nm [15].

The output signals from the detectors are pulses of transistor-transistor-logic (TTL) like amplitude levels which are routed to a two channel counter, in order to have the number of events on single channel, and to a time to amplitude converter (TAC) circuit, followed by a single channel analyser, for selecting and counting coincidence events.

A very interesting degree of freedom of this configuration is given by the fact that by tuning the pump intensity between the two crystals, one can easily tune the value of f, which determines how far from a maximally entangled state ($f = 1$) the produced state is.

This is a fundamental property, which permits to select the most appropriate state for the experiment.

The main problem of this configuration is the alignment, which is of utmost relevance for a high visibility. The solution of this problem lies in a technique, that had been already applied in our laboratory [15] for meteorological studies, namely the use of an optical amplifier scheme, where a solid state laser is injected into the crystals together with the pumping laser, an argon laser at 351 nm wavelength (see figure 1). If the angle of injection is selected appropriately, a stimulated emission along the correlated direction appears, allowing an easy identification of the two correlated directions. Then, stopping the stimulated emission of the first crystal, and rotating the polarisation of the diode laser one obtaines the stimulated emission in the second crystal and can check the superposition with the former.

We think that the proposed scheme will lead to a further step towards a conclusive experimental test of non-locality in quantum mechanics. The analysis of the experiments realised up to now [9] shows in fact that visibility of the wanted effect (essentially visibility of interference fringes) and overall quantum detection efficiency are the main parameters in such experiments. One advantage of the proposed configuration with respect to most of the previous experimental set-ups is that all the entangled pairs are selected (and not just $<$ 50% as with beams splitters); furthermore, it does not require delicate compensations for the optical paths of the ordinary and extraordinary rays emerging from the crystal.

For time being, the results which we are going to present are still far from a definite solution of the detection loophole; nevertheless, being the first test of Bell inequalities using a non-maximally entangled state, they represent an important step in this direction. Furthermore, this configuration allows to use any pair of correlated frequencies and not only the degenerate ones. We have thus realised this test using for a first time two different wave lengths (at 633 and 789 nm).

An experiment which presents analogies with ours, has been realised recently in ref. [16]. The main difference between the two experiments is that in [16] the two crystals are very thin and in contact with orthogonal optical axes: this allows a ‘partial’ superposition of the two emissions with opposite polarisation. This overlapping is mainly due to the finite dimension of the pump laser beam, which reflects into a finite width of each wavelength emission. A much better superposition can be obtained with the present scheme, by fine tuning the crystals’ and optics’ positions and using the parametric amplifier trick. Furthermore, in the experiment of ref. [16] the value of f is in principle tunable by rotating the polarisation of the pump laser; however, this reduces the power of the pump producing PDC already in the first crystal, while in our case the whole pump power can always be used in the first crystal, tuning the PDC produced in the second one.

As a first check of our apparatus, we have measured the interference fringes, varying the setting of one of the polarisers, while leaving the other fixed. We have found a high visibility, $V = 0.973 \pm 0.038$.

Our results are summarised by the value obtained for the Clauser-Horne sum,

$$\begin{aligned}\mathrm{CH} = {} & N(\theta_1, \theta_2) - N(\theta_1, \theta_2') + N(\theta_1', \theta_2) + N(\theta_1', \theta_2') \\ & - N(\theta_1', \infty) - N(\infty, \theta_2)\end{aligned} \tag{2}$$

which is strictly negative for local realistic theory. In (2), $N(\theta_1, \theta_2)$ is the number of coincidences between channels 1 and 2 when the two polarisers are rotated to an angle θ_1 and θ_2 respectively (∞ denotes the absence of selection of polarisation for that channel).

On the other hand, quantum mechanics predictions for CH can be larger than zero: for a maximally entangled state the largest value is obtained for $\theta_1 = 67^\circ.5$, $\theta_2 = 45^\circ$, $\theta_1' = 22^\circ.5$, $\theta_2' = 0^\circ$ and corresponds to a ratio

$$R = \frac{[N(\theta_1,\theta_2) - N(\theta_1,\theta_2') + N(\theta_1',\theta_2) + N(\theta_1',\theta_2')]}{[N(\theta_1',\infty) + N(\infty,\theta_2)]} \tag{3}$$

equal to 1.207.

For non-maximally entangled states the angles for which CH is maximal are somehow different and the maximum is reduced to a smaller value. The angles corresponding to the maximum can be evaluated maximising eq. (2) with

$$\begin{aligned} N[\theta_1,\theta_2] = \Big[& \epsilon_1^{||}\epsilon_2^{||}\left(\sin[\theta_1]^2\cdot\sin[\theta_2]^2\right) + \epsilon_1^{\perp}\epsilon_2^{\perp}\left(\cos[\theta_1]^2\cdot\cos[\theta_2]^2\right) \\ & \times\left(\epsilon_1^{\perp}\epsilon_2^{||}\sin[\theta_1]^2\cdot\cos[\theta_2]^2 + \epsilon_1^{||}\epsilon_2^{\perp}\cos[\theta_1]^2\cdot\sin[\theta_2]^2\right) \\ & +|f|^2 * (\epsilon_1^{\perp}\epsilon_2^{\perp}\left(\sin[\theta_1]^2\cdot\sin[\theta_2]^2\right) + \epsilon_1^{||}\epsilon_2^{||}\left(\cos[\theta_1]^2\cos[\theta_2]^2\right) \\ & + \left(\epsilon_1^{||}\epsilon_2^{\perp}\sin[\theta_1]^2\cdot\cos[\theta_2]^2 + \epsilon_1^{\perp}\epsilon_2^{||}\cos[\theta_1]^2\cdot\sin[\theta_2]^2\right) \\ & +(f+f^*)((\epsilon_1^{||}\epsilon_2^{||} + \epsilon_1^{\perp}\epsilon_2^{\perp} - \epsilon_1^{||}\epsilon_2^{\perp} - \epsilon_1^{\perp}\epsilon_2^{||}) \\ & \times(\sin[\theta_1]\cdot\sin[\theta_2]\cdot\cos[\theta_1]\cdot\cos[\theta_2])\Big]/(1+|f|^2), \end{aligned} \tag{4}$$

where (for the case of non-ideal polariser) $\epsilon_i^{||}$ and $\epsilon_i^{\perp}$ correspond to the transmission when the polariser (on the branch i) axis is aligned or normal to the polarisation axis respectively.

In our case, we have generated a state with $f \simeq 0.4$ which corresponds, for $\theta_1 = 72^\circ.24$, $\theta_2 = 45^\circ$, $\theta_1' = 17^\circ.76$ and $\theta_2' = 0^\circ$, to $R = 1.16$.

Our experimental result is CH $= 512 \pm 135$ coincidences per second, which is almost four standard deviations different from zero and compatible with the theoretical value predicted by quantum mechanics. In terms of the ratio (3), our result is 1.082 ± 0.031.

For the sake of comparison, one can consider the value obtained with the angles which optimise Bell inequalities violation for a maximally entangled state (given before eq. (3)). The result is CH $= 92 \pm 89$, which, as expected, shows a smaller violation than the value obtained with the correct angles setting.

In summary, this is the first measurement of the violation of the Clauser-Horne inequality (or other Bell inequalities) using a non-maximally entangled state and thus represents an interesting result as a first step in the direction of eliminating the detection loophole. Further developments in this sense are the purpose of this collaboration.

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