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Interacting Winds in Eclipsing Symbiotic Systems – The Case Study of EG Andromedae

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Abstract. We report the mathematical representation of the so called eccentric eclipse model, whose numerical solutions can be used to obtain the physical parameters of a quiescent eclipsing symbiotic system. Indeed the nebular region produced by the collision of the stellar winds should be shifted to the orbital axis because of the orbital motion of the system. This mechanism is not negligible, and it led us to modify the classical concept of an eclipse. The orbital elements obtained from spectroscopy and photometry of the symbiotic EG Andromedae were used to test the eccentric eclipse model. Consistent values for the unknown orbital elements of this symbiotic were obtained. The physical parameters are in agreement with those obtained by means of other simulations for this system.

Key words. Symbiotic stars—stellar winds—eclipse condition—EG Andromedae.

1. Introduction

Symbiotic stars are stellar objects, whose spectra are characterized by absorbing lines of advanced spectral lines (Kenyon 1986), such as TiO bands and metal lines (Calabrò & Mammano 1992), emission lines of HeI and HeII. Some symbiotics have also shown forbidden lines such as [OIII] and [NeIII] (Mammano & Martini 1969; Mikolajewska 2012). Moreover, their luminosity is variable in a period of a few years, within a range close to 2 or 3 magnitudes (Skopal *et al.* 1995). Symbiotic phenomenon was widely reviewed in Mikolajewska & Kenyon (1992).

The most adopted physical modeling for many symbiotic stars is that of interacting binaries: a cool giant, a hot compact object whose energy distribution appears often in the UV, and a large nebula in which the components are embedded.

Eventual minima in the light curve might be performed by an eclipse phenomenon between the two stellar components. However, the application of the classical eclipse condition to these systems does not usually provide any real solution (Calabrò 1990).

Nevertheless, several models regarding symbiotics proposed up to now, suppose the presence of a third region in the system. It may have been built on the giant surface, such as that proposed by Chochol & Vittone (1986) for V1329 Cyg. On the other hand, a hot spot may form a ring around the giant, because of its evaporation state; for instance, this could happen in RS Oph (Garcia 1986). Finally, a disk may form accreting onto a compact object such as the one proposed by Garcia (1986) for CI Cyg and by Anderson *et al.* (1981) for AG Dra, or after an outburst as in the case of Z Andromedae (Sokoloski *et al.* 2006).

Many authors have suggested the presence of a third nebular region in a symbiotic, produced by the collision of two stellar winds (Vogel 1991, 1993; Girard & Willson 1987; Huang & Weigert 1982; Isliker *et al.* 1989).

In this paper the hypothesis of occultation phenomena, due to the presence of this region, has been proposed. The unknown parameters such as the inclination angle of the orbital plane, distances, masses and radii of the stars can be found by means of mathematical expressions derived from this modeling.

First of all, there are indications that during the evolution of symbiotics both stars are affected by mass loss (Allen & Wright 1988; Michalitsianos *et al.* 1988; Calvet *et al.* 1992; Vogel 1993). Therefore, in the nebular environment an energetic zone must form, produced by the collision of the stellar winds.

In fact, the two winds cannot diffuse into one another, as the free path length of a wind particle through the opposing stellar wind is too short (Huang & Weigert 1982).

The strength of the winds can be represented by their momenta, expressed in terms of the mass loss rate of the stars (M) and of the stellar winds' velocities. Hence, the momenta of the hot and cool winds are given by $M_{\text{hot}}v_{\text{hot}}$ and $M_{\text{cool}}v_{\text{cool}}$, respectively.

The location where the shocked region is formed due to the wind collision can be calculated by the expression $M_{\text{cool}}v_{\text{cool}} = kM_{\text{hot}}v_{\text{hot}}$ (Vogel 1993).

Kilpio & Bisikalo (2009) showed that contribution from the shocked region formed in the area of wind collision is significant especially at short wavelenghts. For instance, Tomov & Tomova (2001) performed the U light curve of the symbiotic AG Peg by the occultation of a bright gaseous region built by colliding winds. In fact, the analysis of Bisikalo *et al.* (2006) showed that the region between the shocks of the winds has a considerably higher temperature than the surrounding medium by a factor of 50. Furthermore, Kenny & Taylor (2005) determined the density at the static point within the interaction shell of this region, obtaining high values comparable with observed density limits in certain outbursting systems, and the ratio of thickness to binary separation (a reasonable parameter for symbiotics is about 0.05), which is comparable with a typical giant radius. Such a result can support the hypothesis of an eclipse by that region, as well.

The rate of mass loss and the corresponding wind velocities, depending on the evolutionary status of the components, determine the location of the shocked region, where the momenta of the winds are equal.

By using the expression kT_S (KeV) $= 1.2v_8^2$ (Luo *et al.* 1990; Prilutskii & Usov 1976; Stevens 1995), where v_8 is the wind's velocity in units of 10^8 cm/s, we find that the spectral band of the colliding giant's wind falls in the visual range. Indeed, the giant's wind values for a symbiotic vary from 20 km/s to 40 km/s. Energies between 0.48 eV and 1.92 eV correspond to them. This range is close to that of the visual band (from 0.4 eV to 4 eV), so that this region can modify the light curve due to its absorption effect.

Some mechanisms we suppose to be important in typical symbiotics are: the projection effects on the line-of-sight of a tidal distortion acting on the giant by the companion (Wilson & Vaccaro 1997), the occultation by a dense region built by the shock of the winds, eccentric as to the orbital axis, Rayleigh scattering in the extended atmosphere of the cool giant, and H^- absorption (Isliker *et al.* 1989).

These occurrences are related to the following conditions:

(1) The distance between the two stellar components should not be too greater than the giant's diameter (Walder 1995).
(2) The location of the shocked region between the stellar winds should be comparable to that of the primary Lagrangian point of the system. In fact, other authors (Garcia 1986; Harmanec 1982), proposed that in the zone of lower effective gravity of the system, the giant could enhance its mass loss, building a dense nebular region, and so developing an accreting shell.
(3) The wind velocity of the giant must be 30 km/s, at least. Indeed Bisikalo *et al.* (2006) showed that a wind velocity close to 20 km/s can produce an accretion disk in the system. Instead, if the wind velocity increases up to 30 km/s, the disk will disrupt and an expanding envelope, a pseudophotosphere or optically thick wind forms in the system. Hence, if the giant wind velocity amounts to 30 km/s, a conical shock must form.

We can also obtain its distance from the giant by assuming that this region forms when the balance of the wind momenta occurs. All the other physical elements of the system can be determined as well, as discussed in the next sections.

2. Opacity effects on the light curve of symbiotics

It is notoriously difficult to adapt a modeling to the spectroscopic observations and to the light curve shape of a symbiotic.

The quiescent symbiotics generally present a sinusoidal light curve with a primary and a secondary minimum. Such a shape can be easily performed in binaries by the reciprocal occultation of two stellar components of different luminosities. In symbiotic systems, instead, we know that the hot star is too small to produce any eclipse, unless a possible pseudo-photosphere is present.

Wilson & Vaccaro (1997) proposed that the double sinusoidal shape of EG Andromedae light curve could be performed by tidal distortions on the giant surface by the hot companion, that change the giant's envelope shape and, therefore, its emitted amount of light on the line-of-sight. Nevertheless, such a modeling does not explain the different depths between primary and secondary minima, neither between U-light curve, B-light curve and V-light curve.

However, besides tidal effects, occultation phenomena by dense winds shocked regions together with Rayleigh scattering could occur in some symbiotics.

Girard & Willson (1987) proposed a model in which the nebular region is built in two parts: a conical part and a spherical one. The conical part surround the area behind the cool component (see Girard & Willson 1987).

The cone apex angle (see Fig. 2 of Girard & Willson 1987) depends on the wind parameters: the product between the ratios of the mass loss rates and the velocities of the two stellar winds mw, where $m = \frac{M_{\rm hot}}{M_{\rm cool}}$ and $w = \frac{v_{\rm hot}}{v_{\rm cool}}$.

If $mw < 1$, the cool wind predominates the hot wind, and the conical surface envelopes the area behind the hot component; and if $mw > 1$, the hot wind predominates the cool wind. This conical surface divides the neutral wind of the giant (H, He) from the ionized wind (H^+, He^+) of the hot star (Vogel 1991).

Isliker *et al.* (1989) found that Rayleigh scattering by hydrogen in the cool wind of the giant can be a source of opacity for the hot star's radiation, especially in the UV continuum. Moreover, Isliker *et al.* (1989) showed that H^- can absorb very strongly. Indeed, in the inner part of the wind, the relatively high radiation temperature prevents the formation of H^-, that could absorb the same giant's radiation, mainly in the UV. These mechanisms could produce opacity effects in interacting systems, especially when physical conditions similar to those described in the previous section occur.

In this scenario, a light curve of sinusoidal shape (two minima alternatively separated by two maxima) typical for quiescent symbiotics, may be performed.

At phase $\Phi = 0$, the giant is at inferior conjunction and two opacity phenomena can occur as follows:

(1) The giant eclipses the shifted shocked region, whose emission must be heavy as proved by Bisikalo *et al.* (2006). Hence, a primary minimum occurs as the line-of-sight passes through the neutral region of the giant's wind (Fig. 1a).
(2) The hot star radiation could be occulted by the giant and the thickness of the shocked region, that is not neglegible as shown by Kenny & Taylor (2005), together with Rayleigh scattering effect by hydrogen of the cool wind. This effect can be weighty as shown by Isliker *et al.* (1989).

At phase $\Phi = 0.50$, the giant is at superior conjunction and the following phenomena can occur.

(1) The shocked region may become a source of opacities for the giant's radiation because of the presence of H^- in the inner cool wind, producing a secondary minimum (Fig. 1c).
(2) A possible pseudo-photosphere of the hot component could be a source of occultation on the shocked region and the giant, as well.

The amount of light increases at the phases 0.25 and 0.75, since the largest surface of the giant is visible there, not being occulted (Figs. 1b and 1d).

The depths of both minima increase from optical to UV, due to scattering mechanisms, as the line-of-sight passes through the densest nebular region of the cool wind.

Furthermore, Girard & Willson (1987) found that the axisymmetry produced in the velocity field of the winds by the orbital motion cannot be neglected. Bisikalo *et al.* (2004) in their 3D gas dynamical simulations for semi-detached binaries, showed the formation of a 'hot line' eccentric as to the orbital axis. Girard & Willson (1987) calculated a consequent variation in the v_{shell} up to 30%, by typical wind parameters for quiescent symbiotics. They inferred that the orbital motion of a symbiotic system may produce effects on the structure of the nebula.

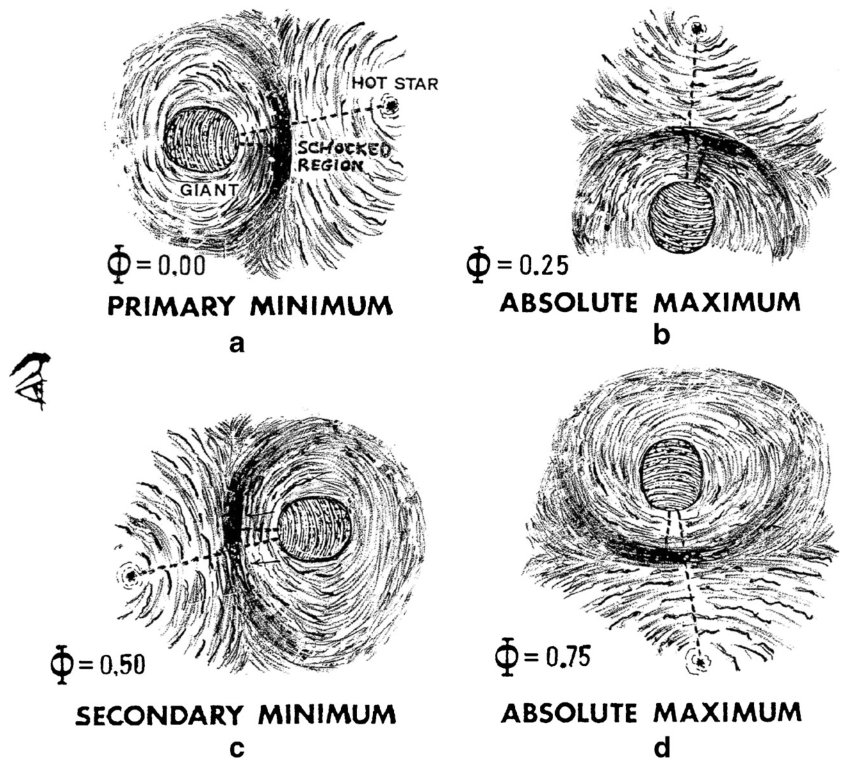

Figure 1(a–d). Eclipse modeling for a typical quiescent symbiotic system at the four orbital phases.

In fact, the typical value of the orbital velocity, close to 10 km/s, does not modify altogether hot wind velocities of several hundred km/s, but it is comparable to the ordinary cool wind velocities of 30 km/s.

Hence, the wind's particles ejected from the giant are shifted towards the direction of the orbital motion with respect to the orbital axis.

The angle between the two directions, pointed out in Fig. 2, is given by the vectorial composition between the cool wind's velocity and the orbital velocity:

$$\gamma = \tan^{-1} \frac{v_{\mathrm{orb}}}{v_{\mathrm{cool}}}, \tag{1}$$

where v_{orb} is the orbital velocity of the system and v_{cool} is the cool wind's velocity.

The densest zone of the cool wind must build in this shifted share, where scattering effects are the heaviest. In the following sections we have defined it as the *'shifted shocked region'*.

The modeling proposed here can be expressed using a mathematical representation from spherical astronomy, that can provide the inclination angle i of the orbital plane as to the normal plane to the line-of-sight and the other stellar parameters related to it.

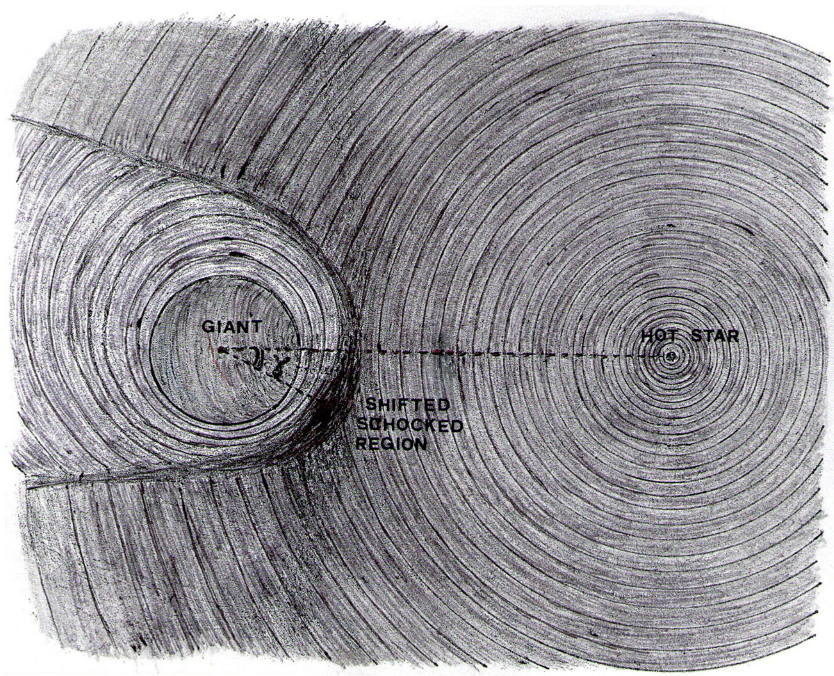

Figure 2. A nebular region forms around the giant, besides the shocked region of the wave train of the winds coming from the two components $mw > 1$. The densest zone is shifted by the angle γ, due to the orbital motion of the system.

3. The eccentric eclipse model

The radial velocities can be inferred from the visual component of a relevant number of spectra and the orbital elements are then derived by using some equations involving the following parameters: period p, eccentricity e, periastron angle W, periastron passage T_0, semi-amplitude K, gamma velocity V_0, mass function F, and the product $q = a_1 \sin i$.

Furthermore, a minimum in the light curve can be detected by several photometric observations. The value of the true anomaly at minimum $V_{\min}$ corresponds to it, that can be found by Kepler's equation.

To specify the value of the inclination angle i, we must consider another condition which the orbital elements must satisfy.

For the binaries in which the components have comparable dimensions, the light minima can be due to their reciprocal occultations. This condition is represented by the classical rigorous eclipse formulation, obtained by the minimization of the projected distance between the two components on the tangent plane, that is, the plane normal to the line-of-sight (Mammano 1979), or by an approximated formulation of the eclipse condition, obtained by a series development (Kopal 1959). Nevertheless, it is generally not suitable to symbiotic systems.

The presence of a nebular region in a symbiotic, not lined up with the two stars, can modify the classical eclipse formulation. Such a region can provide high density values (Kenny & Taylor 2005); this physical condition produces an occultation phenomenon on the giant which is possible.

Furthermore, the same region of H^-, formed in the inner part of the shifted nebular region can contribute to absorb the giant's radiation (Isliker *et al.* 1989), as the line-of-sight passes through it. Hence, the projected distance on the tangent plane between the giant and the shifted shocked region (rather than the two stellar components) must become the smallest at a secondary minimum (Fig. 1c).

The following schematization was obtained using concepts from spherical astronomy, whose techniques were successfully applied in previous studies (Calabrò 2011).

By assuming a system of spherical coordinates whose origin is on the barycentre G, the vector radii of the giant and of the central point of the shocked region (related to the orbital elements) have, respectively, the following expressions:

$$r = \frac{a_1(1-e^2)}{1+e\cos(V+W)}, \tag{2}$$

$$r' = \frac{r - a_h\cos\gamma}{\cos\theta}, \tag{3}$$

where W is the periastron angle, V is the giant's true anomaly, a_1 is the major semi-axis of the orbit, a_h is the distance between the centres of the giant and the shocked region, and

$$\theta = \tan^{-1}\left(\frac{a_h\sin\gamma}{\frac{q}{\sin i} - a_h\cos\gamma}\right) \tag{4}$$

is the angle joining the giant's true anomaly V, to obtain the true anomaly of the centre of the shifted shocked region. These quantities are represented in Fig. 3, where the giant and the hot star were indicated using the letters L and H, respectively. The orbital plane of the symbiotic system also contains the shocked region, which is shifted by the angle γ with respect to the stellar semi-axis, as represented in Fig. 3. The inclination angle i of the orbital plane as to the tangent plane (i.e., the normal plane to the line-of-sight) is pointed out in Fig. 3, as well. Finally, $q = a_1\sin i$ is a factor related to the orbital elements that can be obtained by spectroscopic observations. The distance a_h is related to the elements defined above by the following expression:

$$a_h = \frac{r\sin\theta}{\sin(\theta+\gamma)}. \tag{5}$$

By using (3) and (5) we have

$$r' = \frac{r\sin\gamma}{\sin(\theta+\gamma)}. \tag{6}$$

The projected distances ρ, ρ', ρ'' on the tangent plane of the distances r, r', a_h, are given as follows:

$$\rho = r\cos(FL), \tag{7}$$

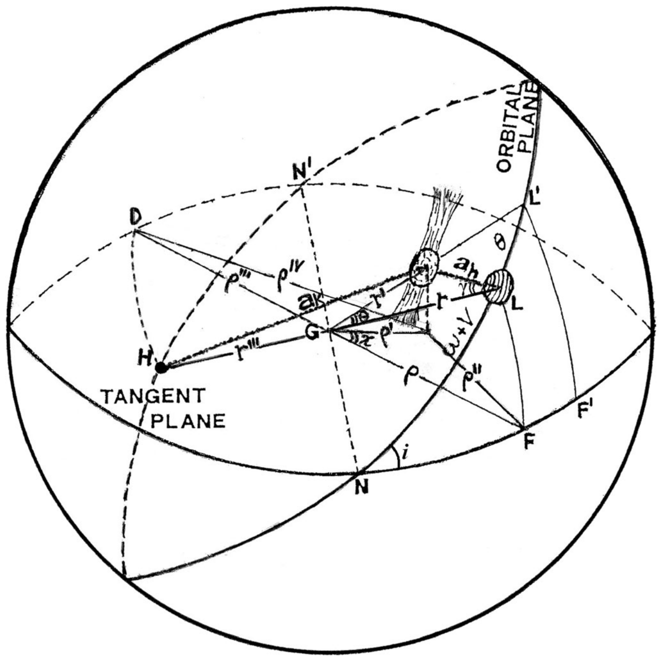

Figure 3. The modified eclipse condition between two bodies, not lined up with the barycentre G of the system: the giant, pointed out L, and the shifted shocked region, whose projection on the giant's orbit is L'.

where

$$\cos(FL) = \sqrt{1 - \sin^2 i \sin^2(W + V)},$$

$$\rho' = r' \cos(F'L'), \tag{8}$$

where

$$\cos\left(F'L'\right) = \sqrt{1 - \sin^2 i \sin^2(W + V + \theta)},$$

$$\rho''^2 = \rho^2 + \rho'^2 - 2\rho\rho' \cos\chi, \tag{9}$$

where (9) is given by the Carnot's theorem, and

$$\chi = \sin^{-1} \frac{r' \sin\theta \cos i}{\rho'} \tag{10}$$

is the angle on the tangent plane between ρ and ρ'. Also these quantities are pointed out in Fig. 3. At the observed secondary minima, their projected distance on the tangent plane, ρ'', must be minimum. At the primary light minima, the projected distance on the tangent plane, ρ^{IV} (between the shifted shocked region and the hot star) must be minimum, as well.

The physical condition that the projected distance, ρ'', between the giant and the shocked region be minimum at a secondary minimum, produces an occultation phenomenon which is possible. This expresses itself in the mathematical condition:

$$\frac{\partial \rho''}{\partial V} = 0. \tag{11}$$

Equation (11) is a transcendent equation in which, through equations (2–4), (7–9), the following terms appear: the orbital elements (known by spectroscopic observations), the true anomaly at minimum V_{min}, and the two unknowns, i and a_h, related to i by (5) and (4). This equation can be solved with respect to i and a_h, by means of numerical methods. Nevertheless, the shifting angle γ, between the direction of the giant's wind of highest momentum and the orbital axis (appearing in equation (11)), contains the giant's orbital velocity, as seen in equation (1), that depends on the variable i, through the expression

$$v_{\text{orb}} = \frac{v_{\text{obs}}}{\sin i}. \tag{12}$$

Equation (11) can be solved by iterative methods, giving arbitrary values to i in (12) and finding the consequent values of i and a_h in (11), until a convergence is reached. Similar opacity effects can occur at a primary minimum, due to the eclipse of the hot star radiation by the same inner part of the cool wind, wherever it forms the shifted nebula region, and the greatest density values are reached (Isliker *et al.* 1989). As the line-of-sight passes through it, the projected distance between the hot star and this region must become the smallest at a primary minimum (Fig. 1a).

The vector radius of the hot star have the expression

$$r''' = \frac{a_2(1 - e^2)}{1 + e\cos(V + W + \pi)} \tag{13}$$

and a_k is the distance between the centres of the hot component and of the shocked region

$$a_k = \sqrt{r'''^2 + r'^2 - 2r'''r' \cos(\pi - \theta)} \tag{14}$$

pointed out in Fig. 3, that is given by Carnot's theorem.

The projected distances on the tangent plane of r' and a_k are given by

$$\rho''' = r''' \cos(DH), \tag{15}$$

where

$$\begin{aligned} \cos(DH) &= \sqrt{1 - \sin^2 i \sin^2(W + V + \pi)} = \cos(FL), \\ (\rho^{\text{IV}})^2 &= \rho'^2 + \rho'''^2 - 2\rho'\rho''' \cos \chi', \end{aligned} \tag{16}$$

where $\chi' = \pi - \chi$.

The physical condition that the projected distance ρ^{IV} between the hot star and the shocked region be minimum at a light minimum is expressed by a mathematical formulation analogue to (11) that can be solved by numerical iterative methods, as well:

$$\frac{\partial \rho^{\mathrm{IV}}}{\partial V} = 0. \tag{17}$$

The observed velocity v_{obs} that appears in (12), must refer to a light minimum, because (11) and (17) represent a minimum condition. Therefore, it has to be related to that point of the orbit where a light minimum occurs,

$$v_{\mathrm{orb}(T_{\mathrm{min}})} = \frac{v_{\mathrm{obs}(T_{\mathrm{min}})}}{\sin i}. \tag{18}$$

As symbiotic binary orbits can be different from circular, we must consider the velocity as variable along the orbit: $v = \mathrm{d}r/\mathrm{d}t$, in which r is the giant's radius vector related to the orbital elements by (2).

The giant's areal velocity must be considered to express it in terms of the orbital elements, that is, the area covered by its radius vector in one unit of time (at time 1 it is $A_1 = \frac{1}{2} r_1^2 V_1$).

By applying Kepler's second law, we obtain the time required to cover the corresponding area 1, $t_1 = T * A_1/A$, where A represents the whole area delimited by the giant's orbit, and T the orbital period. An orbital time $t_i = T * A_i/A$ and an angular velocity $\omega_i = (\mathrm{d}v/\mathrm{d}t)_i$ corresponds to a true anomaly value V_i. In the orbital point P_i in which $V = V_{\mathrm{minimum}}$, we have the radial orbital velocity $v_{\mathrm{orb}(T_{\mathrm{min}})} = \omega_{\mathrm{min}} r_{\mathrm{min}}$. Other constraints for this eclipse are

$$\rho^{\mathrm{IV}} \leq R_{\text{shocked region}}, \tag{19}$$

$$\rho'' \leq R_{\text{giant}} + R_{\text{shocked region}}, \tag{20}$$

so that the projected distance ρ'' may be considered as a lower limit for the radius of the giant. By giving a value to the mass of a stellar component, the mass value of the other component can be obtained using the mass function

$$f(m_1) = \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}. \tag{21}$$

The sum of the semi-axes $a = a_1 + a_2$, and the Lagrangian point, are related to them.

4. Applying the eccentric eclipse model to the symbiotic EG Andromedae

4.1 *The state-of-the-art for EG Andromedae*

EG Andromedae (HD 4174) is a symbiotic system in which the optical component is a giant M3III star (Mürset & Schmidt 1999), as shown by low excitation lines in its spectra. A hot compact object in the system must be present, as its continuum increase in the UV. The radiation temperature close to 75000 K and a luminosity $10L_\odot \leq L \leq 20L_\odot$ (Munari 1993; Murset *et al.* 1991; Vogel *et al.* 1992) were deduced. Moreover, the whole system is embedded in a nebula, as inferred by forbidden lines of [OIII] and [NeIII] (Wilson 1950; Skopal *et al.* 1991).

Although the form of EG Andromedae light curve led us to consider an occultation phenomenon, by using the classical eclipse conditions of Kopal (1959) or

Mammano (1979), we cannot find any real solution for the inclination angle. In fact, with the orbital elements of Munari (1993) at the epoch of the primary minimum of 2446337 JD, we obtained $(\sin i)^2 = -1.5 \pm 1.3$; an analogue negative result is obtained with the orbital elements of Skopal *et al.* (1991), which is $(\sin i)^2 = -4.4 \pm 2.2$. This fact could find its explanation in the small dimensions of the hot companion. Neither can a hot spot, lined up with the stellar axis, produce any eclipse, because it must verify the classical eclipse condition, as well.

The modeling proposed in the previous section refers to those quiescent symbiotics in which the presence of a nebular region, built by the collision of the stellar winds, can be considered.

Vogel (1993) proved for EG Andromedae the existence of a wind coming from the hot star, while Torbett & Campbell (1989) evaluated a wind terminal velocity for the giant of 30 km/s. Incidentally, Bisikalo *et al.* (2006) showed that when the giant's wind velocity amounts to 30 km/s, at least, a conical shock forms, represented by an optically thick wind. Hence, the existence of a nebular region can be assumed, produced by the collision of the stellar winds. Furthermore, the orbital velocity of this system, close to 7 km/s, is comparable to the cool wind's velocity assumed to be 30 km/s, so that the shocked region must be shifted with respect to the orbital axis of the system. This circumstance led us to apply the modeling of an eccentric eclipse to this quiescent symbiotic.

We know that the shocked region builds in that points where the two winds momenta become equal, $\dot{M}_{\text{cool}} v_{\text{cool}} = k \dot{M}_{\text{hot}} v_{\text{hot}}$ (Vogel 1993).

Using the values of mass loss and wind terminal velocity of $\dot{M}_{\text{cool}} = 1.5 * 10^{-8} M_{\odot}/\text{yr}$ and $V_{\text{cool}} = 30$ km/s for the giant (Torbett & Campbell 1989; Vogel 1991), and of $\dot{M}_{\text{hot}} = 5 * 10^{-9} M_{\odot}/\text{yr}$ and $V_{\text{hot}} = 500$ km/s for the hot component (Vogel 1993), we found $k = \frac{1}{5}$. Thus, the shocked region must be located at 1/5 of the distance between the two components (from the giant's surface). By calculating the wind parameters of Girard & Willson (1987), $m = \frac{\dot{M}_{\text{hot}}}{\dot{M}_{\text{cool}}}$ and $w = \frac{v_{\text{hot}}}{v_{\text{cool}}}$, it was found that $mw > 1$. Therefore, the hot wind predominates over the cool wind, and the conical surface envelops the area behind the giant, as schematized in Fig. 2.

4.2 *Verification of the eccentric eclipse modeling applied to EG Andromedae*

Equation (11) is relative to an occultation phenomenon by the shifted nebular region on the giant's radiation, due to absorption mechanisms such as H^-, formed in the inner part of the shocked region (Isliker *et al.* 1989). It was solved for EG Andromedae by numerical methods, using a software. As secondary minimum, the epoch of 2448530 JD (Calabrò 1993; Blanco & Mammano 1995) was used; the true anomaly $V'_{\text{min}} = 117° \pm 22°$ corresponds to it. The set of orbital elements of Munari (1993) was chosen, which is similar to that of Garcia (1986) and Fekel *et al.* (2000), except in the case of $e = 0$. A set of values of a_h (the distance between the shocked region and the giant) and i (the inclination angle of the orbital plane) was used, computing the corresponding quantities appearing in the equation, and the relative new values a_h and i, until a convergence is reached.

The obtained sets of a_h and i have been reported in Table 1.

As explained in the previous section, equation (17) is relative to an occultation of the hot star by hydrogen in the inner part of the cool wind at the shocked region, due to Rayleigh scattering (Isliker *et al.* 1989). It was solved by using the orbital

Table 1. Two sets of solutions of the expression of the eccentric eclipse modeling computed at a secondary minimum of EG Andromedae, where the giant is eclipsed by the shifted nebular region.

First solution		Second solution	
$a_h(R_\odot)$	i (deg)	$a_h(R_\odot)$	i (deg)
50	24.5	50	–
60	24.5	60	–
70	25	70	–
80	25.6	80	–
90	26.3	90	–
100	27.3	100	–
110	28.5	110	–
120	30.3	120	–
130	31.2	130	84.5
140	31.5	140	78
150	30.8	150	73
160	30	160	69.8
170	29	170	67.5
180	28	180	65.7

elements of Munari (1993); the value of 2454057 JD of Skopal *et al.* (2007) was chosen as the epoch of primary minimum. The true anomaly $V_{\min} = -114° \pm 20°$ corresponds to it.

As in equation (17) three unknowns appear (the ratio of the stellar masses $M_{1,2}$, the distance a_h, and the inclination angle i). Reasonable values were given to a_h and $M_{1,2}$, obtaining the relative value of the angle i. Hence, the values of a_h obtained from the previous simulation, together with a set of typical values of the ratio $M_{1,2} = m_{\text{giant}}/m_{\text{hot star}}$ were used. The obtained inclination angles are reported in the third column of Table 2. By using the mass function $f(m_1) = 0.020$ and $q = a_1 \sin(i) = 70.4$ of Belczynski *et al.* (2000) and Munari (1993), and the set of values i reported in Table 2, the relative masses of the components and their semi-axes were found:

$$m_2 = f(m_1)\frac{\left(1 + \frac{m_1}{m_2}\right)^2}{\sin^3 i}, \tag{22}$$

$$a_2 = \frac{m_1}{m_2}a_1. \tag{23}$$

The corresponding semi-axis $a = a_1 + a_2$ is related to the giant's radius R_g by the condition that the shifted shocked region is close to 1/5 of the distance of the hot star from the giant's surface (Vogel 1993), i.e., $d_h = \frac{a - R_g}{5}$.

As a_h represents the distance of the shocked region from the centre of the giant,

$$a_h = \frac{a - R_g}{5} + R_g, \tag{24}$$

an evaluation of the giant's radius can be obtained as follows:

$$R_g = \frac{5a_h - a}{4}. \tag{25}$$

Table 2. The solutions of the equation that represents the eccentric eclipse modeling computed at the primary minimum of EG Andromedae: inclination angles i as to a set of distances a_h, and the relative stellar mass and giant's radius values.

$a_h(R_\odot)$	$M_{1,2}(M_\odot)$	i(deg)	$m_1(M_\odot)$	$m_2(M_\odot)$	$a_2(R_\odot)$	$a(R_\odot)$	$R_{\text{giant}}(R_\odot)$
60	1	81	0.083	0.083	71	142	39
60	1.5	85.4	0.19	0.12	106	176	31
60	> 2	no solut.	–	–	–	–	–
70	1	78.2	0.085	0.085	72	144	51.5
70	1.5	83	0.19	0.13	106	177	43
70	2	86	0.36	0.18	141	212	34.5
70	> 2.5	no solut.	–	–	–	–	–
80	1	75.5	0.088	0.088	73	146	63
80	1.5	81	0.19	0.13	107	178	55
80	2	84	0.36	0.18	141	212	47
80	> 2.5	no solut.	–	–	–	–	–
90	1	73	0.091	0.091	73	147	75
90	1.5	78.5	0.20	0.13	108	180	67
90	2	82.4	0.37	0.18	142	213	59
90	> 2.5	no solut.	–	–	–	–	–
100	1	69.5	0.097	0.097	75	150	87
100	1.5	76.5	0.20	0.13	108	181	80
100	2	80.5	0.37	0.19	143	214	71
100	2.5	83.3	0.62	0.25	177	248	63
110	1	67	0.102	0.102	76	153	99
110	1.5	74	0.21	0.14	110	183	92
110	2	79	0.38	0.19	143	215	84
110	2.5	82	0.63	0.25	178	249	75
120	1	64	0.11	0.11	78	156	111
120	1.5	72	0.22	0.14	111	185	104
120	2	77	0.39	0.19	144	217	96
120	2.5	80.5	0.64	0.26	178	250	87
120	3	83	0.98	0.33	213	284	79
130	1	61.4	0.12	0.12	80	160	122
130	1.5	70.3	0.22	0.15	112	187	116
130	2	74.8	0.40	0.20	146	219	108
130	2.5	78.5	0.65	0.26	180	251	100
130	3	81.6	0.99	0.33	213	284	91
140	1	60	0.12	0.12	81	162	134
140	1.5	68.5	0.23	0.16	113	189	128
140	2	73	0.41	0.21	147	221	120
140	2.5	76.5	0.67	0.27	181	253	111
140	3	80	1	0.34	214	286	103
150	1.5	66.3	0.25	0.16	115	192	139
150	2	71	0.42	0.21	149	223	131
150	2.5	75.5	0.67	0.27	182	254	124
150	3	79	1.01	0.34	215	287	116
160	1.5	64	0.26	0.17	117	196	151
160	2	69.2	0.44	0.22	150	226	143
160	2.5	74.1	0.69	0.28	183	256	136
160	3	77.8	1.02	0.34	216	288	128
170	1.5	62	0.27	0.18	119	199	162
170	2	67.6	0.46	0.23	152	228	155
170	2.5	73	0.70	0.28	184	257	148
170	3	76.7	1.04	0.35	217	289	140
180	2	65	0.48	0.24	155	233	167
180	2.5	70.8	0.73	0.29	186	261	160
180	3	74.5	1.07	0.36	219	292	152

The physical quantities reported in Table 2 represents a set of values compatible with the hypothesis of an eccentric eclipse for EG Andromedae at the primary minimum.

4.3 *Discussion*

Spectroscopy carried out on EG Andromedae showed that emission lines such as HeI and HeII, reach their maximum at $\Phi = 0.50$ (Skopal *et al.* 1991) and greatly reduce their intensity during the primary minimum (Crowley *et al.* 2008), a circumstance which confirms the scenario proposed here, according to the hot star, at that phase $\Phi = 0.50$, faces the observer.

Some solutions reported in Table 2 provided mass values too small for the stellar components, that should imply a state of evaporation for the giant and a neutron star as the hot component, assuming $0.28M_\odot$ as the smallest mass for a white dwarf (Ritter 1984).

However, it is a reasonable assumption that acceptable values are obtained by the intersection between the set of solutions relative to the primary minimum, and that relative to the secondary minimum. Indeed, the inclination angle of the orbital plane with respect to the tangent plane must be unique at the primary and secondary minimum, as well. These two sets of solutions are represented in Fig. 4, whose intersection provides the final set of values of the inclination of the orbit of EG Andromedae. The physical quantities given in Table 3 are related to them.

The mass values of the hot component, corresponding to the first two rows of Table 3, are in agreement with the hypothesis that it can be a pre-white dwarf proposed in the literature. Indeed Vogel *et al.* (1992) proposed a pre-white dwarf with

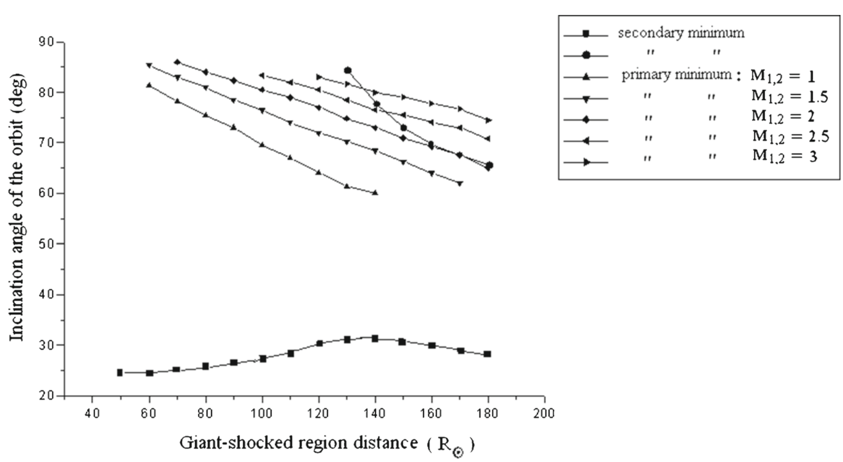

Figure 4. The verification of the eccentric eclipse modeling for EG Andromedae: ordinate represents the inclination angle values i, corresponding to a set of distances between the shifted shocked region and the giant, which are given on a_h, the abscissa. The lines joining the triangle points refer to values relative to the primary minimum (2454057 JD), as to some ratio value of the stellar masses $M_{1,2}$; the two lines with squares and circles refer to the secondary minimum (2448530 JD), as well. The intersection of these lines, as real solutions, provide the possible inclination angle values for EG Andromedae.

Table 3. The physical parameters derived from the hypothesis of an eccentric eclipse for EG Andromedae.

$a_h(R_\odot)$	$M_{1,2}(M_\odot)$	i(deg)	$m_1(M_\odot)$	$m_2(M_\odot)$	$a_2(R_\odot)$	$a(R_\odot)$	$\rho''(R_\odot)$	$R_{\rm giant}(R_\odot)$
135	3	81	0.99	0.33	214	285	60	97
140	2.8	78	0.88	0.32	201	273	65	106
150	2.3	73	0.57	0.25	169	243	74	126
160	2.1	69.8	0.49	0.23	157	232	83	142
170	2	67.5	0.46	0.23	152	228	91	155
180	2	65	0.48	0.24	155	233	100	167

$M_{\rm hot} < 0.5M_\odot$ as the hot star in EG Andromedae, otherwise its temperature would be higher than that observed.

In addition, the low mass values that were found for the giant are consistent with the observed $C^{12}/C^{13} \approx 10$ (Schild *et al.* 1992).

Finally, Vogel *et al.* (1992) showed in their simulation that the most probable value of the inclination of the orbit of EG Andromedae is close to 90° with a confidence of 72%, a value that is similar to that found in this study and reported in the first row of Table 3. The relative values of the giant's radius are at the limit of $100R_\odot$, the maximum value for a single giant suggested by Kenyon & Gallagher (1983).

Nevertheless, the interaction state between the two stars can give rise to the building of an extended giant's envelope, bringing occultation phenomena to distances greater than the photosferic radius, in agreement with the Rayleigh scattering found by Vogel (1993). This is also the case of the giant of the symbiotic V 2116 Oph, for which it was estimated to be $260R_\odot$ (McClintook & Leventhal 1989).

Other solutions, instead, provide values too small for both components, that would force us to assume a neutron star as hot component. It was proved, as a further constraint on the eccentric eclipse modeling, that the projected distance on the tangent

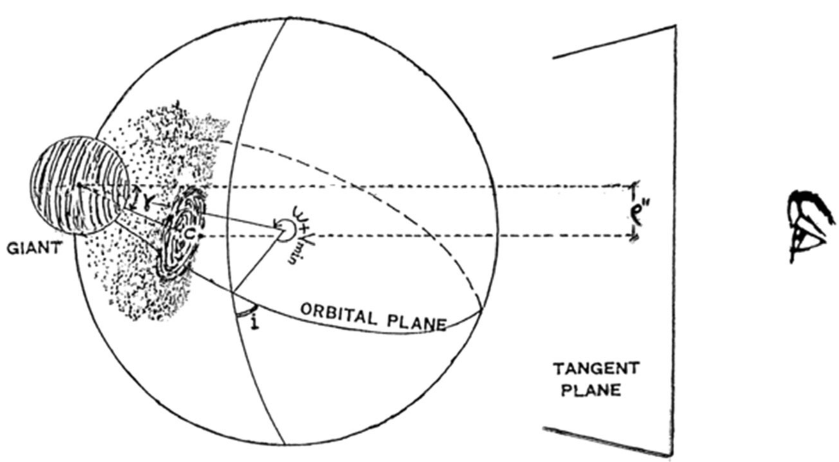

Figure 5. A further constraint on the eccentric eclipse modeling. The projected distance on the tangent plane between the surface of the giant and the shifted shocked region, ρ'', must be less than their dimensions.

plane between the surface of the giant and the shifted shocked region, ρ'', be less than their dimension (see Fig. 5).

Looking at the last two columns of Table 3, it appears that for the mass values resulted, opacity effects by the shocked region are possible, because one always has $\rho'' \leq R_{\text{giant}}$.

5. Conclusions

The presence of a dense nebular region in a symbiotic system, produced by the collision of the stellar winds, can perform the light curve of a typical quiescent symbiotic star. The densest zone of this nebular region should be eccentric as to the orbital axis, due to the orbital motion of the system, whose velocity is comparable to the wind cool velocity. This circumstance led us to modify the classical eclipse condition providing other expressions obtained by the minimization of the distances between the shifted shocked region and the giant, and between this region and the hot star. These equations allow us to obtain the orbital elements that cannot be obtained from spectroscopic observations: the inclination angle, the orbital axes and the other related physical parameters.

The existence of stellar winds in the symbiotic EG Andromedae led us to consider the presence of a nebular region produced by the winds collision that is shifted as to the orbital axis due to the orbital motion of the system. Hence, the modeling of an eccentric eclipse was applied to this symbiotic.

A simulation provided two sets of values of the distance between the giant and the shocked region, and the inclination angle of the orbit at primary and secondary minima, whose intersection provided orbital inclination angle values around 80°.

The related physical quantities of the stellar components are represented by a mass value for the giant close to $1M_\odot$ whose envelope extends, at least, as far as $100R_\odot$, and a mass value for the hot star close to $0.33M_\odot$, in agreement with the hypothesis of a status of pre-white dwarf, suggested by other authors.

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