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# Axially Symmetric Bianchi Type-I Bulk-Viscous Cosmological Models with Time-Dependent $\Lambda$ and $q$

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**Abstract.** The present study deals with spatially homogeneous and anisotropic axially symmetric Bianchi type-I cosmological model with time variable cosmological term $\Lambda$ in the presence of bulk viscous fluid. The Einstein's field equations are solved explicitly by time varying deceleration parameter $q$. Consequences of the four cases of phenomenological decay of $\Lambda$ have been discussed which are consistent with observations. Physical and kinematical parameters of the models are discussed.

*Key words.* Bianchi space-time—bulk viscosity—variable cosmological term and deceleration parameter.

## 1. Introduction

The cosmological picture that emerges after the observations of SN 1a by the HZT and SCP teams reveals that at present we are residing in an accelerating Universe (Perlmutter *et al.* 1998; Riess *et al.* 1998), whose geometry is Euclidean in nature (Sievers *et al.* 2003). This speeding up of the Universe started about 7 Gyr ago (Kirshner 2003) and some kind of repelling forces, termed as dark energy is supposed to be responsible for catapulting the once decelerating Universe into an accelerating one. Recently, there are many variants of dark energy which can be responsible for this accelerated Universe and variation in the forms of dark energy also exhibit variation in expansion rate in different eras. So, there may be more than one candidate which can be stamped as dark energy. For example, one may select the so called cosmological constant, introduced and later abandoned by Einstein, as dark energy. But selection of the cosmological constant as dark energy faces a serious fine-tuning problem which demands that the value of $\Lambda$ must be 123 orders of magnitude and 55 orders of magnitude larger respectively in the Planck scale $\left(T \sim 10^{19}\,\text{GeV}\right)$ and $\left(T \sim 10^{2}\,\text{GeV}\right)$ than its presently observed value. Some of the recent discussions on cosmological constant 'problem' and consequence on cosmology with a time-varying cosmological constant are investigated by Ratra & Peebles (1988), Dolgov (1983, 1997) and Sahni & Starobinsky (2000). Cosmological scenarios with a time varying $\Lambda$ have been proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during the last two decades (Chen & Wu 1990; Pavon 1991; Carvalho *et al.* 1992; Lima & Maia 1994; Lima & Trodden 1996; Arbab & Abdel-Rahman 1994; Vishwakarma 2001; Cunha & Santos 2004; Carneiro & Lima 2005).

Most cosmological models assume that the matter in the Universe can be described by 'dust' (a pressure-less distribution) or at best a perfect fluid. A realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. Nevertheless, there is a good reason to believe that at the early stage of the Universe when the radiation in the form of photons as well as neutrons decoupled from matter, it behaved like a viscous fluid (Weinberg 1971; Nightingle 1973; Heller & Klimek 1975). Misner (1967a, b), investigated the electron–neutrino scattering and the subsequent decoupling of neutrinos, and came to the conclusion that the viscosity of neutrinos can essentially reduce the initial anisotropy of the Universe. Bulk viscosity associated with the Grand Unified Theory (GUT) may lead to an inflationary cosmology. Grøn (1990) studied inflationary Bianchi type models with bulk viscosity and shear. The model presented by Murphy (1973) has an interesting feature in that the Big Bang type of singularity of infinite space time curvature does not possess a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is quite not acceptable for large density. Collins & Stewart (1971) have studied the effect of viscosity on the formation of galaxies. Ribeiro & Sanyal (1987) have presented Bianchi type $VI_0$ models containing a viscous fluid in the presence of an axial magnetic field. Bianchi type-I solutions in the case of stiff matter with shear viscosity being the power function of energy density were obtained by Banerjee *et al.* (1985). Banerjee & Santos (1984) have studied nonviscous and viscous fluids in Bianchi type-II, VIII and IX space-times under the assumption that the ratio of shear scalar and expansion be constant. The solutions for open, closed and flat Universe models have been founded by Santos *et al.* (1985) under the assumption that the bulk viscosity ($\zeta$) is a power function of energy density ($\rho$). Costa & Makler (2008) have discussed the connection among three distinct classes of models in the presence of decaying cosmological term, bulk viscous pressure and non linear fluids. Bulk viscosity in Brans–Dicke theory, leading to an accelerated phase of the Universe today have been investigated by Mak & Harko (2003) and Sen *et al.* (2001). The effect of bulk-viscosity leading to an accelerated phase of the Universe today has been studied by Fabris *et al.* (2006). Lima & Germano (1992), Belinskii & Khalotnikov (1976, 1977), Banerjee *et al.* (1986), Banerjee & Sanyal (1986, 1988), Zimdahl & Pavon (1993, 1994), Silva *et al.* (2002), Gariel & Denmat (1995) and Zimdahl *et al.* (1996) have studied the effect of bulk viscosity on cosmological evolution in the framework of general theory of relativity.

The simplest model of the observed Universe is well represented by Friedmann–Robertson–Walker (FRW) models, which are both spatially homogeneous and isotropic. These models in some sense are good global approximation of the present day Universe. But on smaller scales, the Universe is neither homogeneous nor isotropic. There are theoretical arguments (Chimento 2004; Misner 1968) and recent experimental data regarding cosmic background radiation anisotropies which support the existence of an anisotropic phase that approaches an isotropic one (Land & Magueijo 2005). Spatially homogeneous and anisotropic cosmological models which provide a richer structure, both geometrically and physically, than the FRW model play a significant role in the description of early Universe. Axially symmetric Bianchi type-I models being anisotropic generalization of flat FRW models are interesting to study. These models are favoured by the available evidences for low density Universe. An axially symmetric Bianchi type-I cosmological model have been investigated by Reddy & Rao (2006) and Reddy *et al.* (2006a) in Brans–Dicke theory while Reddy *et al.* (2006b) have discussed the same in Saez–Ballester scalar–tensor theory. Reddy *et al.* (2007) have studied an axially symmetric Bianchi type-I space-time cosmological model with a negative constant deceleration parameter in scale-covariant theory of gravitation. Singh & Kale (2009) have discussed an axially symmetric Bianchi type-I, Kantowski-Sachs and Bianchi type-III cosmological models filled with bulk viscous fluid together with variables G and $\Lambda$. Recently, Pradhan *et al.* (2006) proposed the variation law of the deceleration parameter (DP), which generates scale factor as increasing functions of time in FRW space-time in Lyra's manifold. Also, Amirhashchi *et al.* (2011) have studied an interacting and non-interacting two fluid dark energy model in FRW Universe with time variable DP.

Motivated by the above discussions, in this paper, the Einstein's field equations have been solved with variation law of DP in an axially symmetric Bianchi type-I space-time in the presence of bulk viscous fluid source and time varying cosmological term $\Lambda$, which provides a scale factor as increasing functions of time. Consequences of the following four cases of phenomenological decay of $\Lambda$ have been discussed:

*Case I*: $\Lambda \sim H^2$.
*Case II*: $\Lambda \sim H$.
*Case III*: $\Lambda \sim \rho$.
*Case IV*: $\Lambda \sim S^{-2}$.

Here $H$, $\rho$ and $S$ are respectively, the Hubble parameter, matter energy density and average scale factor of the axially symmetric Bianchi type-I space-time. The dynamical laws for decay of $\Lambda$ have been widely studied by Arbab (1997, 1998), Carvalho *et al.* (1992), Chen & Wu (1990), Schutzhold (2002a, b), Vishwakarma (2000), Singh *et al.* (2008), Singh & Baghel (2009) to name a few.

## 2. The metric and field equations

We consider the homogeneous and anisotropic, axially symmetric Bianchi type-I space-time which is given by the line element

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + B^2(t)\,(dy^2 + dz^2), \tag{1}$$

where $A(t)$ and $B(t)$ are cosmic scale factors.

We assume the cosmic matter consisting of bulk viscous fluid given by the energy momentum tensor as

$$T_{ij} = (\rho + \bar{p})\, v_i v_j + \bar{p} g_{ij}, \tag{2}$$

with

$$\bar{p} = p - \zeta v^i_{;i}, \tag{3}$$

where $\bar{p}$ is the effective pressure, $\zeta$ is the coefficient of bulk viscosity, $p$ is the isotropic pressure, $\rho$ is the energy density of matter and $v^i$ is the fluid four-velocity vector satisfying $v_i v^i = -1$. The semicolon stands for covariant differentiation. On thermodynamical grounds bulk viscous coefficient $\zeta$ is positive, assuring that the viscosity pushes the dissipative pressure $\bar{p}$ towards negative values. But correction to the thermodynmical pressure $p$ due to bulk viscous pressure is very small. Therefore, the dynamics of cosmic evolution does not change fundamentally by the inclusion of viscous term in the energy momentum tensor. The Einstein's field equations with time varying $\Lambda(t)$ are given by

$$R_i^j - \frac{1}{2}g_i^j R = -8\pi G T_i^j + \Lambda g_i^j. \tag{4}$$

The field equations (4) for the metric (1) are

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi G\rho + \Lambda, \tag{5}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -8\pi G\bar{p} + \Lambda, \tag{6}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} = -8\pi G\bar{p} + \Lambda, \tag{7}$$

where an overhead dot $(\cdot)$ denotes ordinary differentiation with respect to cosmic time $t$. In view of the vanishing divergence, Einstein tensor gives

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{8}$$

From (8), we observe that in case of constant $\Lambda$, we recover the continuity equation of matter. In order to satisfy energy conservation, a decaying vacuum term $\Lambda$ transfers energy continuously to the matter component. The effective time-dependent cosmological term is regarded as second fluid component with energy density $\rho_v = \Lambda(t)/8\pi G$, where $\rho_v$ is the vacuum energy density. We assume that the non-vacuum component of matter obeys the equation of state

$$p = \omega\rho, \qquad \omega \in [0, 1]. \tag{9}$$

We define the average scale factor $S$ as

$$S^3 = \sqrt{-g} = AB^2. \tag{10}$$

From equations (6) and (7) we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{S^3}, \tag{11}$$

where $k_1$ is a constant of integration.

We introduced the volume expansion $\theta$ and shear scalar $\sigma$ as

$$\theta = v^i_{;i}, \qquad \sigma^2 = \tfrac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{12}$$

$\sigma^{ij}$ being the shear tensor. For axially symmetric Bianchi type-I metric, expressions for dynamical scalars are

$$\theta = 3\frac{\dot{S}}{S}, \tag{13}$$

$$\sigma = \frac{k_2}{\sqrt{3}S^3}, \tag{14}$$

where $k_2$ ($> 0$) is a constant.

The Hubble parameter $H$ and the deceleration parameter $q$ are defined as

$$H = \frac{\dot{S}}{S}, \tag{15}$$

$$q = -\frac{S\ddot{S}}{\dot{S}^2}. \tag{16}$$

Equations (5), (6), (7) and (8) can be written in terms of $H$, $\sigma$ and $q$ as

$$8\pi G\rho + \Lambda = 3H^2 - \sigma^2, \tag{17}$$

$$8\pi G\bar{p} - \Lambda = (2q - 1)\, H^2 - \sigma^2 \tag{18}$$

and

$$\dot{\rho} + 3\,(\rho + \bar{p})\, H + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{19}$$

For constant value of $\Lambda$, it is to note that energy density of the Universe is a positive quantity. It is believed that at the early stages of the evolution when the average scale factor $S$ was close to zero, the energy density of the Universe was infinitely large. On the other hand, with the expansion of the Universe i.e. with increase of $S$ the energy density decreases and an infinitely large $S$ correspond to a $\rho$ close to zero. Also, equation (17) yields

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}. \tag{20}$$

Therefore, $0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3}$ and $0 \leq \frac{8\pi G\rho}{\theta^2} \leq \frac{1}{3}$ for $\Lambda \geq 0$. Thus, the presence of a positive $\Lambda$ puts restriction on the upper limit of anisotropy whereas a negative $\Lambda$ contributes to the anisotropy. From equations (17) and (18), we obtain

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = -4\pi G(\rho + 3p) - 2\sigma^2 - \frac{1}{3}\theta^2 + 12\pi\theta\zeta G + \Lambda, \tag{21}$$

which is the Raychaudhuri equation for the given distribution. We observe that for $\Lambda \leq 0$ and $\zeta = 0$, the Universe will always be in decelerating phase provided the strong energy condition (Hawking & Ellis 1975) holds. In this case, we have

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} \leq -\frac{\theta^2}{3}, \tag{22}$$

which integrates to give

$$\frac{1}{\theta} \geq \frac{t}{3} + \frac{1}{\theta_0}, \tag{23}$$

where $\theta_0$ is the initial value of $\theta$. If $\theta_0 < 0$, $\theta$ will diverge ($\theta \to -\infty$) for $t < 3/|\theta_0|$. From equation (21), one also concludes the presence of viscosity and a positive $\Lambda$ will slow down the rate of decrease of volume expansion. Also, from equation (14), we get

$$\dot{\sigma} = -\sigma\theta. \tag{24}$$

Thus, the energy density associated with the anisotropy $\sigma$ decays rapidly in an evolving Universe and it becomes negligible for infinitely large $S$. The pressure and density are intimately connected to the motion of the fluid which they describe. This can be appreciated by looking at the general equation of motion. From equations (17) and (18), we obtain

$$\frac{\ddot{S}}{S} = -\frac{4\pi G}{3}(\rho + 3p) - \frac{2}{3}\sigma^2 + 4\pi\zeta\theta G + \frac{\Lambda}{3}. \tag{25}$$

We observe that the positive cosmological term and bulk viscosity contribute positively in driving the acceleration of the Universe. Also, from equation (17), we get

$$\frac{3\dot{S}^2}{S^2} = \sigma^2 + 8\pi G\rho + \Lambda. \tag{26}$$

When $\Lambda \geq 0$, each term on the right-hand side of (26) is non-negative. Thus $\dot{S}$ does not change sign and we get ever-expanding models. For $\Lambda < 0$, however, we can get Universe models that expand and then recontract. From equation (8), we obtain

$$S^{-3(1+\omega)}\frac{\mathrm{d}}{\mathrm{d}t}\{\rho S^{3(1+\omega)}\} = 9\zeta H^2 - \frac{\dot{\Lambda}}{8\pi G}. \tag{27}$$

Thus, decaying vacuum energy and viscosity of the fluid lead to matter creation.

We define the deceleration parameter $q$ as

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b(t), \quad \text{say}. \tag{28}$$

The motivation to choose such time-dependent DP is behind the fact that the Universe has accelerated expansion at present as observed in recent observations of type 1a supernova (Riess *et al.* 1998, 2004; Perlmutter *et al.* 1998; Tonry *et al.* 2003; Clocchiatti 2006), CMB anisotropies (Bennett *et al.* 2003; de Bernadis *et al.* 2000; Hanany *et al.* 2000) deceleration expansion in the past. Also the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating at past and accelerating at present, the DP must show signature flipping (see Padmanabhan & Roychowdhury 2003; Amendola 2003; Riess *et al.* 2001). So, there is no scope for a constant DP at the present epoch. In general, the DP is not constant but varies with time.

Equation (28) may be rewritten as

$$\frac{\ddot{S}}{S} + b\frac{\dot{S}^2}{S^2} = 0. \tag{29}$$

In order to solve equation (29), we assume $b = b(S)$. It is important to note here that one can assume $b = b(t) = b(S(t))$, as $S$ is also a time-dependent function. It can be done only if there is a one-to-one correspondence between $t$ and $S$. But this is only possible when one avoids singularity like Big Bang or Big Rip because $t$ and $S$ are increasing functions.

The general solution of (29) with the assumption that $b = b\,(S)$ is given by

$$\int \mathrm{e}^{\int \frac{b}{S}\mathrm{d}S}\mathrm{d}S = t + m, \tag{30}$$

where $m$ is an integrating constant.

One cannot solve (30) in general as $b$ is a variable. So, in order to solve the problem completely, we have to choose $\int \frac{b}{S}\mathrm{d}S$ in such a manner that equation (30) is integrable without any loss of generality. Hence, we consider

$$\int \frac{b}{S}\mathrm{d}S = In\, L(S)\,, \tag{31}$$

which does not affect the nature of generality solution. Hence, from (30) and (31), we obtain

$$\int L(S)\,\mathrm{d}S = t + m. \tag{32}$$

Of course, the choice of $L(S)$ in (32) is quite arbitrary but, since we are looking for physically viable models of the Universe consistent with observation, we consider

$$L\,(S) = \frac{1}{\alpha\sqrt{1+S^2}}, \tag{33}$$

where $\alpha$ is an arbitrary constant.

In this case, on integrating, (32) gives the exact solution

$$S\,(t) = \sinh\,(\alpha T)\,, \tag{34}$$

where $T = t + m$. From equations (10), (11), (13) and (34), we get

$$A = k_3 \sinh\,(\alpha T) \exp\left[\frac{2k_1}{3}\int \frac{\mathrm{d}T}{\{\sinh\,(\alpha T)\}^3}\right] \tag{35}$$

and

$$B = k_4 \sinh\,(\alpha T) \exp\left[-\frac{k_1}{3}\int \frac{\mathrm{d}T}{\{\sinh\,(\alpha T)\}^3}\right] \tag{36}$$

where $k_3$ and $k_4$ are integration constants such that $k_3 k_4^2 = 1$.

For this solution, the metric (1) is reduced to the following form using suitable transformation of co-ordinates

$$\begin{aligned} \mathrm{d}s^2 \;=\; & -\mathrm{d}T^2 + k_3^2\,\{\sinh\,(\alpha T)\}^2 \exp\left[\frac{4k_1}{3}\int \frac{\mathrm{d}T}{\{\sinh\,(\alpha T)\}^3}\right]\mathrm{d}X^2 \\ & + k_4^2\,\{\sinh\,(\alpha T)\}^2 \exp\left[-\frac{2k_1}{3}\int \frac{\mathrm{d}T}{\{\sinh\,(\alpha T)\}^3}\right](\mathrm{d}Y^2 + \mathrm{d}Z^2). \end{aligned} \tag{37}$$

## 3. Discussion

We now discuss the models resulting from different dynamical laws for the decay of $\Lambda$.

The deceleration parameter $q$, volume expansion $\theta$, Hubble parameter $H$ and shear scalar $\sigma$ for this model are

$$q = -\tanh^2(\alpha T)\,, \tag{38}$$

$$\theta = 3H = 3\alpha\ \coth(\alpha T)\,, \tag{39}$$

$$\sigma = \frac{k_2}{\sqrt{3}\sinh^3(\alpha T)}. \tag{40}$$

For this model, $\sigma/\theta \to 0$ as $T \to \infty$. Therefore, the model approaches isotropy asymptotically.

3.1 *Case I.* We consider

$$\Lambda = 3\beta H^2, \tag{41}$$

where $\beta$ is a constant of the order of unity. Here $\beta$ represents the ratio between vacuum and critical densities. From equations (9), (17) and (18), we obtain

$$8\pi G\rho = 3\alpha^2(1-\beta)\coth^2(\alpha T) - \frac{k_2^2}{3\sinh^6(\alpha T)}, \tag{42}$$

$$\begin{aligned} 24\pi G\zeta &= \alpha\ \coth(\alpha T)\,[(3\omega+1) - 3\beta\,(1+\omega)] \\ &\quad + \frac{2\alpha}{\coth(\alpha T)} + \frac{(1-\omega)\,k_2^2 \sec h\,(\alpha T)}{3\alpha \sinh^5(\alpha T)}, \end{aligned} \tag{43}$$

$$\Lambda = 3\beta\alpha^2\coth^2(\alpha T)\,. \tag{44}$$

We observe that the model has singularity at $T = 0$. It starts with a Big Bang from its singular state at $T = 0$ and continues to expand till $T = \infty$. At $T = 0$, $\rho,\ p,\ \Lambda,\ \zeta$ are all infinite and they become negligible for large values of $T$. Therefore, for large times the model represents a non-rotating, shearing and expanding Universe which approaches isotropy asymptotically. In this case density parameter $\Omega$ for the model (37) yields

$$\Omega = \frac{\rho}{\rho_c} = 1 - \beta - \frac{k_2^2 \sec h^2(\alpha T)}{9\alpha^2\sinh^4(\alpha T)}.$$

The ratio between vacuum and critical densities is given by

$$\frac{\rho_v}{\rho_c} = \beta.$$

3.2 *Case II.* We consider

$$\Lambda = aH, \tag{45}$$

where $\alpha$ is a positive constant of order $m^3$ where $m \approx 150$ MeV is the energy scale of the chiral phase transition of QCD (Borges & Carneiro 2005). In this case, we get

$$8\pi G\rho = \alpha\,[3\alpha \coth(\alpha T) - a] \coth(\alpha T) - \frac{k_2^2}{3\sinh^6(\alpha T)}, \tag{46}$$

$$\begin{aligned} 24\pi G\zeta \; &= \; \alpha\, \coth(\alpha T)\,(3\omega+1) - a\,(1+\omega) + \frac{2\alpha}{\coth(\alpha T)} \\ &\quad + \frac{(1-\omega)\, k_2^2 \sec h\,(\alpha T)}{3\alpha \sinh^5(\alpha T)}, \end{aligned} \tag{47}$$

$$\Lambda = a\alpha \coth(\alpha T)\,. \tag{48}$$

This model also has singularity at $T \to 0$. It evolves from its singular state at $T \to 0$ with $\rho,\ \ p,\ \ \Lambda,\ \ \zeta$ all diverging, and expansion of the model becomes zero for $T \to \infty$. In this case the vacuum energy decays slowly with time. The density parameter $\Omega$ is

$$\Omega = \frac{\rho}{\rho_{\mathrm{c}}} = \frac{\{3\alpha \coth(\alpha T) - a\}}{3\alpha \coth(\alpha T)} - \frac{k_2^2 \sec h^2(\alpha T)}{9\alpha^2 \sinh^4(\alpha T)}$$

and the ratio between vacuum and critical densities is given by

$$\frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{c}}} = \frac{a}{3\alpha \coth(\alpha T)}.$$

3.3 *Case III.* We now consider

$$\Lambda = \frac{8\pi G\gamma\rho}{3}, \tag{49}$$

where $\gamma$ is a constant. From equations (9), (17) and (18) we obtain

$$8\pi G\left(1+\frac{\gamma}{3}\right)\rho = 3\alpha^2 \coth^2(\alpha T) - \frac{k_2^2}{3\sinh^6(\alpha T)}, \tag{50}$$

$$\begin{aligned} 24\pi G\zeta\,(3+\gamma) \; &= \; (9\omega + 3 - 2\gamma)\,\alpha\, \coth(\alpha T) + \frac{2\alpha\,(3+\gamma)}{\coth(\alpha T)} \\ &\quad + \frac{(3 - 3\omega + 2\gamma)\, k_2^2 \sec h\,(\alpha T)}{3\alpha \sinh^5(\alpha T)}, \end{aligned} \tag{51}$$

$$\left(1+\frac{3}{\gamma}\right)\Lambda = 3\alpha^2 \coth^2(\alpha T) - \frac{k_2^2}{3\sinh^6(\alpha T)}. \tag{52}$$

In this case also our model starts expanding with a Big Bang singularity at $T \to 0$ with $\rho,\ \ p,\ \ \Lambda$ and $\zeta$ all infinite, and expansion in the model ceases at $T \to \infty$. The bulk viscosity coefficient $\zeta$, matter density $\rho$ and cosmological term $\Lambda$ also decrease due to expansion and become zero for large times. The density parameter $\Omega$ for this model is given by

$$\Omega = \frac{3}{(3+\gamma)}\left[1 - \frac{k_2^2 \sec h^2(\alpha T)}{3\alpha^2 \sinh^4(\alpha T)}\right]$$

and the ratio between vacuum and critical densities is obtained as

$$\frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{c}}} = \frac{3}{(3+\gamma)}\left[1 - \frac{k_2^2 \sec h^2(\alpha T)}{9\alpha^2 \sinh^4(\alpha T)}\right].$$

3.4 *Case IV.* Finally, we consider the case

$$\Lambda = \frac{\delta}{S^2}, \tag{53}$$

where $\delta$ is a constant. In this case from (9), (17) and (18), we obtain

$$8\pi G\rho = 3\alpha^2 \coth^2(\alpha T) - \frac{k_2^2}{3\sinh^6(\alpha T)} - \frac{\delta}{\sinh^2(\alpha T)}, \tag{54}$$

$$\begin{aligned} 24\pi G\zeta \;=\; & (3\omega+1)\,\alpha \coth(\alpha T) + \frac{(1-\omega)\,k_2^2 \sec h(\alpha T)}{3\alpha \sinh^5(\alpha T)} \\ & + 2\alpha \tanh(\alpha T) - \frac{(1+\omega)\,\delta \sec h(\alpha T)}{\alpha \sinh(\alpha T)}, \end{aligned} \tag{55}$$

$$\Lambda = \frac{\delta}{\sinh^2(\alpha T)}. \tag{56}$$

Here we observe that this model also has singularity at $T = 0$. It starts from a Big Bang singularity with $\rho$, $p$, $\zeta$ and $\Lambda$ all infinite. The bulk viscosity coefficient $\zeta$, matter density $\rho$, cosmological term $\Lambda$ and pressure $p$ decrease as cosmic time increases and they become zero for large times. The density parameter $\Omega$ for this model is given by

$$\Omega = 1 - \frac{k_2^2 \sec h^2(\alpha T)}{9\alpha^2 \sinh^4(\alpha T)} - \frac{\delta}{3\alpha^2 \cosh^2(\alpha T)}.$$

The ratio between vacuum and critical densities is given by

$$\frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{c}}} = \frac{\delta}{3\alpha^2 \cosh^2(\alpha T)}.$$

## 4. Conclusion

In this paper, we have investigated homogeneous and anisotropic, axially symmetric Bianchi type-I space-time with bulk viscous matter and time-dependent cosmological term $\Lambda$ in general relativity. The field equations have been solved explicitly

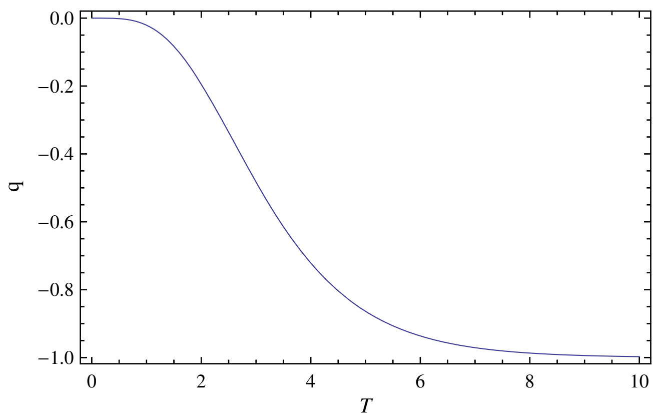


**Figure 1.** The plot of deceleration parameter $q$ vs. $T$ for $\alpha = 1$.

by choosing time-dependent deceleration parameter $q$. Four different decays for the cosmological term have been discussed in the context of models obtained. The cosmological term in this model is a decreasing function of time and this approaches a small value as time increases (i.e. present epoch). The value of the cosmological 'term' for this model is found to be small and positive which is supported by the results from recent supernovae observations obtained by the high-$Z$ supernova team and supernova cosmological project (Perlmutter *et al*. 1997, 1998, 1999; Riess *et al*. 1998, 2004; Garnavich *et al*. 1998a, b; Schmidt *et al*. 1998). These observations on magnitude and red-shift of type 1a supernova suggest that our Universe may be an accelerating one with induced cosmological density through the cosmological $\Lambda$-term. From equation (40), one concludes that the value of deceleration parameter '$q$' is negative, and it gives rise to an accelerating Universe. Recent observations (Perlmutter *et al*. 1997, 1998, 1999; Riess *et al*. 1998, 2004; Tonry *et al.* 2003; John 2004, Knop 2003) reveal that the value of deceleration parameter ($q$) is confined in the range $-1 \leq q < 0$ and the present day Universe is undergoing an accelerated expansion. Figure 1 shows that the value of $q$ lies in the range $-1 \leq q < 0$ which is consistent with recent observations. Thus, the derived model (37) represents a non-rotating, shearing and accelerating Universe which becomes isotropic for large times.

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