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Effect of nonthermal ion distribution and dust temperature on nonlinear dust acoustic solitary waves

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Abstract. The effects of nonthermal ion distribution and finite dust temperature are incorporated in the investigation of nonlinear dust acoustic waves in an unmagnetized dusty plasma. Sagdeev pseudopotential method which takes into account the full nonlinearity of plasma equations, is used here to study solitary wave solutions. Possibility of co-existence of refractive and compressive solitons as a function of Mach number, dust temperature and concentration of nonthermal ions, is considered. For the fixed value of nonthermal ions, it is found that the effect of increase in dust temperature is to reduce the range of co-existence of compressive and refractive solitons. Particular concentration of nonthermal ions results in disappearance of refractive solitons while the decrease in dust temperature, at this concentration restores the lost refractive solitons.

Keywords. Solitons; dust acoustic wave; Sagdeev potential; dusty plasma; nonthermal ion.

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1. Introduction

In recent years, there has been considerable interest in understanding different type of collective processes in plasmas containing electrons, ions and charged micron-sized grain particles. Such plasmas occur frequently in many astrophysical systems including the interplanetary medium, planetary rings, asteroids, cometary tails, intersteller clouds, nebulae, aurora etc. and they are also produced in plasma discharges, optical fibres, dusty crystals, semiconductors as well as regions of hot fusion plasma and in devices for plasma-assisted material processing [1–8]. For low frequency modes, the grain dust can be described as negative ions with large mass and large charge. The ion and dust acoustic modes have been investigated recently by several authors. In particular it has been shown that a dusty plasma with inertial dust fluid and Boltzmann distributed ions admit only negative solitary potentials associated with nonlinear dust acoustic wave [9]. They ignored dust temperature which may not be negligible [10]. The dust charge, although a dynamical variable, is assumed here to be constant for the sake of simplicity. The presence of nonthermal ion distribution with cold dust, leads to the possibility of co-existence of large amplitude compressive as well as refractive dust acoustic solitary waves [11]. In this research paper, we investigate large amplitude solitary waves not only with finite dust temperature but also incorporating the effect of nonthermal ion distribution. The latter may arise due to quasilinear modifications introduced by dust-acoustic fluctuations themselves.

Most of the studies in multi-species plasmas have focused on deriving $KdV/MKdV$ equations using reductive perturbation technique [12]. The method although an elegant one and most frequently used, has limitations as it does not take into account the full non-linearity of plasma wave equations. Sagdeev pseudopotential method [13], which takes complete nolinearity into consideration, is used here to set up an energy integral to characterize the dust acoustic solitary wave. The structure of the pseudopotential can be explored over a wide range of parameter regime for large amplitude refractive/compressive solitary waves.

2. Basic formulation

One dimensional equations governing the dynamics of the dusty plasma are

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_d)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + 3\sigma n_d \frac{\partial n_d}{\partial x} = \frac{\partial \Phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = n_d - \mathrm{e}^{-\Phi}. \tag{3}$$

We have taken the equation of state as

$$p = n_d^{\gamma} p_0. \tag{4}$$

Here γ is the ratio of specific heats and taken as $\gamma = 3$, $\sigma = T_d/T_i$. Further n_d is the dust particle density normalized by unperturbed dust density n_{d0}, u_d is dust particle velocity normalized by dust acoustic speed $c_d = \sqrt{T_i/m_d}$ and Φ is the electrostatic wave potential normalized by T_i/e, T_i being the ion temperature. The time and space variables are in the units of $\omega_{pd}^{-1} = (\sqrt{m_d/4\pi n_{d0} Z_d e^2})$ the dust plasma period and the Debye length $\lambda_d = (\sqrt{T_i/4\pi Z_d n_{d0} e^2})$ respectively [10]. To study the effect of a nonthermal ion distribution on dust acoustic wave, we choose a more general class of ion distribution [14] which includes the population of nonthermal ions. Thus we take

$$f_i(v) = 1/(\sqrt{2\pi}) \frac{1 + \alpha v^4}{1 + 3\alpha} \mathrm{e}^{-v^2/2}, \tag{5}$$

where α is a parameter which determines the population of nonthermal ions in our plasma model. To include the effect of disturbances on the equilibrium, we replace v^2 by $v^2 + 2\Phi$ in eq. (5) and integrate the resulting distribution over velocity space to get

$$n_i = (1 + \beta\Phi + \beta\Phi^2)\mathrm{e}^{-\Phi}, \tag{6}$$

where $\beta = 4\alpha/(1 + 3\alpha)$ and is used in our calculations. In order to investigate the properties of large amplitude stationary dust acoustic solitons, we assume that all the dependent variables in nonlinear equations (1–4) depend only on a single variable $\xi = x - Mt$ where M being the soliton velocity normalized by c_d. Equations (1) and (2) in the stationary frame can be integrated to give

$$n_d = \frac{\sqrt{2}M}{\sqrt{M^2 + 2\Phi + 3\sigma + \sqrt{(M^2 + 2\Phi + 3\sigma)^2 - 12M^2\sigma}}}, \tag{7}$$

where we have used boundary conditions for localized disturbance, viz, $\Phi \to 0$, $u_d \to 0$, $n_d \to 1$ as $\xi \to +\infty$. Substituting n_d from eq. (7) in eq. (3) and following Sagdeev's pseudopotential method along with appropriate boundary conditions, we obtain

$$\frac{1}{2}\left(\frac{\mathrm{d}\Phi}{\mathrm{d}\xi}\right)^2 + V(\Phi) = 0, \tag{8}$$

where

$$\begin{aligned} V(\Phi) = {} & 1 + 3\beta - (1 + 3\beta + 3\beta\Phi + \beta\Phi^2)\mathrm{e}^{-\Phi} \\ & + \frac{M(12\sigma M^2)^{1/4}}{\sqrt{2}}\left[\mathrm{e}^{1/2\cosh^{-1}(\eta/\sqrt{b})} + \frac{1}{3}\mathrm{e}^{(-3/2)\cosh^{-1}(\eta/\sqrt{b})}\right] \\ & - \frac{M(12\sigma M^2)^{1/4}}{\sqrt{2}}\left[\mathrm{e}^{1/2\cosh^{-1}(\chi/\sqrt{b})} + \frac{1}{3}\mathrm{e}^{(-3/2)\cosh^{-1}(\chi/\sqrt{b})}\right]. \end{aligned}$$

Here

$$\begin{aligned} \chi &= M^2 + 2\Phi + 3\sigma, \\ b &= 12\sigma M^2, \\ \eta &= M^2 + 3\sigma. \end{aligned} \tag{9}$$

3. Discussion

Equation (8) can be regarded as an 'energy integral' of an oscillating particle of unit mass with a velocity $\mathrm{d}\Phi/\mathrm{d}\xi$ and position Φ in a potential $V(\Phi)$. Further it is clear that $V(\Phi) = 0$ and $\mathrm{d}V(\Phi)/\mathrm{d}\Phi = 0$ at $\Phi = 0$. Solitary wave solution for eq. (8) exists if $\mathrm{d}^2V/\mathrm{d}\Phi^2 \leq 0$ at $\Phi = 0$, so that the zero as a fixed point is unstable. All the specified conditions are satisfied. Besides that $V(\Phi)$ should be negative between $\Phi = 0$ and Φ_m where Φ_m is some maximum or minimum potential for compressive or refractive solitons respectively. The value of $V(\Phi)$ as a function of Φ for three values of σ and for diffirent range of M is calculated numerically satisfying the criteria for the existence of compressive/refractive solitons and the results are tabulated as shown in table 1.

As apparent from table 1, the range of existence of compressive/refractive solitons shifts. For very low dust temperature, refractive solitons exist over a wider range of M values. Nonetheless, there is a range of σ and M where co-existence of refractive as well as compressive solitons is possible. Figure 1 displays the graph of $V(\Phi)$ vs Φ for fixed value of dust temperature $\sigma = 0.02$ and four different values of concentration of nonthermal ion distribution parameter β. It is observed that compressive and refractive solitons coexist only for specific $\beta = 0.48$. Once we start increasing the value of β, compressive solitons immediately disappear. However, refractive solitons persist till the value of $\beta = 0.51$.

Table 1. Effect of σ, the dust temperature.

M	1.4	1.41	1.42	1.43	1.44	1.45	1.46	1.47	1.48	1.49	1.50	1.51	1.52	1.53	1.54
$\sigma =$	R	R	R	R	R	R	R								
10^{-4}			C	C	C	C	C	C	C	C	C	C	X		
$\sigma =$	R	R	R	R	R	R	R	R	R	R	R	R	R		
10^{-7}			C	C	C	C	C	C	C	C	C	C			
$\sigma = 3.5$	X	X	X	X	X	R	R	R	R						
$\times 10^{-2}$							C	C	C	C	C	C	C	C	C

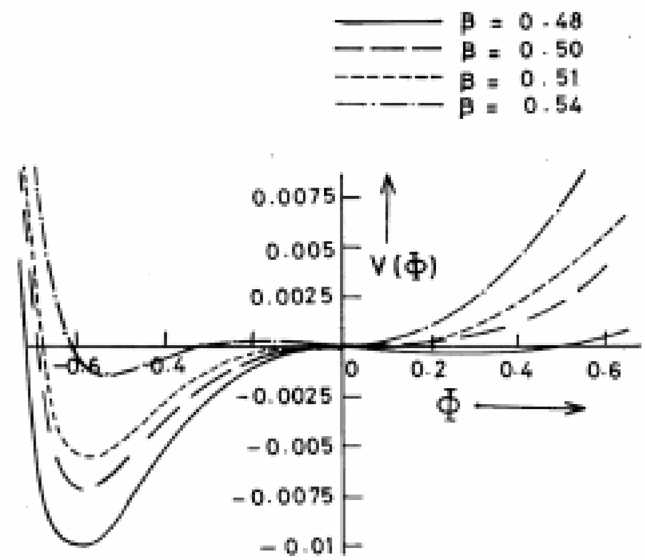

Figure 1. The Sagdeev potential $V(\Phi)$ vs Φ for fixed value of $M = 1.45$ and $\sigma = 0.2$ and four different values of β.

As soon as the value of β is further increased, neither compressive nor refractive solitons exist which is obvious from the graph for $\beta = 0.54$. Since temperature of the dust and nonthermal ion distribution are the main parameters considered here, it is useful to investigate the dynamics of solitons as function of variation of these parameters. We have considered this case in figure 2, where we have plotted the value of $V(\Phi)$ as a function of Φ by choosing the previous value of $\beta = 0.54$ but varying the value of dust temperature. It is obvious from the graphs in figure 2 that decrease in dust temperature does not significantly affect the compressive solitons. However the dynamics of refractive solitons is drastically influenced. There is a shift in the value of $V(\Phi_m)$ as well as Φ_m as σ is decreased. It is also observed that the refractive solitons which had disappeared (cf figure 1) for $\beta = 0.54$, have reappeared on decreasing the value of $\sigma = 0.02$ to $\sigma = 0.002$. Clearly the functional dependence of $V(\Phi)$ is very sensitive to the variation in parameters σ, β, M and Φ. Thus on using Sagdeev pseudopotential approach it is possible to investigate

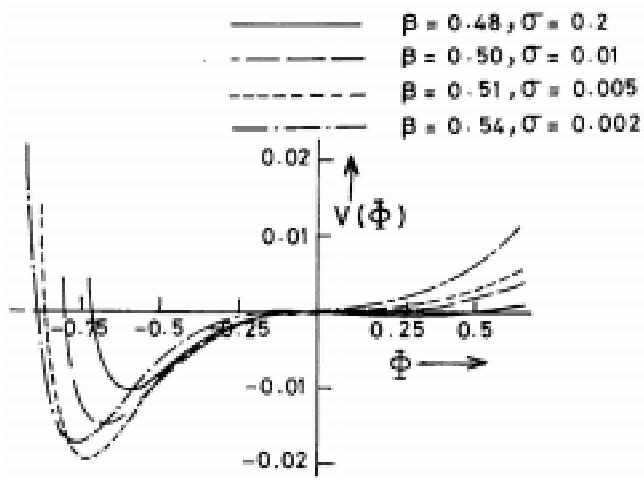

Figure 2. The Sagdeev potential $V(\Phi)$ vs Φ for fixed value of $M = 1.45$ and different values of σ and β.

the nonlinear wave structures over a wider range of parameter space. The limitation of the present analysis is that the dust charge is assumed to be constant. However if variable dust charge is considered, it might add a new scenario to the soliton dynamics.

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