Cosmological Solutions of Tensor–Vector Theories of Gravity by Varying the Space-Time–Matter Coupling Constant

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Abstract. We consider tensor–vector theories by varying the space-time–matter coupling constant (varying Einstein velocity) in a spatially flat FRW universe. We examine the dynamics of this model by dynamical system method assuming a ΛCDM background and we find some exact solutions by considering the character of critical points of the theory and their stability conditions. Then we reconstruct the potential $V(A^2)$ and the coupling $Z(A^2)$ by demanding a background ΛCDM cosmology. Also we set restrictions on the varying Einstein velocity to solve the horizon problem. This gives a selection rule for choosing the appropriate stable solution. We will see that it is possible to produce the background expansion history $H(z)$ indicated by observations. Finally we will discuss the behavior of the speed of light (c_{E}) for those solutions.

Key words. Speed of light—tensor–vector theories—dynamical system method—horizon problem.

1. Introduction

Late-time cosmic acceleration, reported in 1998 (Riess *et al.* 1998; Perlmutter *et al.* 1999) based on the type Ia Supernovae (SN Ia) observations, has led cosmologists to a new field of research. The source of this acceleration, called dark energy (Huterer & Turner 1999), is still an unsolved problem. Independent observational data such as SN Ia (Astier *et al.* 2006; Riess *et al.* 2004, 2007; Wood-Vasey *et al.* 2007; Davis *et al.* 2007; Kowalski *et al.* 2008), cosmic microwave background (CMB) (Spergel *et al.* 2003, 2007; Komatsu *et al.* 2009), and baryon acoustic oscillations (BAO) (Eisenstein *et al.* 2005; Percival *et al.* 2007, 2010) have confirmed that about 70 per cent of total content of the present Universe consists of dark energy. Cosmological constant Λ is the simplest candidate for dark energy. In the standard theory the cold dark matter model with cosmological constant ΛCDM describe an effective epoch. Therefore assuming spatial flatness, the predicted cosmic history is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2[\Omega_{0\mathrm{m}}(1+z)^3 + \Omega_{0\mathrm{r}}(1+z)^4 + \Omega_\Lambda] \tag{1}$$

which offers the best fit to the observational data. In the above equation, $a(t)$ is the scale factor and t is the cosmic time. The redshift z is defined by $z = (\frac{a_0}{a}) - 1$, where a_0 is the scale factor at present. Also $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $\Omega_{0\mathrm{r}} = \frac{\rho_{0\mathrm{r}}}{\rho_{\mathrm{c}}} \simeq 10^{-4}$, $\Omega_{0\mathrm{m}} = \frac{\rho_{0\mathrm{m}}}{\rho_{\mathrm{c}}} \simeq 0.3$ and $\Omega_\Lambda = 1 - \Omega_{0\mathrm{m}} - \Omega_{0\mathrm{r}}$ are the density parameters. Despite the fact that the above-proposed model can explain the late-time acceleration of the Universe, it suffers from the cosmological constant problem. Two classes of models have been proposed in the hope of solving this problem. In the first class, dark energy is responsible for the accelerating expansion (Fujii 1982; Ford 1987; Fujii & Nishioka 1990; Copeland *et al.* 1993) and its role can be played by some scalar fields (like quintessence model) or vector dark energy or by Chaplygin gas, topological defects, holographic dark energy, etc. In the second class, modification of general relativity on cosmological scales tries to explain recent accelerating expansion. Scalar–tensor theories (Boisseau *et al.* 2000; Esposito-Fare‘se & Polarski 2001; Uzan 1999; Gannouji *et al.* 2006; Capozziello *et al.* 2006; Riazuelo & Uzan 2002), $f(R)$ modified gravity theories (Nojiri & Odintsov 2004; Soussa & Woodard 2004; Faraoni & Nadeau 2007) and braneworld models are some examples of these models. Models of present dark energy based on $f(R)$ gravity were first proposed in Capoziello *et al.* (2003) and Carroll *et al.* (2004). However, first viable models of such type satisfying most present observational data were constructed only in Hu & Sawicki (2007), Appleby & Battye (2007) and Starobinsky (2007).

Vector fields are recognized as a mediator for some of the non-gravitational interactions. Therefore, it is possible that a vector field could be the source of the present cosmic acceleration. Various cosmological models based on vector fields propose a suitable model for both the inflationary and the recent acceleration eras (Koivisto & Mota 2008; Armendariz-Picon 2004; Boehmer & Harko 2007; Jimenez & Maroto 2008, 2009). Vector fields also play a key role in different extensions of general relativity, with the vector-tensor theories and so on (Hellings & Nordtvedt 1973; Will & Nordtvedt 1972; Will 1993).

On the other hand, there is a claim that the effective fine structure constant $\alpha = \frac{e^2}{\hbar c}$ has a temporal variation. The variation of α, which is constrained by the Oklo natural fission reactor is given by $-0.9\times 10^{-7} < \Delta\alpha/\alpha \equiv (\alpha - \alpha_0)/\alpha_0 < 1.2\times 10^{-7}$ (according to Damour and Dyson (1996)) at the redshift $z \approx 0.16$, where α_0 is the value of α at present (Fujii *et al.* 2000). Other constraints obtained from the absorption line spectra of distant quasars are $\Delta\alpha/\alpha = (0.574 \pm 0.102) \times 10^{-5}$ over the redshift range $0.2 < z < 3.7$ (Murphy et al. 2001; Webb *et al.* 2001) and $\Delta\alpha/\alpha = (0.06 \pm 0.06) \times 10^{-5}$ for $0.4 < z < 2.3$ (Chand *et al.* 2004). These claims may provide important conclusions for the existence of a light scalar or vector field related with dark energy. However the possibility of systematic errors still remains (Murphy *et al.* 2003).

Furthermore there are many theoretical and experimental proposals that claim that the values of fundamental constants are not actually constant and may be space and time dependent (Shlyakhter 1976; Uzan 2003; Bekenstein 1982; Barrow 2003). One of these constants, which have attracted model makers' attention is the speed of light (Magueijo 2003) that has arisen from the possibility of varying fine structure constant. These results imply the question: which of e, $\hbar$ and c might be responsible for any observed change in α?

The first modern varying speed of light (VSL) theory was by Moffat (1993). Albrecht & Magueijo (1999) and Barrow (1999) took these models as an alternative to the inflation theory in order to solve some puzzles of the Big-Bang cosmological models (Albrecht & Magueijo 1999; Magueijo & Baskerville 2000).

VSL can be attained via a pre-set function for the speed of light (Barrow 1999; Barrow & Magueijo 1999), or by considering a dynamical term in the Lagrangian for the varying speed of light (Magueijo 2000). Magueijo has suggested a generalized varying speed of light theory, having general covariance and local Lorentz invariance by introducing a time-like coordinate x^0 which is not necessarily equal to ct. The physical time t can only be defined when $\mathrm{d}x^0/c$ is integrable.

As a matter of fact there are different constants, which can be interpreted as the velocity of light. Ellis & Uzan (2005) have shown that one has to discriminate between c_{EM} (the electromagnetic wave velocity), c_{ST} (the space–time causal structure constant), c_{GW} (the gravitational wave velocity) and c_{E} (the space–time–matter coupling constant appearing at the right hand side of Einstein's equations). Taking the standard Lagrangian of the electromagnetism and general relativity and having the correct Newtonian limit, one has $c_{\mathrm{E}}/c_0 = c_{\mathrm{GW}}/c_0 = c_{\mathrm{EM}}/c_0 = c_{\mathrm{ST}}/c_0 = 1$, where c_0 is a constant of dimension velocity and is equal to 3×10^8 m/s in the MKS units. All these velocities except for the Einstein velocity have been investigated for some modified gravities in Palatini formalism in Izadi & Shojai (2009) and we have shown that they are not equal to each other in the local inertial frame. In other words, after relaxing the constancy of the speed of light, different facets of c will not coincide necessarily. Therefore considering any modified gravity and/or electromagnetism theory involves in re-examining the meaning and the relation between these different concepts of the speed of light. In addition, in order to formulate a theory in which the speed of light is varying, it is crucial to specify which kind of speeds is going to vary. For that reason it is significant to express clearly which quantities are kept fixed when one or the other aspects of c is assumed to vary.

In what follows, we will take the Einstein velocity as a field that varies like what we have done in Izadi & Shojai (2010). Here we will examine a vector field as the candidate for this field. Vector-like dark energy displays a series of properties that make it phenomenologically interesting. We will assume such a VSL model with a nonminimal coupling to gravity, just like what we have done in Izadi & Shojai (2009, 2010). However, we assume that the Einstein speed of light is obtained by dynamical properties expressed through the coupling $Z(A^2)$ in the action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} Z(A^2) R + \xi F_{\mu\nu} F^{\mu\nu} - V(A^2) + L_{\mathrm{M}}(g_{\mu\nu}, \Phi) \right], \tag{2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $A^2 = g^{\mu\nu} A_\mu A_\nu$. Also $L_{\mathrm{M}}(g_{\mu\nu}, \Phi)$ stands for matter fields. As a matter of fact letting a constant vary implies substituting it by a dynamical field consistently. In reality we must consider only the variation of dimensionless quantities. Before we proceed further with the speed of light, let us make some general comments about tensor–vector theories of gravity. In such theories, one considers a Lagrangian like what we proposed in the above form. The function Z is a dimensionless dynamical field. The dynamics of A^2 depends on Z, V and a kinetic term that by a redefinition of the field A^2 can be set equal to 1. Putting potential term may seem rather *ad hoc*, but we will reconstruct it in such a way that the dynamics of A^2 produce the desired cosmic history. Although, only the variation of dimensionless constants is physically acceptable and this action can represent $8\pi G/c^4$ varying theory, but setting $8\pi G = c_0^4 = 1$, it is equivalent to having $Z(A^2) = (\frac{c}{c_0})^4$ as a varying quantity and thus representing a VSL model.

In actual fact several observational puzzles can be solved by VSL. With appropriate additional observations, the redshift dependence in α could be taken as a result of the varying Einstein velocity c_{E}. However, there are some obstacles in all tests of a varying c. One of them is that the predicted effects are either out of the reach of the present technology, or on the threshold.

One of the best observations wich is relevant to VSL, is the evidence for a redshift dependence in the fine structure constant, which claims that the value of α was lower in the past. Another evidence is the recent observational data for SN Ia and so on. As a matter of fact with a higher c in the past, objects with the same look-back time is further away. Therefore, in VSL theories it looks as though the Universe is accelerating. In addition there is a strange coincidence between the redshifts at which the Universe starts accelerating and those based on variations in α. VSL theories can explain this coincidence. In Barrow & Magueijo (2000), both the Webb and supernovae results are fitted using the same set of parameters.

In section 2, we will investigate the equations governing VSL cosmology in the framework of vector–tensor theories. In section 3, we will examine the dynamics of this theory via the dynamical system method assuming a ΛCDM cosmic history. Considering the character of the critical points of the theory and their stabilities, we find the generic evolution of the system. The forms of the potential V and the coupling Z is reconstructed in section 4. One can set restrictions on the varying speed of light to solve the horizon problem. This gives a selection rule for choosing the appropriate stable solution. Also observational constraints put some restriction on the parameters appeared in the c function.

2. The model

Starting from (2), variation with respect to the metric gives the modified Einstein equations:

$$Z(A^2)G_{\mu\nu} = \overset{(m)}{T}_{\mu\nu} + \overset{(A)}{T}_{\mu\nu} + \overset{(Z)}{T}_{\mu\nu}. \tag{3}$$

Since the matter fields are minimally coupled to the metric, the weak equivalence principle holds and therefore

$$\nabla_\mu \overset{(m)}{T}{}^{\mu}{}_{\nu} = 0, \tag{4}$$

where the energy momentum tensor of matter is defined as usual:

$$\overset{(m)}{T}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g_{\mu\nu}}. \tag{5}$$

The other parts of the energy momentum tensor are defined as

$$\overset{(A)}{T}_{\mu\nu} = g_{\mu\nu}(\xi F_{\rho\sigma} F^{\rho\sigma} - V(A^2)) - 4\xi F_{\mu\rho} F_{\nu}{}^{\rho} + 2\frac{\mathrm{d}V}{\mathrm{d}A^2} A_\mu A_\nu \tag{6}$$

and

$$\overset{(Z)}{T}_{\mu\nu} = (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^\rho \nabla_\rho) Z(A^2) - R \frac{\mathrm{d}Z}{\mathrm{d}A^2} A_\mu A_\nu. \tag{7}$$

Varying the action with respect to the vector field A_μ, one obtains

$$\xi \nabla_\nu F^{\mu\nu} = \frac{1}{4} \left[\frac{\mathrm{d}Z}{\mathrm{d}A^2} R - 2 \frac{\mathrm{d}V}{\mathrm{d}A^2} \right] A^\mu \tag{8}$$

which is a generalization of Maxwell equations.

Let us confine ourselves to the case of a spatially flat, homogeneous and isotropic FRW universe with metric

$$\mathrm{d}s^2 = -\mathrm{d}x^{0^2} + a(x^0)^2 (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2). \tag{9}$$

Using the above field equations and assuming the matter field as a perfect fluid in this cosmological context, we will obtain the c_E-variable Friedman equations and the conservation law respectively as

$$3Z(A^2)H^2 = \overset{(m)}{\rho} + \overset{(A)}{\rho} + \overset{(Z)}{\rho}, \tag{10}$$

$$-2Z\dot{H} = \left(\overset{(m)}{\rho} + \overset{(m)}{p}\right) + \left(\overset{(A)}{\rho} + \overset{(A)}{p}\right) + \left(\overset{(Z)}{\rho} + \overset{(Z)}{p}\right), \tag{11}$$

$$\dot{\overset{(m)}{\rho}} + 3H\left(\overset{(m)}{\rho} + \overset{(m)}{p}\right) = 0, \tag{12}$$

where a dot over any quantity indicates derivative with respect to the time-like coordinate x^0. $R = 6(\dot{H} + 2H^2)$ is the Ricci scalar, the parameter $H(x^0) = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}x^0}$ is the Hubble parameter, and ρ and p are the 00 and ii components of the energy–momentum tensor. The time-like coordinate x^0 is related to the cosmic time by the relation

$$\mathrm{d}t = \frac{\mathrm{d}x^0}{c} = \frac{\mathrm{d}x^0}{Z(A^2)^{\frac{1}{4}}}. \tag{13}$$

In the cosmological context, since $\frac{\mathrm{d}x^0}{c}$ is integrable, the physical time will be obtained simply. In addition, the physical Hubble parameter $H_\mathrm{p}(t) = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}$ can be evaluated as $H_\mathrm{p}(t) = H(x^0)\frac{\mathrm{d}x^0}{\mathrm{d}t}$. Replacing $\frac{1}{H(x^0)}\frac{\mathrm{d}}{\mathrm{d}x^0}$ by $\frac{1}{H_\mathrm{p}(t)}\frac{\mathrm{d}}{\mathrm{d}t}$ in eq. (12) gives

$$\frac{\mathrm{d}\rho^{(m)}}{\mathrm{d}t} + 3H_\mathrm{p}\left(\overset{(m)}{\rho} + \overset{(m)}{p}\right) = 0 \tag{14}$$

which shows the validity of the conservation equation of matter in terms of the cosmic time in this model. We are restricted to consider either a purely time-like vector, $A_\mu = (A(x^0), 0)$, or a space-like vector with $A_\mu = (0, \vec{A}(x^0))$ since the symmetries of the metric (9) do not permit a velocity field T_{0i}. The existence of a spatially non-vanishing vector can break the isotropy of a FRW universe. If a vector field was introduced as a candidate for the dark energy, as long as it remains subdominant, this violation would be observationally irrelevant. Once dark energy becomes dominant, one would expect an anisotropic expansion of the Universe, in conflict with the remarkable isotropy of the CMB. Due to this fact, in order to avoid violations of isotropy, the vector has to be part of a cosmic triad or in spherical coordinates; this means that it should have only the radial component.

Let us now investigate the explicit forms of the equations for these two cases separately.

2.1 *Time-like vector model* $A_\mu = (A(x^0), 0)$

At first we consider the time-like case, namely:

$$A_\mu = A(x^0)\delta^0_\mu. \tag{15}$$

The energy densities and pressures are given by

$$\overset{(A)}{\rho} = V(A^2) + 2\frac{\mathrm{d}V}{\mathrm{d}A^2}A^2, \tag{16}$$

$$\overset{(A)}{p} = -V(A^2) \tag{17}$$

and

$$\overset{(Z)}{\rho} = -3H\dot{Z} - R\frac{\mathrm{d}Z}{\mathrm{d}A^2}A^2, \tag{18}$$

$$\overset{(Z)}{p} = \ddot{Z} + 3H\dot{Z}. \tag{19}$$

Moreover the generalized Maxwell equations for this model would get the form

$$\frac{\mathrm{d}Z}{\mathrm{d}A^2}R - 2\frac{\mathrm{d}V}{\mathrm{d}A^2} = 0. \tag{20}$$

Substituting these equations back in the generalized Friedman equations, we have

$$3Z(A^2)H^2 = \rho_\mathrm{m} + \rho_\mathrm{r} + V(A^2) - 3H\dot{Z}, \tag{21}$$

$$-2Z(A^2)\dot{H} = \rho_\mathrm{m} + \frac{4}{3}\rho_r + \ddot{Z}, \tag{22}$$

in which ρ_m and ρ_r are matter and radiation contributions to $\rho^{(m)}$.

Dynamical system method is a way to find out some exact solutions of cosmological models (Goliath & Ellis 1999). Choosing appropriate variables, the field equations of the desired theory can be converted to a set of autonomous differential equations. Afterwards the *critical points* of this autonomous system describe interesting exact solutions. Furthermore this method can be used to check the stability conditions. In section 3 this method will be used to obtain some exact solutions of this model.

2.2 *Space-like vector model* $A_\mu = (0, \vec{A}(x^0))$

For the case of a space-like vector model, isotropy and homogeneity of the space-time forces one to the following form of the vector field:

$$A_\mu = aA(x^0)\delta^1_\mu. \tag{23}$$

The energy densities and pressures are given by

$$\overset{(A)}{\rho} = -2\xi(\dot{A} + AH)^2 + V(A^2), \tag{24}$$

$$\overset{(A)}{p} = +2\xi(\dot{A} + AH)^2 - V(A^2) + 2\frac{\mathrm{d}V}{\mathrm{d}A^2}A^2(x^0) \tag{25}$$

and

$$\overset{(Z)}{\rho} = -3H\dot{Z}, \tag{26}$$

$$\overset{(Z)}{p} = \ddot{Z} + 3H\dot{Z} - R\frac{\mathrm{d}Z}{\mathrm{d}A^2}A^2(x^0). \tag{27}$$

Substituting these equations in the generalized Friedman equations, we have

$$3Z(A^2)H^2 = \rho_\mathrm{m} + \rho_\mathrm{r} - 2\xi(\dot{A} + HA)^2 + V(A^2) - 3H\dot{Z} \tag{28}$$

$$-2Z(A^2)\dot{H} = \rho_\mathrm{m} + \frac{4}{3}\rho_\mathrm{r} + \ddot{Z} + \left[2\frac{\mathrm{d}V}{\mathrm{d}A^2} - R\frac{\mathrm{d}Z}{\mathrm{d}A^2}\right]A^2(x^0). \tag{29}$$

Also the generalized Maxwell equations for this model is given by

$$\xi[\ddot{A} + 3H\dot{A} + A\dot{H} + 2H^2A] = \frac{1}{4}\left[\frac{\mathrm{d}Z}{\mathrm{d}A^2}R - 2\frac{\mathrm{d}V}{\mathrm{d}A^2}\right]A(x^0). \tag{30}$$

3. Dynamics of the models

As mentioned previously, the dynamical system method describes the cosmological dynamics of these models. Since here we have two arbitrary functions, i.e. $Z(A^2)$ and $V(A^2)$, there is one degree of freedom, m in time-like model and two degrees of freedom, m and n in space-like model that we will introduce in the next subsection. In reality, if we do not try to reconstruct the function $Z(A^2)$, this function can be fixed at first. In such a case $H(N)$ would be determined by the autonomous system. However, in our reconstruction approach, it would be fixed at first. Adopting the dynamical system method, we have to use the Hubble parameter of ΛCDM cosmology, given by eq. (1) as an *input parameter* of the model and in addition to the c_E-variable Friedmann equation (21) or (28), the following conservation equations have to be considered:

$$\dot{\rho}_\mathrm{m} + 3H\rho_\mathrm{m} = 0, \tag{31}$$

$$\dot{\rho}_\mathrm{r} + 4H\rho_\mathrm{r} = 0. \tag{32}$$

To study the cosmological dynamics implied by Friedmann equation we express it as an autonomous system of first-order differential equations. For the case of time-like vector field, equation (21) can be written in the dimensionless form as

$$1 = \frac{\rho_{\mathrm{m}}}{3ZH^2} + \frac{\rho_{\mathrm{r}}}{3ZH^2} + \frac{V(A^2)}{3ZH^2} - \frac{Z'}{Z}, \tag{33}$$

while for space-like vector field equation (28) has the form

$$1 = \frac{\rho_{\mathrm{m}}}{3ZH^2} + \frac{\rho_{\mathrm{r}}}{3ZH^2} - \frac{2\xi}{3}\frac{(A'+A)^2}{Z} + \frac{V(A^2)}{3ZH^2} - \frac{Z'}{Z}, \tag{34}$$

where

$$' \equiv \frac{\mathrm{d}}{\mathrm{d}\ln a} \equiv \frac{\mathrm{d}}{\mathrm{d}N} = \frac{1}{H(x^0)}\frac{\mathrm{d}}{\mathrm{d}x^0} = \frac{1}{H_p(t)}\frac{\mathrm{d}}{\mathrm{d}t}. \tag{35}$$

The average effective equation of state is

$$w_{\mathrm{eff}} = -\frac{2}{3}\frac{H'}{H} - 1 \tag{36}$$

which describes the general expansion rate of the Universe.

3.1 *Time-like model*

For this model according to (33) we can define the dimensionless variables $x_1,\ x_2,\ x_3$ as

$$x_1 := \frac{-Z'}{Z}, \tag{37}$$

$$x_2 := \frac{V(A^2)}{3ZH^2}, \tag{38}$$

$$x_3 := \frac{\rho_r}{3ZH^2} = \Omega_{\mathrm{r}}. \tag{39}$$

In fact x_3 is Ω_{r} and $x_1 + x_2 \equiv \Omega_{\mathrm{DE}}$ is the curvature dark energy. Defining $\Omega_{\mathrm{m}} \equiv \frac{\rho_{\mathrm{m}}}{3ZH^2}$, eq. (33) can be written as

$$\Omega_{\mathrm{m}} = 1 - x_1 - x_2 - x_3. \tag{40}$$

Using the defined dimensionless variables, eq. (22) can be expressed as

$$x_1' = 3 - 3x_1 - 3x_2 + x_3 + x_1^2 + (2 - x_1)\frac{H'}{H}. \tag{41}$$

Differentiating x_2, x_3 with respect to N we have

$$x_3' = -4x_3 + x_3x_1 - 2x_3\frac{H'}{H} \tag{42}$$

and

$$x_2' = x_2\left[x_1(1-m) - 2\frac{H'}{H}\right], \tag{43}$$

where

$$m = \frac{\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}A^2}}{\frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}A^2}}. \tag{44}$$

The potential $V(A^2)$ is restricted to satisfy the equation (44), where m is an arbitrary constant in order to study the dynamical features of the model with a general coupling $Z(A^2)$. With the above assumption, it is worthy of attention that without specifying neither the coupling nor the potential, it is possible to find analytically the class of inflationary attractors. As a matter of fact potentials like exponential, power-laws and so on have been chosen in different models, based either on simplicity or on some particle physics model. However there are no observations or fundamental principles to lead our investigation.

The cosmological dynamics of the model is described by the autonomous dynamical system (41), (42) and (43). It should be emphasized that in our reconstruction approach, $H(N)$ is taken fixed, while $\frac{Z_{,A^2}}{Z}$ (and its perturbations) are allowed to vary. In addition, it is significant to bear in mind that $\frac{H'}{H}$ is not always constant and for this reason the dynamical equations are not generally autonomous. On the other hand for ΛCDM background, $\frac{H'}{H}$ is approximately constant in the radiation ($w_{\rm eff} = 1/3$), matter ($w_{\rm eff} = 0$) and de Sitter ($w_{\rm eff} = -1$) eras and therefore the dynamical system method can be used for these three different eras. This approach comes up with the form of both the coupling and the potential. In addition, exact solutions can be derived by first integrals of motion. The results of our investigation do not depend on using any particular form of $H(z)$, except that the Universe should go through the radiation, matter and acceleration eras. In order to achieve the definiteness, a specific form for $H(z)$ corresponding to a ΛCDM cosmology (1) will be assumed, which in terms of N will take the form

$$H(N)^2 = H_0^2\big[\Omega_{0\rm m}\mathrm{e}^{-3N} + \Omega_{0\rm r}\mathrm{e}^{-4N} + \Omega_\Lambda\big], \tag{45}$$

where $N = \ln a = -\ln(1+z)$ and $\Omega_\Lambda = 1 - \Omega_{0\rm m} - \Omega_{0\rm r}$.

If a ΛCDM behavior for H is applied, the model will agree with observation. As a matter of fact, observations from SN Ia, CMB and BAO are all based on the determination of $r(z)$, where r is the comoving distance to a certain object located at redshift z. The comoving distance is obtained from the null geodesics of the Robertson–Walker metric as $r(z) = \int \frac{\mathrm{d}x^0}{a(x^0)} = \int \frac{\mathrm{d}z}{H(z)}$, where $H(z)$ is the standard Hubble parameter. Therefore in order to reproduce the background expansion history indicated by observations, it would be appropriate to impose condition on $H(z)$ rather than on $H_{\rm p}(z)$. In order to find the critical points and their stability in each one of these three eras, dynamics of the system can be described by setting $x_i' = 0$.

Equation (45) can be used to find $\frac{H'(N)}{H(N)}$, which is a quantity needed in our relations

$$\frac{H'(N)}{H(N)} = \frac{-3\Omega_{0\rm m}\mathrm{e}^{-3N} - 4\Omega_{0\rm r}\mathrm{e}^{-4N}}{2[1 - \Omega_{0\rm m} - \Omega_{0\rm r} + \Omega_{0\rm m}\mathrm{e}^{-3N} + \Omega_{0\rm r}\mathrm{e}^{-4N}]}.$$

The values of the above term in the three different eras are

$$\frac{H'(N)}{H(N)} = -2, \qquad N < N_{\text{r}-\text{m}}$$

$$\frac{H'(N)}{H(N)} = \frac{-3}{2}, \quad N_{\text{r}-\text{m}} < N_{\text{m}-\Lambda}$$

$$\frac{H'(N)}{H(N)} = 0, \qquad N > N_{\text{m}-\Lambda}$$

in which $n_{\text{r}-\text{m}} \simeq -\ln\frac{\Omega_{0\text{m}}}{\Omega_{0\text{r}}}$ and $N_{\text{m}-\Lambda} \simeq \frac{-1}{3}\ln\frac{\Omega_{\Lambda}}{\Omega_{0\text{m}}}$ are the N radiation–matter and matter–de Sitter transition values. For $\Omega_{0\text{m}} = 0.3$ and $\Omega_{0\text{r}} = 10^{-4}$, N takes these values respectively: $N_{\text{r}-\text{m}} \simeq -8$ and $N_{\text{m}-\Lambda} \simeq -0.3$. Although the transition between these three eras is model dependent, this transition is rapid and so it will not play an important role in our analysis. It is remarkable to emphasize that this dynamical system can be approximated as such during the radiation, matter and de Sitter eras when $\frac{H'(N)}{H(N)}$ is roughly constant, even though this dynamical system is not autonomous at all times. The results for time-like model are shown in Table 1.

In general, any other parameter which could be related to $Z_{,A}$ and $Z_{,A^2}$ should be added beside the above m. In actuality such a function could be fixed from the beginning, if we did not try to reconstruct the function $Z(A^2)$, and the corresponding

Table 1. The critical points of the system and their eigenvalues in each one of the three eras. In the above table $\zeta = \sqrt{\left(1+\frac{8}{1-m}\right)^2 - 4(1-m)\left(\frac{4(5-m)}{(1-m)^2}-1\right)}$ and $\chi = \frac{1}{2}\sqrt{\left(\frac{3(5-m)}{2(1-m)}\right)^2 - \frac{18(3-m)}{(1-m)}}$.

Era	Critical point (x_1, x_2, x_3)	Eigenvalues
Radiation	$R_1 = (0, 0, 1)$	$\left(4, \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$
$\frac{H'}{H} = -2$	$R_2 = \left(\frac{1+\sqrt{5}}{2}, 0, 0\right)$	$\left(\sqrt{5}, 4+\frac{(1+\sqrt{5})(1-m)}{2}, \frac{1+\sqrt{5}}{2}\right)$
	$R_3 = \left(\frac{1-\sqrt{5}}{2}, 0, 0\right)$	$\left(-\sqrt{5}, 4+\frac{(1-\sqrt{5})(1-m)}{2}, \frac{1-\sqrt{5}}{2}\right)$
	$R_4 = \left(\frac{-4}{1-m}, \frac{1}{3}\left(\frac{4(5-m)}{(1-m)^2}-1\right), 0\right)$	$\left(\frac{-4}{1-m}, \frac{-\left(1+\frac{8}{1-m}\right)+\zeta}{2}, \frac{-\left(1+\frac{8}{1-m}\right)-\zeta}{2}\right)$
Matter	$M_1 = (0, 0, 0)$	$(-1, -3/2, 3)$
$\frac{H'}{H} = -\frac{3}{2}$	$M_2 = \left(\frac{3}{2}, 0, 0\right)$	$\left(\frac{1}{2}, \frac{3}{2}, \frac{9-3m}{2}\right)$
	$M_3 = \left(1, 0, \frac{1}{2}\right)$	$\left((4-m), 1, \frac{-1}{2}\right)$
	$M_4 = \left(\frac{-3}{1-m}, \frac{3(3-m)}{2(1-m)^2}, 0\right)$	$\left(\frac{-4+m}{1-m}, \frac{3(m-5)}{4(1-m)}+\chi, \frac{3(m-5)}{4(1-m)}-\chi\right)$
de Sitter	$d_1 = (0, 1, 0)$	$\left(-4, \frac{-3+\sqrt{3(4m-1)}}{2}, \frac{-3-\sqrt{3(4m-1)}}{2}\right)$
$\frac{H'}{H} = 0$	$d_2 = (4, 0, -7)$	$\left(4(1-m), \frac{5+i\sqrt{3}}{2}, \frac{5-i\sqrt{3}}{2}\right)$

parameter would be, for example, $\frac{Z_{,A^2}}{A^2}$. In this approach, which is different from our reconstruction approach, $H(N)$ would not be fixed but the autonomous system would have to determine it.

According to the above table, it is obvious that for $m < \frac{9-\sqrt{5}}{(\sqrt{5}-1)}$, R_3 is an attractor and also R_4 is an attractor provided that $m < 1$ and $\zeta < 1 + \frac{8}{1-m}$. Additionally for $m < 4$ and $\chi < \frac{3(m-5)}{4(1-m)}$, M_4 is an attractor. As a matter of course for $m < 1$, the critical point d_1 is an attractor.

In the stability analysis of the cosmological dynamical systems, a specific cosmological model (e.g. a particular form of $Z(A^2)$ or m) is usually assumed and then the stability of the cosmic history, $H(N)$, is investigated. In this circumstance a stable cosmic history is the one chosen by the model. However, the stability analysis has a very different meaning in the reconstruction approach. The model functions $Z(A^2)$ and $V(A^2)$ are not fixed, but we fix the cosmic history and permit the functions $Z(A^2)$ and $V(A^2)$ to be specified so that the model predict the desired cosmic history. Critical points are the physically interesting quantities in each era in the context of the ΛCDM cosmic history. These lead us to the possible forms of $Z(A^2)$ and $V(A^2)$ that can produce a ΛCDM cosmic history.

Although some critical points are not stable, this does not indicate that these points are cosmologically irrelevant. In actual fact these instabilities are not instabilities of the trajectory $H(N)$ (which we kept fixed). On the contrary these are instabilities of the $Z(A^2)$ and $V(A^2)$ forms which are permitted to vary. Thereby they are not so relevant physically for the reason that $Z(A^2)$ is assumed to be fixed from the beginning in a physical context.

The effective equation of state $w_{\rm eff}$ is applied on the dynamical system, acquired from (36) using (45).

An interesting characteristic of the critical points of Table 1 is that in all the situations, which differ from GR, dark energy (DE $= 1$) induce the expansion rate in each era, which implies that having the proper perturbation properties at early times, the vector field could also play the role of dark matter.

We could follow the behavior of critical points for this model to verify the dynamical evolution indicated by the attractors of table 1 by using ansatz $\frac{H'(N)}{H(N)}$ with $\Omega_{0\rm m} = 0.3$ and $\Omega_{0\rm r} = 10^{-4}$. This ansatz gives rise to the $w_{\rm eff}(N)$ as shown in Fig. 1. If the system initially were set up, on R_4 in the radiation era, with $m = \frac{1}{4}$, it would follow the evolution from R_4 to the matter era (M_4) and ultimately to the de Sitter Era (d_1). It is examined that if we did not choose the initial conditions exactly coinciding with any of the other critical points then the system would be captured by the R_4 attractor and then follows the above trajectory (see Fig. 2). In the literature, it is common to use the power-law or exponential forms for the potential $V(A^2)$ and coupling $Z(A^2)$ which also give a constant m.

3.2 *Space-like model*

According to (34) dimensionless variables x_1, x_2, x_3, x_4 can be defined as

$$x_1 := -\frac{Z'}{Z}, \tag{46}$$

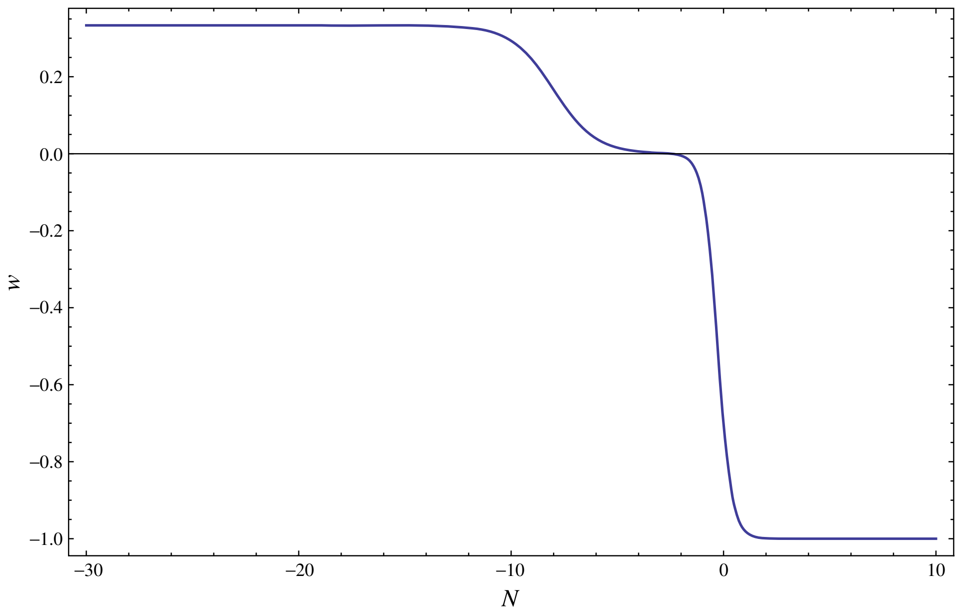

Figure 1. The effective equation of state $w_{\rm eff}$ imposed on the dynamical system.

$$x_2 := \frac{V}{3ZH^2}, \tag{47}$$

$$x_3^2 := \frac{2(A'+A)^2}{3Z}, \tag{48}$$

$$x_4 := \frac{\rho_{\rm r}}{3ZH^2}. \tag{49}$$

Considering $\Omega_{\rm m} = \frac{\rho_{\rm m}}{3ZH^2}$ we can write (34) as

$$1 = \Omega_{\rm m} + x_1 + x_2 - \xi x_3^2 + x_4. \tag{50}$$

As one can see x_4 is in fact $\Omega_{\rm r}$ and $x_1 + x_2 - \xi x_3^2 \equiv \Omega_{\rm DE}$ is again related to the curvature dark energy. Using the defined dimensionless variables, we can express eq. (29) as

$$x_1' = 3(1-x_1-x_2+\xi x_3^2)+x_4+\frac{A^2}{ZH^2}\left(2\frac{\mathrm{d}V}{\mathrm{d}A^2} - R\frac{\mathrm{d}Z}{\mathrm{d}A^2}\right)-\frac{H'}{H}x_1+x_1^2+2\frac{H'}{H}. \tag{51}$$

Also, differentiating x_2, x_3 and x_4 with respect to N we have

$$x_2' = x_2\left[x_1(1-m) - 2\frac{H'}{H}\right], \tag{52}$$

$$(x_3^2)' = x_1 x_3^2 + \frac{4(A+A')(A'+A'')}{3Z} \tag{53}$$

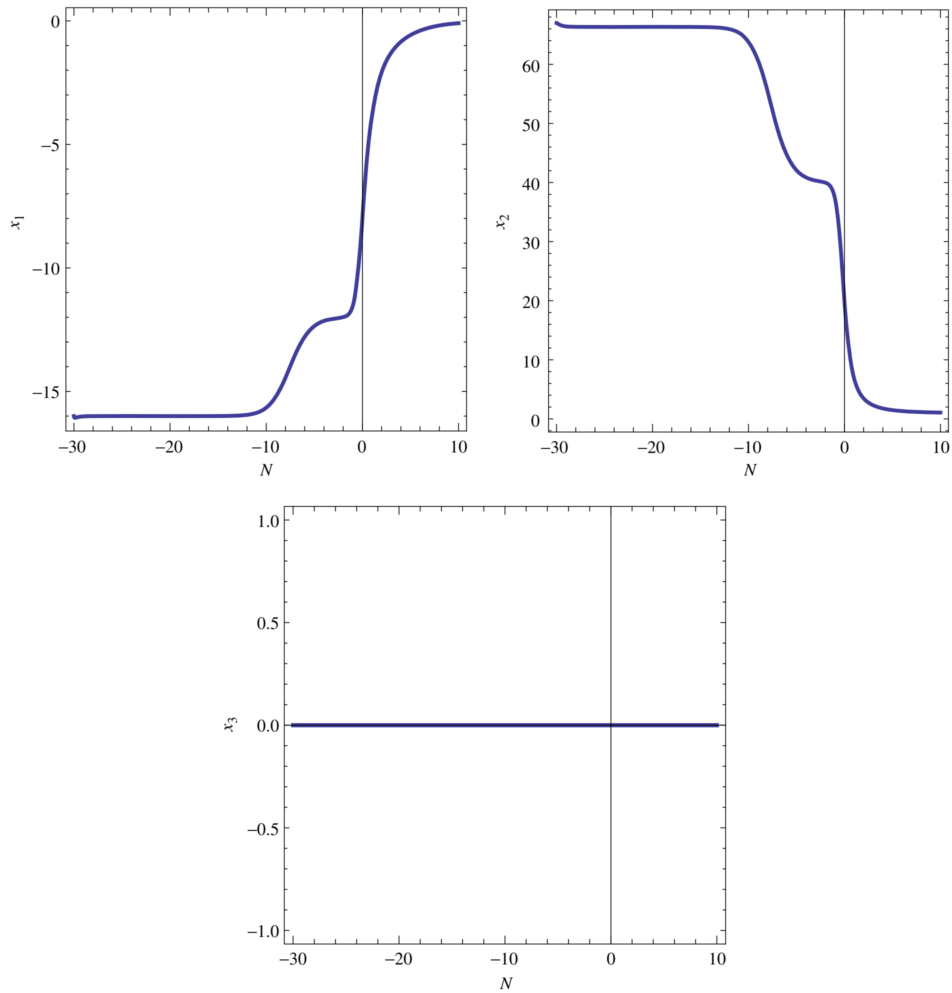

Figure 2. The evolution of the variables $x_1(N)$, $x_2(N)$ and $x_3(N)$. The system follows the evolution of the attractor through the three eras.

and

$$x_4' = -4x_4 + x_4 x_1 - 2x_4 \frac{H'}{H}. \tag{54}$$

As it is mentioned earlier, we are going to investigate the dynamical characteristics of models with a general coupling $Z(A^2)$, without restricting ourselves to a particular choice, except for the condition that the potential $V(A^2)$ satisfies equation (44), where m is an arbitrary constant. Using $R = 6(\dot{H} + 2H^2)$ and the generalized Maxwell equation as a constraint equation, these forms for x_1' and $(x_3^2)'$ will be given as

$$x_1' = 3(1 - x_1 - x_2 + \xi x_3^2) + x_4 - x_1 \frac{A}{2A'} \left[6mx_2 - 6\frac{H'}{H} - 12\right] + \frac{H'}{H}(2 - x_1) + x_1^2 \tag{55}$$

and

$$(x_3^2)' = x_1 x_3^2 - 2x_3^2\left(\frac{H'}{H}+2\right) + \frac{1}{6\xi}\left(1+\frac{A}{A'}\right)\left(6mx_2 - 6\frac{H'}{H} - 12\right)x_1. \quad (56)$$

Again we impose the general form for physical Hubble parameter (ΛCDM cosmology), as eq. (1). Also we confine ourselves to a specific choice in which $\frac{A}{A'} = n$ where n is an arbitrary constant. This assumption makes the equations much easier to solve.

The autonomous dynamical system (55), (52), (56) and (54) describes the cosmological dynamics of the space-like vector model. It is straightforward to study the dynamics of this system by setting $x_i' = 0$ to find the critical points. Putting $x_i' = 0$, one obtains Table 2.

A specific cosmological model is taken in the typical stability analysis of the cosmological dynamical systems (e.g. a form of $Z(A^2)$, m or n) and in the context of this physical law, the stability of cosmic histories $H(N)$ is examined. Here the cosmic history has been fixed and $Z(A^2)$ and $V(A^2)$ are allowed to vary in order to anticipate the required cosmic history. In the context of the ΛCDM cosmic history the values of the critical points in each era are represented in Table 2. These points denote the possible physical functions $Z(A^2)$ and $V(A^2)$ that can reproduce a

Table 2. The critical points of the system in each one of the three eras. Here $\zeta = \sqrt{1-24\xi^2(1+n)+6n+9n^2}$, $\chi = -4+(5-3\xi^2)m+(3\xi^2-1)m^2+(12-3\xi^2)mn+3(\xi^2-n)m^2n$, $\delta = 3(12+\xi^2-(7+2\xi^2)m+(1+\xi^2)m^2+(\xi^2-4)n+(5-2\xi^2)mn+(\xi^2-1)m^2n)$ and x_1 and x_3 in de-Sitter era satisfies these equations: $3(1-x+\xi z^2)+6nx+x^2=0$ and $(x-4)z^2-\frac{2}{\xi}(1+n)x=0$.

Era	Critical point $(x_1, x_2, {x_3}^2, x_4)$
	$R_1\left(0, 0, \frac{1}{\sqrt{3\xi}}, 0\right)$
Radiation	$R_2\left(0, 0, -\frac{1}{\sqrt{3\xi}}, 0\right)$
$\frac{H'}{H}=-2$	$R_3\left(\frac{1+\sqrt{5}}{2}, 0, 0, 0\right)$
	$R_4\left(\frac{1-\sqrt{5}}{2}, 0, 0, 0\right)$
	$R_{5,6}\left(\frac{-4}{1-m}, \frac{-19+47m-48m^2+16m^3}{3(-1+m^2(1+n)+mn(3+m))}, \mp\frac{\sqrt{-m(1+n)(-19+47m-48m^2+16m^3)}}{\sqrt{3\xi(-1+m^2(1+n)+mn(3+m))}}, 0\right)$
	$M_1(0, 0, 0, 0)$
Matter	$M_{2,3}(\frac{1}{4}\left(5+3n-\zeta, 0, \mp\frac{1}{2\sqrt{3\xi}}\sqrt{1+3n+6\xi^2(1+n)+\zeta}\right)$
$\frac{H'}{H}=-\frac{3}{2}$	$M_{4,5}(\frac{1}{4}\left(5+3n+\zeta, 0, \mp\frac{1}{2\sqrt{3\xi}}\sqrt{1+3n+6\xi^2(1+n)-\zeta}\right)$
	$M_{6,7}\left(\frac{-3}{1-m}, \frac{3\delta}{2(m-1)\chi}, \mp\frac{\sqrt{3\xi/2}\sqrt{(1+n)(1-11m+4m^2)}}{\sqrt{(m-1)\chi}}, 0\right)$
de Sitter	$d_1(0, 1, 0, 0)$
$\frac{H'}{H}=0$	$d_2(x_1, 0, x_3, 0)$

ΛCDM cosmic history. Calculations of eigenvalues for this model are too long, but in what follows we do not need them. Therefore we will skip them.

4. Reconstruction of $Z(A^2)$, $V(A^2)$

We can now reconstruct the form of the functions $Z(A^2)$ and $V(A^2)$ corresponding to each one of the critical points of the system shown in Tables 1 and 2 for these two models. These reconstructions are effectively an approximation of these functions in the neighborhood of each critical point.

4.1 *Time-like model*

Consider a critical point of the form $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$. Using (37), we find that

$$Z(N) = Z_0 \mathrm{e}^{-\bar{x}_1 N}, \tag{57}$$

where Z_0 is the present value of Z. Now we have to find the relation between N and A^2 in order to reconstruct $Z(A^2)$. Due to the fact that in the time-like case, we have $F_{\mu\nu} = 0$, the field A_μ does not appear as a dynamical variable in the action (2). As a result it is not possible to determine the function $N(A^2)$. Since, now the Maxwell equation (20) is a non-dynamical equation, it puts only a constraint on the fixed points

$$(2-\beta)\bar{x}_1 = \bar{x}_2(\bar{x}_1 + 2\beta) \tag{58}$$

in which β is 2, 3/2 and 0 for radiation, matter and de Sitter era respectively. Under these conditions, only some of the fixed points in Table 1 are acceptable. In fact in radiation era, R_1, R_2 and R_3 are acceptable but R_4 is acceptable provided that $m = 0$. In addition, in matter era, M_1 is acceptable but M_4 is acceptable provided that $m = \frac{11 \pm \sqrt{105}}{8}$. Finally in de Sitter era, d_1 is acceptable.

As we mentioned earlier, there is no limitation in determination of functional form of $N(A^2)$. Using the relation

$$\frac{\mathrm{d}N}{\mathrm{d}A^2} = \frac{\mathrm{d}N}{\mathrm{d}x^0}\frac{\mathrm{d}x^0}{\mathrm{d}A^2} = H\frac{\mathrm{d}x^0}{\mathrm{d}A^2} \tag{59}$$

and equation (45), we can find N in terms of x^0 and using equation (57) one obtains Z in terms of x^0 for each era:

$$Z(x^0) = \alpha x_0^{\left(\frac{-\bar{x}_1}{\lambda - \frac{1}{4}\bar{x}_1}\right)}, \tag{60}$$

where

$$\lambda = 2 \quad \text{(radiation era)} \tag{61}$$

$$-\frac{3}{2} \quad \text{(matter era)} \tag{62}$$

$$0 \quad \text{(de-Sitter era)} \tag{63}$$

and

$$\alpha = Z_0 \left[\left(\lambda - \frac{1}{4}\bar{x}_1 \right) H_0 \sqrt{\Omega_{0i}} \right]^{\frac{-\bar{x}_1}{\lambda - \frac{1}{4}\bar{x}_1}}, \tag{64}$$

where Ω_{0i} is related to each era, namely $\Omega_{0\mathrm{r}}$, $\Omega_{0\mathrm{m}}$ and $\Omega_{0\Lambda}$. It has to be noted, that since the Maxwell equation does not fix the form of A we have the freedom in choosing any form $A = g(x_0)$. The field A now can be assumed as a clock field and thus naturally we can choose it as $A^2 \equiv x^0$, thus

$$Z(A^2) = \alpha A^{2\left(\frac{-\bar{x}_1}{\lambda - \frac{1}{4}\bar{x}_1}\right)}. \tag{65}$$

Since $\frac{c}{c_0} = Z^{\frac{1}{4}}$, we have

$$\frac{c}{c_0} = Z_0^{\frac{1}{4}} a^{\frac{-\bar{x}_1}{4}}. \tag{66}$$

Depending on the value of $\bar{x}_1$ given in the table, this can lead to a constant, decreasing or increasing speed of light with respect to the scale factor.

Horizon problem is one of the cosmological problems, which motivated physicists to suggest some models like inflation and VSL theories. The story of horizon problem goes back to the fact that the present causally connected comoving horizon, breaks down to several causally connected regions, which were completely disconnected to one another in the past. These disconnected regions in the early Universe can not describe the large-scale properties that can be observed today. Mathematically the comoving horizon is $r = \frac{c}{\dot{a}}$, and one should put forward r as an increasing function of cosmic time in order to have a causally connected large region observed now. Hence

$$\frac{\ddot{a}}{\dot{a}} - \frac{\dot{c}}{c} > 0$$

that is indicated by an accelerating expansion, or a decreasing speed of light, or a combination of both. Using the horizon criteria, some restrictions on $\bar{x}_1$ and thus on m can be obtained. As a means to solve the horizon problem of the standard cosmology, the horizon criteria for the early Universe (see Shlyakhter 1976; Uzan 2003; Bekenstein 1982; Barrow 2003) should be set and one has $\dot{a} > 0$ as well. For the radiation era this condition leads to $\bar{x}_1 > 4$. In addition we have $\bar{x}_1 > 2$ for the matter era, and in the end $\bar{x}_1 > -4$ for the de-Sitter era. This can be used as a selection rule for the model parameter m. The result is shown in Table 3.

In fact M_4 satisfies both *Maxwell equation* and *horizon problem* provided that $m = \frac{11-\sqrt{105}}{8}$ and only d_1 satisfies them in de Sitter era.

Furthermore the observational constraints such as SN Ia data put some extra constraints on Z_0.

Finally let us reconstruct the form of the potential V. From equation (38) we have

$$V(N) = 3\bar{x}_2 Z(N) H^2(N). \tag{67}$$

Using (65), we can write $V(A^2)$ as

$$V(A^2) = 3Z_0 \bar{x}_2 H_0^2 \Omega_{0i} \left[H_0 \Omega_{0i}^{\frac{1}{2}} \left(\lambda - \frac{1}{4}\bar{x}_1 \right) A^2 \right]^{2\beta}, \tag{68}$$

Table 3. The horizon problem sets some selection rules on the model.

Critical point	Horizon problem
R_1	Not Okay
R_2	Not Okay
R_3	Not Okay
R_4	Okay, provided $m < 2$ (see the text)
M_1	Not Okay
M_2	Not Okay
M_3	Not Okay
M_4	Okay, provided $m < \frac{5}{2}$ (see the text)
d_1	Okay (see the text)
d_2	Okay (see the text)

where

$$\beta = \frac{-8-\bar{x}_1}{8-\bar{x}_1} \quad \text{(radiation era)} \tag{69}$$

$$\frac{-6-\bar{x}_1}{6-\bar{x}_1} \quad \text{(matter era)} \tag{70}$$

$$1 \quad \text{(de-Sitter era).} \tag{71}$$

If we put these reconstructed functions, $Z(A^2)$ and $V(A^2)$ in (21) and (22), equation (45) will consistently satisfy these modified Friedman equations.

4.2 *Space-like model*

Here we consider a critical point of the form $(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4)$. Using (46), we find that

$$Z(N) = Z_0 \mathrm{e}^{-\bar{x}_1 N}, \tag{72}$$

where Z_0 is the present value of Z. And from (47) we have

$$V(N) = 3\bar{x}_2 H^2 Z_0 \mathrm{e}^{-\frac{1}{2}\bar{x}_1 N}. \tag{73}$$

For reconstructing $Z(A^2)$, we have to find the relation between N and A^2. Using the definition of x_3^2 we get

$$x_3^2 = \frac{2(A'+A)^2}{3Z} = \frac{2(n+1)^2}{3Z} A'^2$$

and thus

$$A(N) = \sqrt{\frac{3z_0}{2}} \frac{\bar{x}_3}{(1+n)} \frac{-2}{\bar{x}_1} \mathrm{e}^{-\frac{1}{2}\bar{x}_1 N} + C.$$

The above relation allows us to eliminate N in favor of A and hence

$$Z(A^2) = \frac{(1+n)^2}{6}\left(\frac{\bar{x}_1}{\bar{x}_3}\right)^2 (A-C)^2. \tag{74}$$

The form of $V(A^2)$ can be obtained using equation (45) as

$$V(A^2) = 3H_0^2 Z_0 \bar{x}_2 \Omega_{0i}\left[\frac{(A-C)^2}{6z_0}(1+n)^2\left(\frac{\bar{x}_1}{\bar{x}_3}\right)^2\right]^{\frac{\frac{1}{2}\bar{x}_2+\lambda}{\bar{x}_1}}, \tag{75}$$

where

$$\lambda = 4 \quad \text{(radiation era)} \tag{76}$$

$$3 \quad \text{(matter era)} \tag{77}$$

$$0 \quad \text{(de-Sitter era)}. \tag{78}$$

Again if we put these reconstructed functions $Z(A^2)$ and $V(A^2)$ in (28) and (29), we will find out that equation (45) consistently satisfies these modified Friedman equations. Since $\frac{c}{c_0} = Z^{\frac{1}{4}}$, we have

$$\frac{c}{c_0} = Z_0^{\frac{1}{4}} a^{\frac{-\bar{x}_1}{4}}. \tag{79}$$

This can lead to a constant, decreasing or increasing speed of light with respect to the scale factor depending on the value of $\bar{x}_1$ given in the table.

Using the horizon criteria, some limitations on $\bar{x}_1$ and thus on n and m can be obtained. For the radiation era this condition results in $\bar{x}_1 > 4$. Moreover for the

Table 4. The horizon problem sets some selection rules on the model.

Critical point	Horizon problem
R_1	Not Okay
R_2	Not Okay
R_3	Not Okay
R_4	Not Okay
R_5	Okay, provided $m < 2$
R_6	Okay, provided $m < 2$
M_1	Not Okay
M_2	Not Okay
M_3	Not Okay
M_4	Okay, provided $n > 1 + \frac{1}{3}\xi$
M_5	Okay, provided $n > 1 - \frac{1}{3}\xi$
M_6	Okay, provided $m < \frac{5}{2}$
M_7	Okay, provided $m < \frac{5}{2}$
d_1	Okay
d_2	Okay, provided $\bar{x}_1 > -4$

matter era we have $\bar{x}_1 > 2$, and for the de-Sitter era it is $\bar{x}_1 > -4$. Again this can be used as a selection rule for model parameters m and n. The result is shown in Table 4.

5. Conclusion

We saw that it is possible to interpret tensor–vector gravitational models as a VSL theory. It has to be stressed that at least four different velocities can be distinguished when one lets the velocity of light to vary according to Ellis & Uzan (2005) and Izadi & Shojai (2009). In the standard theory all these four velocities are equal to the constant $c_0 = 3 \times 10^8$ m/s. The first one appears in the coupling constant of gravity and matter and this is what is chosen to be varying and related to the dynamical vector field here. Calling it c_E we have $\frac{c_E}{c_0} = Z^{\frac{1}{4}}$. The other velocities are the gravitational wave velocity c_{GW}, the electromagnetic wave velocity c_{EM}, and the space–time causal structure constant or the information velocity that we have not considered here.

Here we have investigated analytically the behavior of tensor–vector gravity with varying space-time–matter coupling constant. We have shown that producing the background expansion history $H(z)$ indicated by observations is possible. We investigate the dynamics of this model by dynamical system method and find some exact solutions by considering the character of the critical points of the theory. After reconstruction we saw that the form of the speed of light is a power-law with respect to the scale factor. In addition, using the horizon criteria some restrictions on the critical points were obtained.

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