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Particle production in higher derivative theory

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Abstract. The effect of particle production on the evolution of the spatially flat Friedmann–Lemaitre–Robertson–Walker cosmological model during the early stages of the universe is analysed in the framework of higher derivative theory. The universe has been considered as an open thermodynamic system where particle production gives rise to a supplementary negative creation pressure in addition to the thermodynamic pressure. The dynamical behaviour of both exponential as well as power law solutions have been discussed.

Keywords. Cosmological models; particle production; higher derivative theory of gravitation.

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1. Introduction

There have been a number of investigations into quantum and theoretical treatment of gravity which have attracted attention in studies of the early universe [1–4]. The inflationary universe model [1,2] in which the universe has undergone a period of exponential expansion, has successfully explained many problems in standard cosmology. Attractive features of these models are that they provide a mechanism to generate the small scale density fluctuations in the universe which are needed to seed galaxy formation [4]. Many different alternative theories have been proposed to explain the cosmological problems of the early universe [5]. Generalized Einstein theory [6] with an additional R^2 term (higher derivative theory) was introduced to regularize ultraviolet divergences in Einstein theory and applied to cosmology to obtain bouncing models, thus avoiding the singularity at the big bang [7]. Subsequently, a number of authors have elaborated the structure and properties of the higher derivative gravitational theory (refer [8] and references therein). Very recently Paul, Mukherjee and Beesham [9] have studied causal viscous cosmological models in higher derivative theory. Although bulk viscous stress in expanding universe has been phenomenologically described in terms of particle production, but thermodynamical behaviour of the universe changes in both cases. In this context it is important to investigate cosmological models with particle production during evolution of the universe in the frame work of the higher derivative theory of gravity.

The idea of particle production in cosmology has been dealt by many authors [10]. Prigogine *et al* (refer [11] and references therein) have studied thermodynamics of open systems in the reference of cosmology and suggested a quantitative expression for the particle production out of gravitational energy. They have presented a new concept of adiabatic transformation from closed to open systems. If we consider the universe as an adiabatic open thermodynamic system, allowing for irreversible matter production from the gravitational field, then the thermodynamic energy conservation equation becomes

$$\mathrm{d}(\rho V) + p\mathrm{d}V - \frac{(\rho+p)}{n}\mathrm{d}N = 0\,, \tag{1}$$

where V is the volume of the system, $n = N/V$ is the particle number density, and N is the number of particles in V. This conservation equation may be written as

$$\mathrm{d}(\rho V) + (p + p_c)\,\mathrm{d}V = 0\,. \tag{2}$$

Here p_c is the supplementary pressure corresponding to creation of matter and expressed as

$$p_c = -\frac{(\rho+p)}{n}\,\frac{\mathrm{d}N}{\mathrm{d}V} = -\frac{(\rho+p)}{N}\,\frac{\mathrm{d}N}{\mathrm{d}t}\,\frac{1}{3H}\,, \tag{3}$$

the creation pressure p_c is negative or zero depending on the presence or absence of particle production. Thus, the effect of production of new particles is equivalent to adding a supplementary pressure term p_c to the thermodynamic pressure p so that the conservation equation for a closed system

$$\mathrm{d}(\rho V) + p\mathrm{d}V = 0 \tag{4}$$

is modified to eq. (2) for an open system.

For an adiabatic open system, the increase in entropy is only due to the creation of matter and since entropy (S) is an extensive property of the system (i.e., S is proportional to the number of particles included in the system), we have the relation

$$\frac{\mathrm{d}S}{S} = \frac{\mathrm{d}N}{N}\,. \tag{5}$$

The second law of thermodynamics requires that $\mathrm{d}S \geq 0$, which imposes the condition that the only particle number variations admitted are such that $\mathrm{d}N \geq 0$. Several authors [12] have studied the thermodynamics of particle production in different contexts.

Cosmological models with constant deceleration parameter have been undertaken by several authors [13] in general relativity (GR). Johri and Kalyani [14] have shown that many known cosmological models of Brans–Dicke theory (BDT) for flat Friedmann–Lemaitre–Robertson–Walker (FLRW) space-time universe are models with constant deceleration parameter. These models may be categorized in two classes, the first category is of singular models where the cosmic expansion is driven by the big-bang impulse; all the matter and radiation energy is produced at the big-bang epoch. In the second category of models the universe has a non-singular origin, the expansion starting from a vacuum fluctuation, and then particle production occurs. Further, constant deceleration parameter measures the deviation of the growth of the scale factor from its linear growth in time [15].

Motivated by the aforesaid studies in GR as well as in BDT, in this paper we have considered constant deceleration parameter models to study the particle production out of the gravitational field in higher derivative theory. The dynamical behaviour of models will also be discussed.

2. Field equations

In order to study classical solutions with particle production, we consider the following generalized action

$$I = \int \sqrt{-g}\left[-\frac{1}{2}(R+\alpha R^2) + L_{\mathrm{matter}}\right]\mathrm{d}^4x\,, \tag{6}$$

where g is the determinant of the 4-dimensional metric, G is the gravitational constant, R is the scalar curvature and α is a positive constant. It is straight forward to write down the field equation from the action (6) as

$$R_{ij} - \frac{1}{2}g_{ij}R + \alpha\left[2R\left(R_{ij} - \frac{1}{2}g_{ij}R\right) + 2(R_{;ij} - g_{ij}\Box R)\right] = -8\pi G T_{ij}\,, \tag{7}$$

where (;) represents the covariant derivative and T_{ij} is the energy-momentum tensor for matter distribution defined as

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}\,. \tag{8}$$

The field equation (7) for a homogeneous and isotropic universe represented by a spatially flat FLRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\ \mathrm{d}\phi^2)] \tag{9}$$

takes the form

$$H^2 - 6\alpha[2H\ddot{H} - \dot{H}^2 + 6H^2\dot{H}] = \frac{8\pi G}{3}\rho \tag{10}$$

together with the conservation equation for the matter

$$\dot{\rho} + 3H(\rho + p) = 0\,, \tag{11}$$

where $H = \dot{a}/a$ is the Hubble parameter. It may be mentioned [11] here that the space–space components of the field equation given by eq. (7) can be obtained using the time–time component (10) and the conservation equation (11), hence the relevant field equations are (10) and (11) respectively [8].

Let us treat the universe as an open thermodynamic system with initially N particles and assume that a random fluctuation in curvature induces a transformation of gravitational energy into matter energy, producing an additional number of particle $\mathrm{d}N$. This increase in the number of particles from N to $N + \mathrm{d}N$ gives rise to a negative supplementary pressure to the thermodynamic pressure and this negative pressure drives the expansion of the universe. Hence, the perfect fluid pressure should be replaced by an effective pressure of the cosmic fluid which is given by

$$p_{\mathrm{eff}} = p + p_c\,. \tag{12}$$

By using eq. (12) and the equation of state

$$p = (\gamma - 1)\,\rho\,, \tag{13}$$

where γ ($1 \leq \gamma \leq 2$) is a constant, the conservation equation (11) reduces to

$$\dot{\rho} + 3H\gamma\rho = -3Hp_c\,. \tag{14}$$

With help of (3), eq. (14) after integration yields

$$N(t) = N_0 a^3(t)\rho^{1/\gamma}\,, \tag{15}$$

where N_0 is an integration constant. Equation (15) gives rise to a relation between the particle number density (n) and energy density (ρ) as

$$n \propto \rho^{1/\gamma}\,. \tag{16}$$

According to Gibbs integrability condition, one cannot independently specify an equation of state for the pressure and temperature [16]. If we consider one barotropic relation then the other relation must be barotropic and hence

$$T \propto \exp \int \frac{\mathrm{d}p}{\rho(p) + p} \tag{17}$$

which, with help of the equation of state (13), gives

$$T = T_0 \rho^{(\gamma-1)/\gamma}\,. \tag{18}$$

Here T_0 is a proportionality constant.

As we have only four independent equations viz. (10), (13), (15) and (18) and five unknown variables viz. a, ρ, p, N and T, we have to assume one more physically plausible relation.

3. Cosmological solutions

In order to study pariticle production during the early stages of evolution, we take the deceleration parameter to be constant (see [13–15])

$$-\frac{\dot{H} + H^2}{H^2} = q\ (\mathrm{constant})\,. \tag{19}$$

The deceleration parameter q measures the deviation from linearity of the growth of the cosmic scale factor. Equation (19) may be written as

$$\dot{H} + (q+1)H^2 = 0 \tag{20}$$

which gives the solutions

$$a(t) = (B + MAt)^{1/M}\,, \qquad M = q + 1 \neq 0 \tag{21}$$

$$a(t) = C\,\mathrm{e}^{Dt}\,, \qquad M = 0 \tag{22}$$

where A, B, C and D are constants. It is worthwhile to mention that matter creation may not be in the contracting phase of the universe. Thus particle production from gravitational energy take place only in the expanding phase ($H > 0$) of the universe provided $M > 0$. Equation (21) represents inflationary models of the universe for $-1 < q < 0$. Now, we shall consider, in turn, the power law and exponential models separately.

3.1 *Power-law model*

Using (21), eq. (10) yields

$$\rho = \frac{3A^2}{8\pi G(B+MAt)^2}\left[1+\frac{18\alpha M(2-M)A^2}{(B+MAt)^2}\right]. \tag{23}$$

For $M \leq 2$, the energy condition $\rho \geq 0$ is satisfied. The energy density is decreasing with time. Substituting the values of $a(t)$ and $\rho(t)$ from eqs (21) and (23) into (15), we get an explicit expression for the particle number N,

$$N = N_1(B+MAt)^{(3\gamma-2M)/\gamma M}\left[1+\frac{18\alpha M(2-M)A^2}{(B+MAt)^2}\right]^{1/\gamma}, \tag{24}$$

where

$$N_1 = N_0\left(\frac{3A^2}{8\pi G}\right)^{1/\gamma}.$$

The second law of thermodynamics suggests that $\mathrm{d}N \geq 0$ which implies $\gamma \geq (2M/3)$.

From eqs (18) and (23), we obtain for the temperature

$$T = T_1(B+MAt)^{(2(1-\gamma))/\gamma}\left[1+\frac{18\alpha M(2-M)A^2}{(B+MAt)^2}\right]^{(\gamma-1)/\gamma}, \tag{25}$$

where $T_1 = T_0(N_1/N_0)^{\gamma-1}$. In this model the number of particles increases. For $M = 1.5$ and large value of t, in the dust model ($\gamma = 1$) of the universe the number of particles becomes constant. The energy density, thermodynamic pressure, creation pressure and particle number density are decreasing functions of time. These results also follow the results of Johri and Kalyani in Brans–Dicke theory [14]. The model solves the entropy problem.

3.2 *Exponential model*

Considering (22), eqs (10) and (15) suggest that the cosmological model has uniform energy density and that the number of particles is directly proportional to the volume of the universe. In this case the strong energy condition $(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)v^\mu v^\nu \geq 0$ is violated.

The field equation (10) permits an alternative exponential solution

$$a(t) = \left[a_0 + \exp\left(\frac{t}{\sqrt{6\alpha}}\right)\right]^{2/3}. \tag{26}$$

The corresponding energy density is

$$\rho(t) = \frac{\exp\left(\frac{4t}{\sqrt{6\alpha}}\right)}{36\pi\alpha G\left[a_0 + \exp\left(\frac{t}{\sqrt{6\alpha}}\right)\right]^4}\,. \tag{27}$$

By using eqs (26) and (27), (15) yields

$$N(t) = \frac{N_0}{(36\pi\alpha G)^{1/\gamma}}\,\frac{\exp\left(\frac{4t}{\gamma\sqrt{6\alpha}}\right)}{\left[a_0 + \exp\left(\frac{t}{\sqrt{6\alpha}}\right)\right]^{(4-2\gamma)/\gamma}}\,. \tag{28}$$

The weak energy condition $T_{\alpha\beta}v^{\alpha}T_{\mu\nu}v^{\mu} \geq 0$ suggests that $a_0 \leq 0$ and the strong energy condition suggests that $a_0 < 0$.

From eqs (18) and (27), we get

$$T(t) = T_0\left[\frac{\exp\left(\frac{4t}{\sqrt{6\alpha}}\right)}{36\pi\alpha G\left[a_0 + \exp\left(\frac{t}{\sqrt{6\alpha}}\right)\right]^4}\right]^{(\gamma-1)/\gamma}\,. \tag{29}$$

4. Discussion

In standard cosmology, the field equation (10) reduces to

$$3H^2 = 8\pi G\rho \tag{30}$$

and the continuity equation (14) remains the same. The energy-density, total particle number, and temperature with scale factor given by (21), take the following form

$$\rho = \frac{3A^2}{8\pi G(B + MAt)^2}\,, \tag{31}$$

$$N = N_0\left(\frac{3A^2}{8\pi G}\right)^{1/\gamma}(B + MAt)^{(3\gamma-2M)/\gamma M}\,, \tag{32}$$

$$T = T_0\left(\frac{3A^2}{8\pi G}\right)^{(\gamma-1)/\gamma}(B + MAt)^{(2(1-\gamma))/\gamma}\,. \tag{33}$$

It can be seen from the above expressions that in higher derivative theory with power law relation between scale factor and time, the universe starts with higher energy density, total number of particles and temperature in comparison with standard cosmology based on general relativity. The additional terms in these physical quantities is decreasing fast and later on reduces to those of general relativity. For $M = 2$ all solutions of higher derivative theory are similar to solutions of standard cosmology.

The horizon distance (proper distance travelled by light emitted at time t_1) [17] for the model is

$$d_H(t) = a(t) \lim_{t_1 \to -\infty} \int_{t_1}^{t} \frac{\mathrm{d}\bar{t}}{a(\bar{t})} = \frac{B + MAt}{A(M-1)} \tag{34}$$

for all $M < 1$ (i.e. $q < 0$). From (34) it can be easily seen that horizon distance is finite throughout and hence causal communication between two observers exist. During expansion of the universe the ratio of potential energy and kinetic energy $\Omega = 8\pi G\rho/3H^2$ becomes

$$\Omega = 1 + \frac{18\alpha M(2-M)A^2}{(B+MAt)^2} \,. \tag{35}$$

This agrees with the fact that universe was curved during early stages of its evolution. For large value of t, the second term on the right hand side is negligible and hence universe becomes flat.

In adiabatic particle production processes the entropy per particle associated with particle creation is constant.

The energy-density, total particle number and temperature with exponential solution given by eq. (22), have the following expressions

$$\rho(t) = \frac{\exp\left(\frac{2t}{\sqrt{6\alpha}}\right)}{36\pi\alpha G\left[a_0 + \exp\left(\frac{t}{\sqrt{6\alpha}}\right)\right]^2} \,, \tag{36}$$

$$N(t) = \frac{N_0}{(36\pi\alpha G)^{1/\gamma}} \, \frac{\exp\left(\frac{2t}{\gamma\sqrt{6\alpha}}\right)}{\left[a_0 + \exp\left(\frac{t}{\sqrt{6\alpha}}\right)\right]^{(2-2\gamma)/\gamma}} \,, \tag{37}$$

$$T(t) = T_0 \left[\frac{\exp\left(\frac{2t}{\sqrt{6\alpha}}\right)}{36\pi\alpha G\left[a_0 + \exp\left(\frac{t}{\sqrt{6\alpha}}\right)\right]^2}\right]^{(\gamma-1)/\gamma} \,. \tag{38}$$

In the case of exponential model also, it is clear that during early stages universe gets more energy-density, total number of particles and temperature in comparison to standard cosmology. After a large time, both the higher derivative theory as well as general relativity become similar and yield uniform energy density and temperature. The total number of particles also remains the same which may be interpreted that particle production is balanced by particle annihilation.

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