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Canonical Ensemble Model for Black Hole Horizon of Schwarzschild–de Sitter Black Holes Quantum Tunnelling Radiation

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Abstract. In this paper, we use the canonical ensemble model to discuss the radiation of a Schwarzschild–de Sitter black hole on the black hole horizon. Using this model, we calculate the probability distribution from function of the emission shell. And the statistical meaning which compare with the distribution function is used to investigate the black hole tunnelling radiation spectrum. We also discuss the mechanism of information flowing from the black hole.

Key words. Schwarzschild–de Sitter black hole: black hole horizon: canonical ensemble model: quantum tunnelling.

1. Introduction

In 2000, Parikh and Wilczek gave a calculation about the particles emission rate, which is an approach of particles tunnelling across the event horizon (Parikh & Wilczek 2000; Parikh 2004). If we consider energy conservation, we can assume the emission process as quantum tunnelling. The tunnelling particle can be assumed as an emission shell or a spherical shell (Zhang 2012; Zhao *et al.* 2006). A corrected spectrum was given (at first-order approximation). Parikh and Wilczek's work supports the conservation of information when the particles emit on the surface of the black hole (Zhang & Zhao 2005, 2006; Zhang 2007). Take Schwarzschild black hole as an example, the corrected emission spectrum can be expressed as

$$\Gamma \sim e^{\Delta S_{\mathrm{BH}}} \approx e^{-\beta\omega}. \tag{1}$$

The most meaningful yield of Parikh and Wilczek's framework is that it supports information conservation.

2. Canonical ensemble model used in black hole horizon

We know that tunnelling particle can be expressed as a spherical shell or *S*-wave (eq. (2)). Consider it as a composite particle which is composed of a lot of particles.

In the Schwarzschild–de Sitter black hole, the tunnelling speed of S-wave on the black hole horizon is

$$\dot{r} = \frac{\mathrm{d}(E-\omega)}{\mathrm{d}p_r} = -\frac{\mathrm{d}\omega}{\mathrm{d}p_r}. \tag{2}$$

Observed the spherical shell in thermal equilibrium. The temperature of the shell is same as the black hole and the shell is a thermodynamical system. If there are N isolated systems that together compose of a mixture ensemble, then the statistical operator will be defined as

$$\hat{\rho} = |\psi_i\rangle P_i \langle\psi_i|. \tag{3}$$

P_i represents the probability of $|\psi_i\rangle$, $|\psi_i\rangle$ is the quantum state of the shell. In canonical ensemble system,

$$P_i = \frac{\Omega_{\mathrm{BH}}(E-E_i)}{\Omega_E}. \tag{4}$$

E is the total energy of the black hole with the shell (the isolated system) and E_i is the energy of the spherical shell. Ω is the microscopic state number, $\Omega_{\mathrm{BH}}(E-E_i)$ of the black hole, and $\Omega(E)$ of the isolated system. As $E_i \ll E$, the term $\ln\Omega_{\mathrm{BH}}(E-E_i)$ can be expanded as a Taylor series:

$$\begin{aligned}\ln\Omega_{\mathrm{BH}}(E-E_i) &= \ln\Omega_{\mathrm{BH}}(E) + \left(\frac{\partial\ln\Omega_{\mathrm{BH}}}{\partial E_{\mathrm{BH}}}\right)_{E_{\mathrm{BH}}=E}(-E_i)\\ &\quad + \frac{1}{2}\left(\frac{\partial^2\ln\Omega_{\mathrm{BH}}}{\partial {E_{\mathrm{BH}}}^2}\right)_{E_{\mathrm{BH}}=E}(-E_i)^2 + \cdots\end{aligned} \tag{5}$$

In the equation below,

$$\left(\frac{\partial\ln\Omega_{\mathrm{BH}}}{\partial E_{\mathrm{BH}}}\right)_{E_{\mathrm{BH}}=E} = \beta = \frac{1}{K_{\mathrm{B}}T}, \tag{6}$$

$$\left(\frac{\partial^2\ln\Omega_{\mathrm{BH}}}{\partial {E_{\mathrm{BH}}}^2}\right)_{E_{\mathrm{BH}}=E} = \frac{\partial\beta}{\partial E_{\mathrm{BH}}} = -\frac{K_{\mathrm{B}}\beta^2}{C_{\mathrm{BH}}}. \tag{7}$$

Calculating it to second order, we find that

$$P_i = \frac{\Omega_{\mathrm{BH}}(E-E_i)}{\Omega_E} = \frac{1}{Z}\mathrm{e}^{-\beta E_i - \frac{K_{\mathrm{B}}}{2C_{\mathrm{BH}}}\beta^2 {E_i}^2}. \tag{8}$$

In this equation

$$Z = \sum \mathrm{e}^{-\beta E_i - \frac{K_{\mathrm{B}}}{2C_{\mathrm{BH}}}\beta^2 {E_i}^2}. \tag{9}$$

So together we have

$$\hat{\rho} = |\psi_i\rangle\frac{1}{Z}\mathrm{e}^{-\beta E_i - \frac{K_{\mathrm{B}}}{2C_{\mathrm{BH}}}\beta^2 {E_i}^2}\langle\psi_i| = \frac{1}{Z}\mathrm{e}^{-\beta\hat{H} - \frac{K_{\mathrm{B}}}{2C_{\mathrm{BH}}}\beta^2\hat{H}^2}. \tag{10}$$

In a canonical system, $\hat{H}$ is the Hamiltonian operator. For Schwarzschild–de Sitter black hole,

$$E = M, \quad T = \frac{\kappa_{\mathrm{H}}}{2\pi k_{\mathrm{B}}}, \quad E_i = \omega, \quad C_{\mathrm{BH}} = -\frac{K_{\mathrm{B}}\beta^2}{8\pi}, \tag{11}$$

where M is the mass of the Schwarzschild–de Sitter black hole, and κ_{H} is the surface gravitation or black hole horizon. We can find that

$$P_i = \Gamma \propto \mathrm{e}^{\Delta S_{\mathrm{BH}}}. \tag{12}$$

As a result, when we use canonical ensemble to deal with an isolated system, then the probability distribution function, P_i, is similar to black hole's emission rate.

3. Situation of entropy, information on emission shell of black hole horizon

The line element of Schwarzschild–de Sitter black hole is

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2. \tag{13}$$

It has a black hole and a cosmological horizon. For black hole horizon, the entropy of S-wave (emission shell) fulfils

$$Im S = Im\int p_r \mathrm{d}r = Im\int\int \mathrm{d}p_r'\mathrm{d}r. \tag{14}$$

Combining with equation (4), we have

$$Im S = Im\int\int \frac{\mathrm{d}\omega'}{\dot{r}}\mathrm{d}r. \tag{15}$$

Considering the emission shell's self-gravitational interaction,

$$\dot{r} = -\frac{1}{2r}\sqrt{\frac{\Lambda r}{3}}\frac{\left(r^3 - \frac{3}{\Lambda}r + \frac{6(M-\omega')}{\Lambda}\right)}{\sqrt{r^3 + \frac{6(M-\omega')}{\Lambda}}} = -\frac{1}{2r}\sqrt{\frac{\Lambda r}{3}}\frac{(r - r_-')(r - r_H')(r - r_C')}{\sqrt{r^3 + \frac{6(M-\omega')}{\Lambda}}}. \tag{16}$$

Combining with equation (15), we have

$$Im S = -Im\int\int \frac{2r\sqrt{r^3 + \frac{6(M-\omega')}{\Lambda}}}{\sqrt{\frac{\Lambda r}{3}}(r - r_-')(r - r_{\mathrm{H}}')(r - r_{\mathrm{C}}')}\mathrm{d}\omega'\mathrm{d}r. \tag{17}$$

In this integral there is an extreme value, $r = r'_{\mathrm{H}}\frac{1}{2}\sqrt{\frac{1}{\Lambda}}\cos\left(\frac{\varphi'}{3}+\frac{\pi}{3}\right)$, where

$$\cos\varphi' = -3(M-\omega')\sqrt{\Lambda}. \tag{18}$$

So, we can calculate

$$ImS = -\frac{6\pi}{\Lambda}\int\frac{r'_H}{(r'_{\mathrm{H}}-r'_{\mathrm{C}})(r'_{\mathrm{H}}-r'_{-})}\mathrm{d}\omega'. \tag{19}$$

From equation (18), we have

$$\mathrm{d}\omega' = \frac{-\sin\varphi'}{3\sqrt{\Lambda}}\mathrm{d}\varphi'. \tag{20}$$

So we have

$$ImS = -\frac{\pi}{2}({r_f}^2 - {r_i}^2) = -\frac{1}{2}\Delta S_{\mathrm{BH}}. \tag{21}$$

In equation (21), $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(M-\omega) - S_{\mathrm{BH}}(M)$ is the change of entropy after the particles emission. The emission rate is

$$\Gamma \sim \exp[-2ImS] = \mathrm{e}^{\Delta S_{\mathrm{BH}}}. \tag{22}$$

The total entropy of the system is greater than the entropy before emission. It supports the conservation law of information.

4. Conclusion

In our work we use a canonical ensemble model to deal with the tunnelling framework of Parikh and Wilczek. The probability distribution function about the canonical ensemble is same as the emission of the particle's tunnelling rate. The quantum tunnelling rate is the same as the probability when the Schwarzschild–de Sitter black holes state with mass M changing into mass $M-\omega$. When the spherical shell emits from the black hole, information will also be taken from the black hole. By using the canonical ensemble model, we can discuss the statistical meaning of quantum tunnelling radiation in a different way. It infers a different discussion of the problem of information in the process of Hawking radiation. We believe that the canonical ensemble model can be extended to other kinds of black hole and get more useful findings.

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