

## Evolution of Primordial Black Holes in Loop Quantum Cosmology

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**Abstract.** In this work, we study the evolution of primordial black holes within the context of loop quantum cosmology. First we calculate the scale factor and energy density of the Universe for different cosmic era and then taking these as inputs, we study evolution of primordial black holes. From our estimation it is found that accretion of radiation does not affect evolution of primordial black holes in loop quantum cosmology. We also conclude that due to slow variation of scale factor, the upper bound on initial mass fraction of presently evaporating PBHs are much greater in loop quantum cosmology than the standard case.

*Key words.* Primordial black holes—loop quantum cosmology—accretion—Hawking evaporation.

### 1. Introduction

The demand for consistency between a quantum description of matter and a geometric description of space–time indicate the necessity of a complete theory of quantum gravity. This theory is expected to provide a new light on singularities present in classical cosmology. Einstein’s theory of general relativity leads to the occurrence of space–time singularities in a generic way. So, one may say, general relativity is severely incomplete and is unable to predict what will come out of a singularity. One of the outstanding problems in classical Einstein cosmology is the Big Bang singularity which is expected to be solved by quantum gravity. Loop Quantum Gravity (LQG) (Thiemann 2001, 2007; Ashtekar & Lewandowski 2004; Han *et al.* 2007; Rovelli 2004) is one of the best motivated theories of quantum gravity. LQG is a background independent, non perturbative approach to quantum gravity. When loop quantum gravity is applied to cosmology to analyse our Universe, it is called Loop Quantum Cosmology (LQC) (Ashtekar *et al.* 2006a, b) (also see

Banerjee *et al.* (2012) for a comprehensive review on LQC). In loop quantum cosmology, the non perturbative effects add a term of  $-\rho^2/\rho_c$  to the standard Friedmann equation (Ashtekar *et al.* 2006a, b; Singh 2006; Copeland *et al.* 2006), where  $\rho$  represents the energy density of the Universe and  $\rho_c$  is the critical density at which the universe is completely filled with a free massless scalar field when the scale factor reaches a minimum. The modification becomes important when the energy density of the Universe approaches critical density ( $\rho_c$ ) and causes the quantum bounce. So the classical Big Bang is replaced by a quantum big bounce in such a quantum theory of gravity. Since inverse volume corrections have not been fully realized in mini superspace models so far (Bojowald 2009), here we neglect that term. Recently more and more researchers have paid attention to LQC inspired by its appealing features, like avoidance of various singularities (Sami *et al.* 2006; Cailleteau *et al.* 2008), inflation in LQC (Zhang & Ling 2007; Bojowald *et al.* 2011), large scale effects (Bojowald *et al.* 2007; Bojowald 2007), present cosmic acceleration (Xiao & Zhu 2010; Jamil *et al.* 2010; Sadjadi & Jamil 2011; Fu *et al.* 2008; Wu *et al.* 2008; Cognola *et al.* 2005) and so on.

Primordial Black Holes (PBHs) are the black holes formed in the early Universe (Carr & Hawking 1974; Carr 2003; Jamil & Qadir 2011; Jamil 2010). A comparison of the cosmological density at any time after the Big Bang with the density associated with a black hole shows that PBHs would have mass of the order of the particle horizon mass at their formation epoch. Thus PBHs could span enormous mass range starting from  $10^{-5}$  g to the typical values of  $10^{15}$  g. These black holes could be formed due to initial inhomogeneities (Zeldovich & Novikov 1967; Carr 1975), inflation (Kholpov *et al.* 1985; Carr *et al.* 1994), phase transitions (Kholpov & Polnarev 1980), bubble collisions (Kodma *et al.* 1982; La & Steinhardt 1989) or the decay of cosmic loops (Polnarev & Zemboricz 1991). In 1974, Hawking discovered that the black holes emit thermal radiation due to quantum effects (Hawking 1975). So the black holes get evaporated depending upon their initial masses. Smaller the initial masses of the PBHs, quicker they evaporate. But the density of a black hole varies inversely with its mass. So high density is required to form lighter black holes and such high densities are available only in the early Universe. So primordial black holes are the only black holes whose masses could be small enough to have evaporated by present time. There have been speculations that PBHs could act as seeds for structure formation (Mack *et al.* 2007) and could also form a significant component of dark matter (Blais *et al.* 2002, 2003). Since the cosmological environment was very hot and dense in the radiation-dominated era, an appreciable amount of energy-matter from the surroundings can be absorbed by black holes. Such accretion is responsible for the prolongation of life time of PBHs (Majumdar 2003; Majumdar & Mukharjee 2005; Majumdar *et al.* 2008; Nayak *et al.* 2009, 2010; Nayak & Singh 2010, 2011; Nayak & Jamil 2012).

In this work, we study the evolution of PBH within the context of loop quantum cosmology. First, we estimate the cosmic scale factor  $a(t)$  and energy density  $\rho(t)$  of the fluid filling the Universe for different cosmic era within the context of loop quantum cosmology. Taking these as inputs, PBH evolution is studied considering both the Hawking evaporation and accretion of radiation by the PBH. The primary aim being to compare the results so obtained with the analyses carried out earlier within the context of general theory of relativity and Brans–Dicke theories.

## 2. Solution of Friedmann equations

For a spatially flat FRW Universe ( $k = 0$ ) with scale factor ( $a$ ), the Friedmann equation in loop quantum cosmology takes the form (Bojowald 2005; Ashtekar *et al.* 2006; Chen *et al.* 2008; Jamil *et al.* 2011; Jamil & Debnath 2011)

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \quad (1)$$

where  $H$  is the Hubble parameter,  $\rho$  is the energy density and  $\rho_c$  represents the critical value of energy density of the universe given by  $\rho_c = \frac{\sqrt{3}}{16\pi^2\gamma^3}\rho_{\text{Pl}}$  with  $\gamma = \frac{\ln 2}{\pi\sqrt{3}}$  the dimensionless Barbero-Immirzi parameter (Ashtekar *et al.* 1998; Domagala & Lewandowski 2004; Meissner 2004) and  $\rho_{\text{Pl}}$  is the energy density of the Universe in Planck time.

The energy conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (2)$$

Here  $p$  is the pressure of the fluid filling the Universe. The pressure and energy density are connected via an equation of state  $p = w\rho$ .

From energy conservation equation, we get the evolution of energy density during radiation-dominated ( $w = 1/3$ ) and matter-dominated ( $w = 0$ ) era respectively as

$$\rho \propto \begin{cases} a^{-4}, & t < t_e, \\ a^{-3}, & t > t_e, \end{cases} \quad (3)$$

where  $t_e$  is the time of radiation–matter equality.

Using this solution in equation (1), one gets the temporal behaviour of the scale factor  $a(t)$  as shown below. For radiation-dominated era ( $t < t_e$ ), the scale factor is

$$a(t)_{t < t_e} = \left[ \frac{\rho_0 a_0^3 a_e}{\rho_c} + \left\{ 2\rho_0^{1/2} a_0^{3/2} a_e^{1/2} \sqrt{\frac{8\pi G}{3}} (t - t_e) + \left( a_e^4 - \frac{\rho_0 a_0^3 a_e}{\rho_c} \right)^{1/2} \right\}^2 \right]^{1/4}, \quad (4)$$

where the subscript 0 indicates the present value of any parameter and  $a_e = a(t_e)$ . Also for matter-dominated era ( $t > t_e$ ), the scale factor varies as

$$a(t)_{t > t_e} = \left[ \frac{\rho_0 a_0^3}{\rho_c} + \left\{ \frac{3}{2}\rho_0^{1/2} a_0^{3/2} \sqrt{\frac{8\pi G}{3}} (t - t_0) + \left( a_0^3 - \frac{\rho_0}{\rho_c} a_0^3 \right)^{1/2} \right\}^2 \right]^{1/3}. \quad (5)$$

Here one can notice that the scale factor in LQC varies at a slower rate than standard cosmology. The absence of Big Bang in LQC may be the reason of this discrepancy. We know that the energy created during Big Bang is responsible for expansion of the universe. But in LQC as the big bang is absent, there is less amount of energy

available for expansion and hence the Universe expands at a slower rate than the standard case.

Using equations (3), (4) and (5), we get

$$\rho(t)_{t < t_e} = \rho_0 \left[ \frac{\rho_0}{\rho_c} + \left\{ 2\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_e) + \frac{3}{2}\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c}\right)^{1/2} \right\}^2 \right]^{-1} \quad (6)$$

and

$$\rho(t)_{t > t_e} = \rho_0 \left[ \frac{\rho_0}{\rho_c} + \left\{ \frac{3}{2}\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_0) + \left(1 - \frac{\rho_0}{\rho_c}\right)^{1/2} \right\}^2 \right]^{-1}. \quad (7)$$

Again differentiating equation (1) and using energy conservation equation (2), we get modified Raychaudhuri's equation as

$$\dot{H} = -4\pi G(\rho + p) \left(1 - \frac{2\rho}{\rho_c}\right). \quad (8)$$

### 3. Accretion of radiation

When a PBH evolves through radiation-dominated era, it can also accrete radiation from the surrounding. The accretion of radiation leads to an increase in mass with the rate given by

$$\dot{M}_{\text{acc}} = 4\pi f r_{\text{BH}}^2 \rho_r, \quad (9)$$

where  $\rho_r$  is the radiation energy density of the surrounding of the black hole,  $r_{\text{BH}}$  is the black hole radius and  $f$  is the accretion efficiency. The value of the accretion efficiency  $f$  depends on the complex physical processes such as the mean free path of the particles comprising the radiation surrounding PBHs. Any peculiar velocity of the PBH with respect to the cosmic frame could increase the value of  $f$  (Majumdar et al. 1995; Guedens et al. 2002). Since the precise value of  $f$  is unknown, it is customary (Page & Hawking 1976) to take the accretion rate to be proportional to the product of the surface area of the PBH and the energy density of radiation with  $f \sim \text{O}(1)$ .

After substituting the expressions for  $r_{\text{BH}} = 2GM$  and  $\rho_r$  given by equation (6) in equation (9), we get

$$\dot{M}_{\text{acc}} = 16\pi f G^2 M_{\text{acc}}^2 \rho_0 \left[ \frac{\rho_0}{\rho_c} + \left\{ 2\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_e) + \frac{3}{2}\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c}\right)^{1/2} \right\}^2 \right]^{-1}. \quad (10)$$

Solving the above equation (10), one can find

$$M_{\text{acc}}(t) = \left\{ M_i^{-1} - 8\pi f G^2 \rho_c^{1/2} \sqrt{\frac{3}{8\pi G}} \tan^{-1} \left[ \frac{\rho_c}{\rho_0} Z(t) - 1 \right]^{1/2} + 8\pi f G^2 \rho_c^{1/2} \sqrt{\frac{3}{8\pi G}} \tan^{-1} \left[ \frac{\rho_c}{\rho_0} Z(t_i) - 1 \right]^{1/2} \right\}^{-1}. \quad (11)$$

Again using horizon mass as initial mass of PBH i.e;  $M_i = M_H(t_i) = G^{-1}t_i$ , we get

$$M_{\text{acc}}(t) = M_i \left[ 1 - 8\pi f G^{1/2} \rho_c^{1/2} t_i \sqrt{\frac{3}{8\pi}} \left\{ \tan^{-1} \left[ \frac{\rho_c}{\rho_0} Z(t) - 1 \right]^{1/2} - \tan^{-1} \left[ \frac{\rho_c}{\rho_0} Z(t_i) - 1 \right]^{1/2} \right\} \right]^{-1}, \quad (12)$$

where  $Z(t) = \frac{\rho_0}{\rho_c} + \{2\sqrt{\frac{8\pi G}{3}}\rho_0^{1/2}(t - t_e) + \frac{3}{2}\sqrt{\frac{8\pi G}{3}}\rho_0^{1/2}(t_e - t_0) + (1 - \frac{\rho_0}{\rho_c})^{1/2}\}^2$ .

#### 4. Evaporation of PBH

As is well known black holes can also lose mass through Hawking evaporation. The rate at which the PBH mass decreases due to evaporation is given by

$$\dot{M}_{\text{evap}} = -4\pi r_{\text{BH}}^2 a_H T_{\text{BH}}^4, \quad (13)$$

where  $a_H \sim$  is the black body constant and  $T_{\text{BH}} \sim$  is the Hawking temperature =  $\frac{1}{8\pi GM}$ . Now equation (13) becomes

$$\dot{M}_{\text{evap}} = -\frac{a_H}{256\pi^3} \frac{1}{G^2 M_{\text{evap}}^2}. \quad (14)$$

Integrating the above equation (14), we get

$$M_{\text{evap}}(t) = [M_i^3 + 3\alpha(t_i - t)]^{1/3}, \quad (15)$$

where  $\alpha = \frac{a_H}{256\pi^3} \frac{1}{G^2}$  and  $M_i$  is the the black hole mass at its formation time  $t_i$ . We can rewrite equation (15) as

$$M_{\text{evap}}(t) = M_i \left[ 1 + \frac{3\alpha}{M_i^3} (t_i - t) \right]^{1/3}. \quad (16)$$

#### 5. PBH dynamics in different era

Primordial black holes, as discussed earlier, are formed only in radiation dominated era. We now study PBHs so formed in two categories: (i) PBHs evaporating in radiation dominated era ( $t_{\text{evap}} < t_e$ ) and (ii) PBHs evaporating in matter dominated era ( $t_{\text{evap}} > t_e$ ).

5.1  $t_{\text{evap}} < t_e$ 

If we consider both evaporation and accretion simultaneously, then the rate at which primordial black hole mass changes is given by

$$\dot{M}_{\text{PBH}} = 16\pi f G^2 M_{\text{PBH}}^2 \rho_0 \left[ \frac{\rho_0}{\rho_c} + \left\{ 2\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_e) + \frac{3}{2}\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c}\right)^{1/2} \right\}^2 \right]^{-1} - \frac{a_H}{256\pi^3} \frac{1}{G^2 M_{\text{PBH}}^2}. \quad (17)$$

Solving the above equation (17) numerically, we construct Table 1 for PBHs which are evaporating in radiation-dominated era. In our calculation, we have used  $\rho_0 = 1.1 \times 10^{-29} \text{ g cm}^{-3}$ ,  $\rho_c = 5.317 \times 10^{94} \text{ g cm}^{-3}$ ,  $G = 6.673 \times 10^{-8} \text{ dyne cm}^2/\text{g}^2$ ,  $t_e = 10^{11} \text{ s}$ ,  $t_0 = 4.42 \times 10^{17} \text{ s}$  and  $M_e = 10^{49} \text{ g}$ .

It is clear from Table 1 that with increase in initial mass, evaporating time increases. However radiation accretion, surprisingly, seems to have little effect on evolution of PBH unlike the results obtained in theories of Einstein or scalar-tensor type (Nayak *et al.* 2009; Nayak & Singh 2011). This is also shown in Fig. 1.

5.2  $t_{\text{evap}} > t_e$ 

Since there is insignificant accretion of radiation in matter dominated era, the first term in the combined equation (17) for variation of  $M_{\text{PBH}}$  with time should be integrated only up to  $t_e$ . Based upon the numerical solution, we construct Table 2 for the PBHs evaporating by the present time.

It is clear from Table 2 that PBH evaporation is again not affected by radiation accretion efficiency.

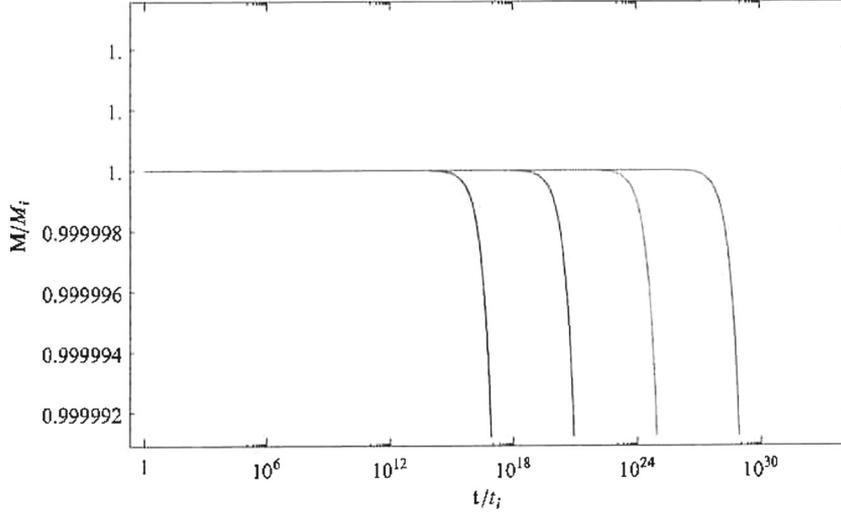
## 6. Constraints on PBH

The fraction of the Universe mass going into PBHs at time  $t$  is (Carr 1975)

$$\beta(t) = \left[ \frac{\Omega_{\text{PBH}}(t)}{\Omega_R} \right] (1+z)^{-1}, \quad (18)$$

**Table 1.** Display of formation times and initial masses of the PBHs evaporating in radiation-dominated era.

$t_i$	$M_i$	$t_{\text{evap}}$	
		$f = 0$	$f = 1$
$10^{-32} \text{ s}$	$10^6 \text{ g}$	$3.333 \times 10^{-11} \text{ s}$	$3.333 \times 10^{-11} \text{ s}$
$10^{-30} \text{ s}$	$10^8 \text{ g}$	$3.333 \times 10^{-5} \text{ s}$	$3.333 \times 10^{-5} \text{ s}$
$10^{-28} \text{ s}$	$10^{10} \text{ g}$	$3.333 \times 10^1 \text{ s}$	$3.333 \times 10^1 \text{ s}$
$10^{-26} \text{ s}$	$10^{12} \text{ g}$	$3.333 \times 10^7 \text{ s}$	$3.333 \times 10^7 \text{ s}$



**Figure 1.** Evaporation of PBHs for different initial masses (i.e.  $10^6$  g,  $10^8$  g,  $10^{10}$  g and  $10^{12}$  g) are shown in the figure where axes are taken in logarithmic scale.

where  $\Omega_{\text{PBH}}(t)$  is the density parameter associated with PBHs formed at time  $t$ ,  $z$  is the redshift associated with time  $t$  and  $\Omega_R$  is the microwave background density having value  $10^{-4}$ . Substituting the value of  $\Omega_R$  in the equation (18), we get

$$\beta(t) = (1 + z)^{-1} \Omega_{\text{PBH}}(t) \times 10^4. \quad (19)$$

For  $t < t_e$ , redshift definition implies

$$(1 + z)^{-1} = \frac{a(t)}{a(t_0)} = \frac{a(t) a(t_e)}{a(t_e) a(t_0)}. \quad (20)$$

But here

$$\frac{a(t)}{a(t_e)} = \left[ \frac{\rho_0 a_0^3}{\rho_c a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} (t - t_e) + \left( 1 - \frac{\rho_0 a_0^3}{\rho_c a_e^3} \right)^{1/2} \right\}^2 \right]^{1/4} \quad (21)$$

**Table 2.** Display of formation times of PBHs which are evaporating now for several accretion efficiencies.

$t_{\text{evap}} = t_0 = 4.42 \times 10^{17}$ s		
$f$	$t_i$	$M_i$
0	$2.3669 \times 10^{-23}$ s	$2.3669 \times 10^{15}$ g
0.25	$2.3669 \times 10^{-23}$ s	$2.3669 \times 10^{15}$ g
0.5	$2.3669 \times 10^{-23}$ s	$2.3669 \times 10^{15}$ g
1.0	$2.3669 \times 10^{-23}$ s	$2.3669 \times 10^{15}$ g

and

$$\frac{a(t_e)}{a(t_0)} = \left[ \frac{\rho_0}{\rho_c} + \left\{ \frac{3}{2} \rho_0^{1/2} \sqrt{\frac{8\pi G}{3}} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c}\right)^{1/2} \right\}^2 \right]^{1/3}. \quad (22)$$

Using numerical values of different quantities in equation (22), we get

$$\frac{a(t_e)}{a(t_0)} = 0.746. \quad (23)$$

Again using equations (21) and (23) in equation (20), one can write

$$(1+z)^{-1} = \left[ \frac{\rho_0 a_0^3}{\rho_c a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} (t - t_e) + \left(1 - \frac{\rho_0 a_0^3}{\rho_c a_e^3}\right)^{1/2} \right\}^2 \right]^{1/4} \times 0.746. \quad (24)$$

Now substitution of equation (24) in equation (19) implies

$$\beta(t) = \left[ \frac{\rho_0 a_0^3}{\rho_c a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} (t - t_e) + \left(1 - \frac{\rho_0 a_0^3}{\rho_c a_e^3}\right)^{1/2} \right\}^2 \right]^{1/4} \times 0.746 \times \Omega_{\text{PBH}}(t) \times 10^4. \quad (25)$$

Using horizon mass as the formation mass of PBH i.e.  $M = G^{-1}t$ , we can write equation (25) to represent the fraction of the Universe going into PBHs' with formation mass  $M$  as

$$\beta(M) = \left[ \frac{\rho_0 a_0^3}{\rho_c a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} G(M - M_e) + \left(1 - \frac{\rho_0 a_0^3}{\rho_c a_e^3}\right)^{1/2} \right\}^2 \right]^{1/4} \times 0.746 \times \Omega_{\text{PBH}}(M) \times 10^4. \quad (26)$$

Observations imply that  $\Omega_{\text{PBH}}(M) < 1$  over all mass ranges for which PBHs have not evaporated yet. But presently evaporating PBHs( $M_*$ ) generate a  $\gamma$ -ray background where most of the energy is appearing at around 100 Mev (Carr 1994). If the fraction of the emitted energy which goes into photons is  $\epsilon_\gamma$ , then the density of the radiation at this energy is expected to be  $\Omega_\gamma = \epsilon_\gamma \Omega_{\text{PBH}}(M_*)$ . Since  $\epsilon_\gamma \sim 0.1$  (Page 1976) and the observed  $\gamma$ -ray background density around 100 Mev is  $\Omega_\gamma \sim 10^{-9}$ , we get  $\Omega_{\text{PBH}} < 10^{-8}$ .

With the use of all these parameters, equation (26) leads to an upper bound

$$\beta(M_*) < 0.746 \times 10^{-4}, \quad (27)$$

where  $M_*$  is the initial mass of PBHs which are evaporating at  $t_0$ . The value of  $M_*$  is calculated by taking  $t_{\text{evap}} = t_0$  which is given in Table 2.

Since scale factor varies at a slower rate in LQC than standard cosmology and the initial mass fraction of PBH strongly depends on variation of scale factor through

$(1+z)^{-1}$ , the initial mass fraction of PBHs differ from the standard result (Nayak *et al.* 2009; Nayak & Singh 2011).

## 7. Conclusion

We have studied PBH evolution in loop quantum cosmology. We have estimated the cosmic scale factor  $a(t)$  and energy density  $\rho(t)$  of the Universe for both radiation- and matter-dominated era and found that scale factor varies at a slower rate in LQC than in standard cosmology. Using these results as inputs we have studied evolution of PBHs using both accretion of radiation and evaporation. We find accretion of radiation has no effect on PBH evaporation in the present formalism. From numerical calculation it is found that the PBHs created before  $1.443 \times 10^{-25}$  s could evaporate completely in radiation dominated era and the accretion efficiency does not affect the evaporation of individual PBHs formed at different times in this era. Further, we found that the upper bounds on initial mass fraction of the presently evaporating PBHs are much greater than all previous analyses even though the formation times are nearly equal. The greater upper bound is due to the slow variation of scale factor, which is a result of the absence of Big Bang, in loop quantum cosmology in comparison with the standard cosmology and scalar-tensor theories.

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