

Effect of rippled laser beam on excitation of ion acoustic wave

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Abstract. Growth of a radially symmetrical ripple, superimposed on a Gaussian laser beam in collisional unmagnetised plasma is investigated. From numerical computation, it is observed that self-focusing of main beam as well as ripple determine the growth dynamics of ripple with the distance of propagation. The effect of growing ripple on excitation of ion acoustic wave (IAW) has also been studied.

Keywords. Self-focusing; ion acoustic wave; growth; collisional plasma.

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1. Introduction

Inertially confined fusion plasmas involve interaction of high power laser beam with long scale length under dense plasma. Aim is to excite some low frequency electrostatic waves in the plasma which may subsequently dump their energies to particles via Landau damping and heat the plasma to thermonuclear temperature. Excitation of plasma waves through different mechanisms has also been reported in a number of investigations [1–7]. The direct and indirect experimental evidences reveal that the smooth looking laser beams have strong intensity spike [8]. In experimental situations where laser or electromagnetic beam travelling through self-focusing media results in multiple filament formation, there is one to one correspondence between filaments and intensity spikes riding with the incident laser beam. Thus plasma as a self-focusing nonlinear medium may lead to filamentary breakup of the incident beam. Self-focusing and filamentation instability have been investigated theoretically as well as experimentally in considerable details [9–20]. The whole beam self-focusing of laser beam may arise on account of variety of nonlinearities resulting e.g. from ponderomotive force, collisional non-uniform heating, relativistic etc. On the other hand origin of filamentation instability may be attributed to small scale density fluctuation (resulting from quasi-neutrality) or small scale intensity spikes associated with the main beam. The physics of the perturbation growing at the cost of the main beam is relevant to the inertially confined fusion plasmas. On account of this instability, the plasma is rendered inhomogeneous and phase matching conditions for the excitation of absorptive instabilities get localised. Furthermore, the filamentary structures also destroy the symmetry of the energy deposition and may trigger the hydrodynamic instabilities of target.

Theoretical investigations of self-focusing of laser beams were carried out by Akhmanov *et al* [9] and Sodha *et al* [12]. They used WKB and paraxial ray approximations. As discussed by Wagner *et al* [21], the trial function is substituted in the evolution equation, where the nonlinear refractive index is Taylor expanded in the transverse direction. The procedure was then generalised to include phase dynamics. Vlasov *et al* [22] developed an alternative approach based on invariants of cubic nonlinear Schrödinger equation and moments of electromagnetic field to arrive at an equation governing the evolution of laser beam width with distance of propagation. The theory also determines the conditions for self-focusing in terms of certain moments. Later on, it was further generalised to non-relativistic case [23] and relativistic self-focusing [24]. However, in spite of its elegant formulation, the inclusion of proper phase description through moment theory is still an open question. Yet another method involving the invariants of nonlinear Schrödinger equation is variational approach used by Anderson *et al* to describe self-focusing in plasmas [25] and optical fibers [26]. Variational method which though approximately analytical but fairly general in nature, has been used recently in a number of investigations [27–32]. Besides that, it has been reported to give correct regularised phase description [27]. Nevertheless, it is of limited scope in investigation of singularity formation and collapse associated with self-focusing phenomenon [33]. Moreover, the trial functions have to be guessed which is sometimes difficult in more complicated cases and also the finer intrinsic details of phase profile are not predicted correctly.

Recently Subbarao *et al* [34] have pointed that the most popular theories for whole beam focusing of Akhmanov *et al* [9], Sodha *et al* [12] and Max [35] are not applicable for higher power laser beam. They further remarked that discrepancy lies with the assumption of using Taylor expansion in the transverse cylindrical co-ordinates. On using non-Taylor power series expansion as a correction to paraxial ray approximation [34] with proper methodology, beam dynamics can put in well known ABCD matrix form [36]. Also corrected version of paraxial ray approximation predicts the same conditions for self-trapping as those obtained from variational approach and moment theory.

In laser plasma experiments, beam with non-uniform intensity distribution, having index of refraction dependent on local field intensity can produce its own dielectric waveguide in which it propagates undergoing self-focusing/defocusing. Small scale intensity fluctuations on such a beam can grow by the same self-focusing mechanism. It has been reported that intensity in individual spikes created by hot spots on incident laser beam, also approaches the threshold for self-focusing [37]. Such a situation is adequately modelled by considering the growth of a radially symmetrical ripple superimposed on a Gaussian laser beam [1,39]. By the way, it also incorporates the dynamics of small scale as well as large scale self-focusing on the growth of the ripple. Further, the effect of growth of spike on the excitation of plasma waves can also be investigated [7,2].

In the present investigation, since we are considering low power rippled laser beam, we therefore choose the most popular theory of Akhmanov *et al* [9] and Sodha *et al* [12]. In this problem authors have used ripple model [1] and the nonlinearity considered here is of collisional type, which ensues from temperature dependent collision frequency. In §2, the equations for the main beam and ripple have been set up and solved for using WKB and paraxial ray approximations [9,12]. The differential equations for the beam width parameters of the pump and ripple are simultaneously solved numerically using Runge-Kutta method. In §3, an expression for the density perturbation associated with the

excited ion acoustic wave is derived. Last section deals with the brief discussion of the important results.

2. Solution of wave equation

Consider the propagation of a laser beam of angular frequency ω_0 in a collisional homogeneous plasma along the z direction. The rippled laser beam can be written as

$$\mathbf{E} = \mathbf{E}_0 \exp [i(\omega_0 t - k_0 z)] + \mathbf{E}_1 \exp [i(\omega_0 t - k_0 z)]. \quad (1)$$

We consider a Gaussian rippled laser beam [1] with initial intensity distribution as

$$\mathbf{E}_0 \cdot \mathbf{E}_0^*|_{z=0} = E_{00}^2 \exp(-r^2/r_0^2) \quad (2)$$

and

$$\mathbf{E}_1 \cdot \mathbf{E}_1^*|_{z=0} = E_{100}^2 (r/r_{10})^{2n} \exp(-r^2/r_{10}^2), \quad (3)$$

where n is a positive number and r_{10} is the width of the ripple [1]. As n increases, the maximum $r_{\max} = r_{10} \sqrt{n}$ of the ripple shifts away from the axis. The total electric vector of the beam \mathbf{E} satisfies the wave equation

$$\nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) + \frac{\omega_0^2}{c^2} \epsilon \mathbf{E} = 0, \quad (4)$$

where

$$\epsilon = \epsilon_0 + \Phi(\mathbf{E} \cdot \mathbf{E}^*).$$

Here ϵ_0 and $\Phi(\mathbf{E} \cdot \mathbf{E}^*)$ are the linear and nonlinear parts of the dielectric constant for non-uniform heating type nonlinearity and are given by [12]

$$\begin{aligned} \epsilon_0 &= 1 - \frac{\omega_p^2}{\omega_0^2}, \\ \Phi(\mathbf{E} \cdot \mathbf{E}^*) &= \frac{\omega_p^2}{\omega_0^2} \left[1 - \frac{N_{0i}}{N_0} \right], \\ N_{0i} &\cong N_{0e} = N_0 \left(1 + \frac{\alpha}{2} \mathbf{E} \cdot \mathbf{E}^* \right)^{(s-2)/2}, \end{aligned}$$

where ω_p and α are respectively the plasma frequency and nonlinearity constant with the usual meanings for all parameters. s is a parameter characterising the nature of collisions; $s = -3$ for collisions between electrons and ions, $s = 2$ for collisions between electrons and di-atomic molecules and $s = 0$ corresponds to the case when collisions are velocity independent. In WKB approximation, the second term in eq. (4) can be neglected and electric vectors of the main beam (\mathbf{E}_0) and ripple (\mathbf{E}_1) satisfy the following equations:

$$\nabla^2 \mathbf{E}_0 + \frac{\omega_0^2}{c^2} \epsilon(\mathbf{E}_0 \cdot \mathbf{E}_0^*) \mathbf{E}_0 = 0 \quad (5)$$

and

$$\begin{aligned} \nabla^2 \mathbf{E}_1 + \frac{\omega_0^2}{c^2} \epsilon (\mathbf{E} \cdot \mathbf{E}^*) \mathbf{E}_1 \\ + \frac{\omega_0^2}{c^2} [\Phi (\mathbf{E} \cdot \mathbf{E}^*) - \Phi (\mathbf{E}_0 \cdot \mathbf{E}_0^*)] \mathbf{E}_0 = 0 \end{aligned} \quad (6)$$

respectively. Equations (5) and (6) are solved using the approach followed in Akhmanov *et al* [9] and Sodha *et al* [12]. The dimensionless beam width parameter f_0 of main beam is governed by the equation:

$$\frac{d^2 f_0}{dz^2} = \frac{1}{k_0^2 r_0^4 f_0^3} - (1 - s/2) \frac{\omega_p^2}{2\epsilon_0 r_0^2 f_0^3 \omega_0^2} \alpha E_{00}^2 \left[1 + \frac{1}{2} \alpha \mathbf{E}_0 \cdot \mathbf{E}_0^* \right]^{(s-4)/2}. \quad (7)$$

The initial conditions for a plane wave front are that $f_0 = 1$ and $df_0/dz = 0$ at $z = 0$. To solve eq. (6), one can express

$$\mathbf{E}_1 = \mathbf{A}_1(r, z) \exp(-ik_0 z), \quad (8)$$

where $A_1(r, z)$ is a complex function of its argument. Substituting eq. (8) in eq. (6) and following Sodha *et al* [1], we can write the solution of eq. (6) as

$$\begin{aligned} E_1^2 = A_{10}^2 = \frac{E_{100}^2}{f_1^2} \left(\frac{r}{r_{10} f_1} \right)^{2n} \exp \left(-\frac{r^2}{r_{10}^2 f_1^2} \right) \\ \times \exp \left[-2 \int_0^z k_i(z) dz \right], \\ S_1 = \frac{r^2}{2} \beta_1 + \Phi_1(z), \quad \beta_1 = \frac{1}{f_1} \frac{df_1}{dz}, \\ k_i(z) \simeq \frac{1}{2} \frac{\omega_0^2}{c^2} \frac{1}{k_0} \frac{\omega_p^2}{\omega_0^2} (s-2)/2 \\ \left(1 + \frac{\alpha}{2} \mathbf{E}_0 \cdot \mathbf{E}_0^* \right)^{(s-2)/2} \sin(2\phi_p). \end{aligned} \quad (9)$$

For an initially plane wave front, $df_1/dz = 0$ and $f_1 = 1$ at $z = 0$, the beam width parameter of ripple f_1 satisfies the following equation

$$\begin{aligned} \frac{d^2 f_1}{dz^2} = \frac{1}{k_0^2 r_{10}^4 f_1^3} - (1 - s/2) \frac{f_1 \omega_p^2}{2\epsilon_0 r_0^2 f_0^4 \omega_0^2} \alpha E_{00}^2 \\ \left[T_1^{s/2-2} \left\{ \exp \left(-\frac{r_{10}^2 f_1^2 n}{r_0^2 f_0^2} \right) + T_3 \left(1 + 2(s/2 - 2) \frac{\alpha E_{00}^2}{f_0^2} \right. \right. \right. \\ \left. \left. \exp \left(-\frac{r_{10}^2 f_1^2 n}{r_0^2 f_0^2} \right) T_1^{-1} \right\} - \cos^2 \phi_p \exp \left(-\frac{r_{10}^2 f_1^2 n}{r_0^2 f_0^2} \right) T_2^{s/2-2} \right. \\ \left. \left. \left\{ 1 + (s/2 - 2) \frac{\alpha}{2} E_{00}^2 T_2^{-1} \left(\exp \left(-\frac{r_{10}^2 f_1^2 n}{r_0^2 f_0^2} \right) + T_3 \right) \right\} \right] \right], \end{aligned} \quad (10)$$

where

$$\begin{aligned}
 T_1 &= 1 + \frac{\alpha}{2} \mathbf{E}_0 \cdot \mathbf{E}_0^* \Big|_{r^2=r_{10}^2, f_1^2 n}, \\
 T_2 &= 1 + \frac{\alpha}{2} \mathbf{E} \cdot \mathbf{E}^* \Big|_{r^2=r_{10}^2, f_1^2 n}, \\
 T_3 &= \cos \phi_p n^{n/2} \frac{f_0 E_{100}}{f_1 E_{00}} \exp \left(- \int_0^z k_i(z) dz \right) \exp \left(- \frac{n}{2} - \frac{r_{10}^2 f_1^2 n}{2 r_0^2 f_0^2} \right).
 \end{aligned}$$

It is apparent from the eq. (9) for A_{10}^2 that the ripple grows/decays inside the plasma and the growth/decay rate is k_i . The growth rate depends on the intensity of the main beam, the phase angle ϕ_p and other plasma parameters of pump wave and plasma.

3. Excitation of ion acoustic wave

A weak ion acoustic wave is nonlinearly coupled to rippled laser beam via modified background density. Thus, self-focusing of main beam and dynamics of the growth of ripple affect the excitation of IAW substantially. The background density of ions is modified due to nonuniform heating. The equation characterising IAW can be derived using

$$\frac{\partial N_\alpha}{\partial t} + \nabla \cdot (N_\alpha \mathbf{V}_\alpha) = 0$$

.... Continuity Equation,

$$\begin{aligned}
 m_\alpha N_\alpha \left(\frac{\partial}{\partial t} + \mathbf{V}_\alpha \cdot \nabla \right) \mathbf{V}_\alpha &= N_\alpha e_\alpha \left(\mathbf{E}_t + \frac{\mathbf{V}_\alpha \times \mathbf{B}}{c} \right) \\
 &\quad - \gamma_\alpha K_B T_\alpha \nabla N_\alpha - 2 \Gamma_\alpha N_\alpha \mathbf{V}_\alpha m_\alpha
 \end{aligned} \tag{11}$$

.... Momentum Balance Equation,

Maxwell's equations and perturbation analysis [2]. We take

$$N_\alpha = N_{0\alpha} + n_\alpha, \mathbf{V}_\alpha = \mathbf{V}_{0\alpha} + \mathbf{v}_\alpha, \mathbf{E}_t = \mathbf{E} + \mathbf{E}' \quad \text{and} \quad \mathbf{B} = \mathbf{B}_0 + \mathbf{B}', \tag{12}$$

where

$$n_\alpha \ll N_{0\alpha}, \quad |\mathbf{v}_\alpha| \ll |\mathbf{V}_{0\alpha}|, \quad |\mathbf{E}'| \ll |\mathbf{E}| \quad \text{and} \quad |\mathbf{B}'| \ll |\mathbf{B}_0|.$$

$\mathbf{V}_{0\alpha}$ is the drift velocity in the pump field, and \mathbf{E} is the total field of the rippled laser beam. In eq. (12), n_α is the density perturbation in the excited ion acoustic wave, \mathbf{v}_α is the drift velocity of the carriers in the electrostatic field E' of the ion acoustic wave. Following procedure of Singh and Singh [2], the equation for the excitation of the ion acoustic wave can be written as

$$\frac{d^2 n_i}{dt^2} - 2 \Gamma_i \frac{\partial n_i}{\partial t} - \frac{\gamma_i k_B T_i}{m_i} \nabla^2 n_i + \omega_{pi}^2 \frac{N_{0i}}{N_0} \frac{k_S^2 \lambda_d^2}{1 + k_S^2 \lambda_d^2} n_i = 0, \tag{13}$$

where λ_d is Debye length. In the absence of laser beam, eq. (13) gives the usual dispersion relation for IAW. The presence of N_{0i} in the last term of eq. (13) gives the coupling of density perturbation associated with IAW to laser beam via modified background density. Equation (13) is solved using standard procedure of Akhmanov *et al* [9] and Sodha *et al* [12] and the equation for the dimensionless beam width parameter f_i of IAW is

$$\begin{aligned} \frac{d^2 f_i}{dz^2} = & \frac{1}{k_S^2 a_0^4 f_i^3} + (s/2 - 1) \frac{k_S^2 \lambda_d^2}{1 + k_S^2 \lambda_d^2} \frac{\omega_{pi}^2}{\gamma_i k_S^2 v_{th}^2} \frac{f_i \alpha E_{00}^2}{f_0^4 2r_0^2} \\ & \left(1 + \frac{\alpha}{2} \mathbf{E} \cdot \mathbf{E}^*\right)^{(s-4)/2} \exp\left(-\frac{r_{10}^2 f_1^2 n}{r_0^2 f_0^2}\right) \left[1 + \frac{f_0 E_{100}}{f_1 E_{00}}\right. \\ & \left. \cos(\phi_p) n^{n/2} \exp\left(-\int_0^z k_i dz\right) \exp\left(-\frac{n}{2} - \frac{r_{10}^2 J_1^2 n}{r_0^2 f_0^2}\right)\right]. \end{aligned} \quad (14)$$

For an initially plane wave front, $df_i/dz = 0$ and $f_i = 1$ at $z = 0$.

4. Discussion

It is apparent from the present analysis that, on account of coupling between the main beam and the ripple [eqs (7) and (10)], the ripple can grow/decay in the plasma. Following set of parameters has been chosen for the numerical solution to the eqs (7), (10) and (14) and results are plotted in the form of graphs.

$$\begin{aligned} \left(\frac{r_0 \omega_p}{c}\right)^2 = 16.0, \quad \frac{\omega_p}{\omega_0} = 0.2, \quad \omega_0 = 1.778 \times 10^{14} \frac{\text{rad}}{\text{s}}, \quad n = 2.0, \quad 2\phi_p = \frac{3\pi}{2}, \\ 2\phi_p = \frac{\pi}{2}, \quad \frac{r_{10}^2}{r_0^2} = 0.85, \quad \frac{r_{10}}{a_0} = 0.3, \quad \alpha E_{00}^2 = 0.2, \quad s = -3 \end{aligned}$$

Excitation of ion acoustic wave, used in laser heating experiments is governed by the eq. (13), where the last term represents the coupling of ion acoustic wave to the modified background density. Figure 1 depicts the variation of the plasma density perturbation associated with IAW with the normalised distance of propagation $\xi (= zc/\omega_0 r_0^2)$ for different values of intensity. As apparent from the expression for k_i (eq. (9)), it is inherently negative and for $2\phi_p = \frac{\pi}{2}$, the growing ripple affects the perturbation associated with IAW. This situation corresponds to increase in nonlinear term viz. second term on right hand side of second order nonlinear differential equation (14) representing the focusing/defocusing of the ion acoustic wave. When $2\phi_p = \pi/2$, the curve 1 is for $\alpha E_{00}^2 = 0.2$ while the curve 2 which is oscillatory growing, corresponds to $\alpha E_{00}^2 = 0.3$. If we choose $2\phi_p = \frac{3\pi}{2}$, we get the curves 3 and 4 respectively for $\alpha E_{00}^2 = 0.2$ and $\alpha E_{00}^2 = 0.3$. The density perturbation decays and does not exhibit oscillatory character. The effect of decaying ripple strongly affects the focusing of IAW. The nonlinear term in eq. (14) in this case becomes drastically weaker with the distance of propagation. Both these results contrast well with our earlier investigations [1].

Figure 2 displays the normalised density perturbation associated with IAW for fixed intensity $\alpha E_{00}^2 = 0.2$ and for three different values of parameter n . The curves 1, 2 and 3 are respectively for various values of 'n' as 2.0, 2.5 and 1.5. As mentioned earlier,

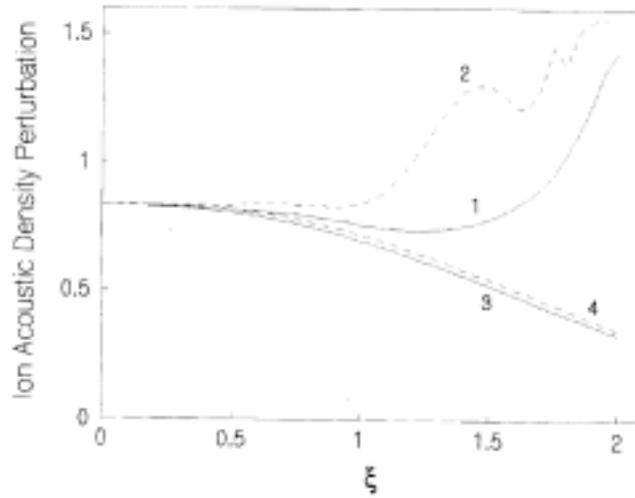


Figure 1. Ion acoustic density perturbation $[n_0^2/n_{00}^2]$ against the normalised distance of propagation $\xi (=zc/\omega_0 r_0^2)$ for the following set of parameters: $(\frac{r_0 \omega_p}{c})^2 = 16.0$, $\frac{\omega_p}{\omega_0} = 0.2$, $\omega_0 = 1.778 \times 10^{14}$ rad/s, $n = 2.0$, $\frac{r_{10}^2}{r_0^2} = 0.85$, $\frac{r_{10}}{a_0} = 0.3$, $s = -3$. The curve 1 is for $\alpha E_{00}^2 = 0.2$ and curve 2 is for $\alpha E_{00}^2 = 0.3$ with $2\phi_p = \frac{\pi}{2}$ and curves 3 and 4 are for $\alpha E_{00}^2 = 0.2$ and 0.3 respectively when $2\phi_p = \frac{3\pi}{2}$.

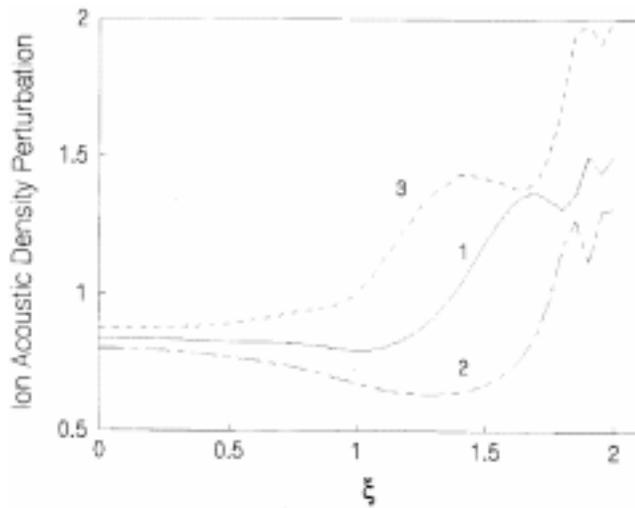


Figure 2. Ion acoustic density perturbation vs the normalised distance of propagation ξ for different values of n and $2\phi_p = \frac{\pi}{2}$ and other parameters are same as mentioned in figure caption 1. The curve 1 is for $n = 2.0$, curve 2 is for $n = 2.5$ and curve 3 is for $n = 1.5$.

larger the value of n , farther away is the ripple from the main beam axis. Thus nonlinear excitation process for IAW is more effective for the smaller n and almost ineffective for larger n . It is thus concluded that the dynamics of growth of laser ripple plays crucial role in self-focusing propagation problem.

References

- [1] M S Sodha, Tarsem Singh, D P Singh and R P Sharma, *Phys. Fluids* **24**, 914 (1981)
- [2] A Singh and T Singh, *Contrib. Plasma. Phys.* **31**, 499 (1991)
- [3] V A Yukhimuk and A K Yukhimuk, *Kinematics Phys. Celest. Bodies* **10**, 54 (1994)
- [4] L Stenflo, *Phys. Plasmas* **3**, 2467 (1996)
- [5] B I Cohen, B F lasinki, A B Lagder and E A Williams, *Phys. Plasmas* **4**, 956 (1997)
- [6] F Do Prado, D M Karfidov, M Virginia Alves and R S Dallaqua, *Phys. Lett.* **A248**, 1186 (1998)
- [7] Nareshpal Singh and Tarsem Singh, *J. Plasma Fusion Res. Series* **2**, 423 (1999)
- [8] S C Abbi and H Mahr, *Phys. Rev. Lett.* **26**, 604 (1971)
- [9] S A Akhmanov, A P Sukhorukov and R P Khokhlov, *Sov. Phys. Usp.* **10**, 609 (1968)
- [10] P K Kaw, G Schmidt and T Wilcox, *Phys. Fluids* **16**, 1522 (1973)
- [11] J F Drake, P K Kaw, Y C Lee, G Schmidt, C S Liu and M N Rosenbluth, *Phys. Fluids* **17**, 778 (1974)
- [12] M S Sodha, V K Tripathi and A K Ghatak, *Prog. Optics* (E Wolf, North Holland) **13**, 171 (1976)
- [13] R D Jones, W C Mead, S V Coggeshall, C H Aldrich, J L Norton, G D Pollak and J M Wallace, *Phys. Fluids* **31**, 1249 (1988)
- [14] A J Schmitt, *Phys. Fluids* **31**, 3079 (1988)
- [15] P E Young, H A Baldis, R P Drake, E M Campbell and K G Estabrook, *Phys. Rev. Lett.* **61**, 2336 (1988)
- [16] B I Cohen, B F Lasinski, A B Langdon, and J C Cummings, *Phys. Fluids* **3**, 766 (1991)
- [17] H A Rose and D F Dubois, *Phys. Fluids* **B5**, 3337 (1993)
R L Berger, B F Lam, T B Kaiser, E A William, A B Langdon and B I Cohen, *Phys. Fluids* **5**, 2243 (1993)
- [18] A Bendib, F Bouzid, K Bendib and G Matthieussent, *Phys. Plasmas* **6**, 4008 (1999)
- [19] C Sulem and P L Sulem, *The nonlinear Schrödinger equation* (Springer, 1999)
- [20] Y S Kivshar and D E Pelinovsky, *Phys. Rep.* **331**, 117 (2000)
- [21] W G Wagner, H A Haus and J H Marburger, *Phys. Rev.* **175**, 256 (1968)
- [22] S N Vlasov, V A Petrishev and V I Talanov, *Sov. Rad. Phys. and Q E* **14**, 1962 (1971)
- [23] J F Lam and B Lippman, *Phys. Fluids* **20**, 1176 (1977)
- [24] P Sprangle and E Esarey, *Phys. Fluids* **B4**, 2241 (1992)
X L Chen and R N Sudan, *Phys. Rev. Lett.* **70**, 2082 (1993)
R N Sudan, Y S Dimant and B Shiraev, *Phys. Plasmas* **4**, 1489 (1997)
L Berge, *Phys. Plasmas* **4**, 1227 (1997)
- [25] D Anderson and M Bonnedal, *Phys. Fluids* **22**, 105 (1979)
- [26] D Anderson, *Phys. Rev.* **A27**, 3135 (1983)
- [27] M Karlsson, D Anderson and M Desiax, *Opt. Lett.* **17**, 22 (1992)
- [28] M Karlsson and D Anderson, *J. Opt. Soc. Am.* **B9**, 9 (1992)
- [29] J C Bhakta, *Phys. Rev.* **E49**, 667 (1994)
- [30] Z Jovansoki and R A Sammut, *Phys. Scr.* **57**, 233 (1998)
- [31] Tarsem Singh and S S Kaul, *Indian J. Pure Appl. Phys.* **37**, 794 (1999)
- [32] T Singh, N S Saini and S S Kaul, *Pramana – J. Phys.* (2000) (in press)
- [33] L Berge, *Phys. Rep.* **303**, 259 (1998)
- [34] D Subbarao, R Uma and H Singh, *Phys. Plasmas* **5**, 3440 (1998) and references therein

- [35] C E Max, *Phys. Fluids* **19**, 74 (1976)
- [36] A Yariv, *Optical Electronics* (Holt-Saunders, New York, 1985)
- [37] V T Tikhonchuk, S Hullar and Ph Mounaix, *Phys. Plasmas* **4**, 4369 (1997)
- [38] A Singh and T Singh, *Plasma Phys. Contl. Fusion* **33**, 123 (1991)
- [39] M V Asthana, A Giulietti, D Varshney and M S Sodha, *J. Plasma Phys.* **62**, 389 (1999)