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On the Jeans Criterion of a Stratified Heat Conducting Gaseous Medium in the Presence of Non-Uniform Rotation and Magnetic Field

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Abstract. The problem of self-gravitational instability of an infinite, homogeneous stratified gaseous medium with finite thermal conductivity and infinite electrical conductivity, in the presence of non-uniform rotation and magnetic field in the Chandrasekhar's frame of reference, is studied. It is found that the magnetic field, whether uniform or non-uniform, has no effect on the *Jeans' criterion* for gravitational instability and remains essentially unaffected. However, the thermal conductivity has the usual stabilizing effect on the criterion that the adiabatic sound velocity occurring in the *Jeans criterion* is replaced by the isothermal sound velocity. Thus, the present analysis extends the results of Chandrasekhar for the case of heat conducting medium and for non-uniform rotation and magnetic field.

Key words. Jeans criterion—gravitational instability—non-uniform magnetic field—thermal conductivity.

1. Introduction

In astrophysical scenarios, the simplest theory that describes the aggregation of masses in space is the *Jeans instability*. The system comprises of particles that can aggregate together depending on the relative magnitude of the gravitational force to pressure force. Whenever the internal pressure of a gas is too weak to balance the self-gravitational force of a mass density perturbation, a collapse occurs. Such a mechanism was first studied by Jeans (1929). In terms of the wavelengths of a fluctuation, the *Jeans criterion* says that instability follows for all perturbations of wave number less than a critical value k_j, where $k_j = \sqrt{\frac{4\pi G\rho_0}{c^2}}$ (here G is the universal gravitational constant, ρ_0 is the unperturbed matter density and c is the velocity of sound). In astrophysical fluids, the collapse of an object is attributed to a self-gravitational force that is responsible for producing the *Jeans instability*. The Jeans instability is of central importance in understanding the process of formation of stars, planets, and other astrophysical objects. For latest and broader view on the formation of stars and gravitational instability, one may refer to Chandrasekhar (1961), Spiegel & Thiffeault (2003), Larson (2003) and McKee & Ostriker (2007).

The gravitational instability of an infinite homogenous self-gravitating medium under varying assumptions of hydrodynamics and hydromagnetics has been studied by many authors. Chandrasekhar (1961) proved that the *Jeans criterion* remains unaffected whether the uniform rotation and the uniform magnetic field acts simultaneously or individually. Nakamura (1984) studied the stability of disks with both rotation and magnetic fields and showed that the effects of rotation and magnetic fields on the growth rate of perturbations are additive in the linear (small amplitude) approximation and showed that a magnetic field, like rotation, always has a stabilizing effect and that shorter-wavelength perturbations are more strongly stabilized. The stability of various kinds of magnetized configurations including sheets, filaments and disks has been studied by many authors including Pacholczyk and Stodolkiewicz (1960), and Nakamura *et al.* (1993).

It is a well-established fact that thermal effects play a very crucial role in astrophysical medium. Field (1965) suggested that the observed filamentary condensations in nebulae may be due to thermal effects. Abbassi *et al.* (2008) considered the possibility of the thermal conduction in the presence of toroidal magnetic field – which had been a largely neglected ingredient before – could affect the global properties of the hot accretion flows substantially and investigated the effect of thermal conduction on the physical structure of advection-dominated accretion flow (ADAF) like accretion flow around a black hole in the presence of a toroidal magnetic field. Brüggen (2003) pointed out that the role of thermal conduction in the Intra-Cluster Medium (ICM) has been the subject of a long debate and, owing to the complex physics of MHD turbulence, the value of the effective conductivity remains uncertain. Originally it has been thought that the magnetic field in clusters strongly suppresses the thermal conductivity because the magnetic fields prevent an efficient transport perpendicular to the field lines. This paradigm has been supported by a number of observations, such as sharp edges at cold fronts, small-scale temperature variations in mergers and sharp boundaries around radio bubbles. However, the theoretical works by Narayan and Medvedev (2001), Malyshkin and Kulsrud (2001) and Rechester and Rosenbluth (1978) have shown that a turbulent magnetic field is not as efficient in suppressing thermal conduction as previously thought.

The analysis of gravitational instability has also been studied by numerous authors under the effect of thermal conductivity. Kato and Kumar (1960) studied the Jeans' problem of *gravitational instability* to include the effect of thermal conductivity of the medium. They remarked that the original *Jeans criterion* of instability was modified due to the presence of thermal conductivity with the only difference that adiabatic sound velocity c is replaced by the isothermal sound velocity c'. Nayyar (1961) considered the effect of finite electrical and thermal conductivity on magneto-gravitational instability and showed that the adiabatic speed of sound is being replaced by the isothermal one, much similar to what happens in the absence of magnetic field. Anand and Khushwaha (1962) extended the problem considered by Bel and Schatzman (1958) to include the effect of heat conduction on the medium and showed that the *Bel and Schatzman criterion* of gravitational instability for non-uniformly rotating self-gravitating medium remains unaffected. Kumar (1960, 1961) also studied the effects of uniform rotation and uniform magnetic field on the problem of gravitational instability of heat conducting medium and found that this *modified Jeans criterion* is valid for these cases also.

Recently, the role of non-uniform magnetic field and rotation in arresting the Jeans collapse has also been studied in detail by Dhiman & Dadwal (2010, 2011) in a thermally conducting/non-conducting axisymmetric gaseous medium.

A central idea in the study of the instability by including various factors is to find ways of arresting the gravitational collapse. All the studies which are mostly based on hydrodynamic models of stratified medium have considered the rotation and magnetic field to be uniform. However, it is a well-known fact that there are many astrophysical situations wherein the rotation and magnetic field may be regarded as non-uniform or variable. Thus, in the light of the above discussion and the importance of various parameters in the process of star formation and gravitational collapse, we discuss the joint effect of thermal conductivity, non-uniform magnetic field and rotation on the gravitational instability of an infinite, homogeneous stratified gaseous medium. The frame of reference chosen here is the same as that considered by Chandrasekhar (1961) in his analysis. The propagation of wave is taken along the longitudinal direction. Since the system of linearized perturbation equations governing the present problem contains variable coefficients, the sufficient condition for local instability following Anand and Khushwaha (1962) and Bel and Schatzman (1958) has been obtained.

2. Mathematical formulation of the problem

Consider an infinite homogeneous, self-gravitating stratified gaseous medium under the simultaneous action of a non-uniform rotation and a non-uniform magnetic field. The medium is assumed to be finitely heat and infinitely electrically conducting. Let $\vec{u} = (u_x, u_y, u_z)$, $\vec{H} = (H_x, H_y, H_z)$ and $\vec{\Omega} = (\Omega_x, \Omega_y, \Omega_z)$ respectively denote the components of the velocity, magnetic field intensity and rotation in the x, y and z directions, in a rectangular co-ordinate system.

The basic hydrodynamical equations that govern the above physical problem for the non-viscous and non-resistive gaseous medium are given by (cf. Chapter V and Chapter XIII of Chandrasekhar (1961) and Dhiman & Dadwal (2011))

$$\rho\left(\frac{\partial u_x}{\partial t} + (\vec{u}\cdot\mathrm{grad})\,u_x\right) - \frac{\mu_e}{4\pi}\left((\vec{H}\cdot\mathrm{grad})H_x - \frac{1}{2}\frac{\partial}{\partial x}(H_x^2 + H_y^2 + H_z^2)\right) = \rho\frac{\partial\phi}{\partial x} - \frac{\partial p}{\partial x} + 2\rho(\Omega_z u_y - \Omega_y u_z), \tag{1}$$

$$\rho\left(\frac{\partial u_y}{\partial t} + (\vec{u}\cdot\mathrm{grad})\,u_y\right) - \frac{\mu_e}{4\pi}\left((\vec{H}\cdot\mathrm{grad})H_y - \frac{1}{2}\frac{\partial}{\partial y}(H_x^2 + H_y^2 + H_z^2)\right) = \rho\frac{\partial\phi}{\partial y} - \frac{\partial p}{\partial y} + 2\rho\,(\Omega_x u_z - \Omega_z u_x), \tag{2}$$

$$\rho\left(\frac{\partial u_z}{\partial t} + (\vec{u}\cdot\mathrm{grad})\,u_z\right) - \frac{\mu_e}{4\pi}\left((\vec{H}\cdot\mathrm{grad})H_z - \frac{1}{2}\frac{\partial}{\partial z}(H_x^2 + H_y^2 + H_z^2)\right) = \rho\frac{\partial\phi}{\partial z} - \frac{\partial p}{\partial z} + 2\rho(\Omega_y u_x - \Omega_x u_y), \tag{3}$$

$$\frac{\partial H_x}{\partial t} + (\vec{u} \cdot \mathrm{grad}) H_x - (\vec{H} \cdot \mathrm{grad}) u_x + H_x\,(\nabla \cdot \vec{u}) - u_x (\nabla \cdot \vec{H}) = 0, \tag{4}$$

$$\frac{\partial H_y}{\partial t} + (\vec{u} \cdot \mathrm{grad}) H_y - (\vec{H} \cdot \mathrm{grad}) u_y + H_y\,(\nabla \cdot \vec{u}) - u_y (\nabla \cdot \vec{H}) = 0, \tag{5}$$

$$\frac{\partial H_z}{\partial t} + (\vec{u} \cdot \mathrm{grad}) H_z - (\vec{H} \cdot \mathrm{grad}) u_z + H_z (\nabla \cdot \vec{u}) - u_z (\nabla \cdot \vec{H}) = 0, \tag{6}$$

$$\frac{\partial \rho}{\partial t} + (\vec{u} \cdot \mathrm{grad})\, \rho + \rho\, (\nabla \cdot \vec{u}) = 0, \tag{7}$$

$$\nabla^2 \phi = -4\pi G \rho, \tag{8}$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0, \tag{9}$$

$$\rho c_{\mathrm{p}} \frac{\partial T}{\partial t} - \frac{\partial p}{\partial t} = \kappa \nabla^2 T, \tag{10}$$

$$p = \rho R T. \tag{11}$$

Here,

$$\vec{u} \cdot \mathrm{grad} = u_x \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z}, \tag{12}$$

$$\vec{H} \cdot \mathrm{grad} = H_x \frac{\partial}{\partial x} + H_y \frac{\partial}{\partial y} + H_z \frac{\partial}{\partial z} \tag{13}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{14}$$

In the above equations, p, ϕ, μ_e, ρ, T R, c_{p}, κ and G respectively denote the pressure, gravitational potential, magnetic permeability, density, temperature, gas constant, specific heat at constant pressure, thermal conductivity (assumed constant) and the gravitational constant.

3. Initial stationary state and its solutions

Initially, $\vec{\Omega}$ is the non-uniform rotation given by $\vec{\Omega} = (\Omega_x, \Omega_y, \Omega_z)$ and the non-uniform magnetic field is given by $\vec{H} = (0, H_y, H_z)$, where H_y, H_z, Ω_x, Ω_y and Ω_z are the functions of z only.

The equilibrium state under discussion is clearly characterized as follows:

$$\begin{aligned} \vec{u} &= (0, 0, 0)\,, \quad \vec{H} = (0, H_y, H_z), \quad \vec{\Omega} = (\Omega_x, \Omega_y, \Omega_z), \\ \rho &= \rho_0, \quad \phi = \phi_0, \quad p = p_0, \quad T = T_0. \end{aligned} \tag{15}$$

The propagation of waves is assumed to be in the z-direction (longitudinal propagation), thus for such a solution $\partial/\partial z$ is the only non-zero component of the gradient.

Under these conditions, equations (1)–(11) allow a stationary (steady) state solution of the form

$$\frac{\mu_e H_z}{4\pi}\left(\frac{\partial H_y}{\partial z}\right) = 0, \tag{16}$$

$$\rho_0 \frac{\partial \phi_0}{\partial z} - \frac{\partial p_0}{\partial z} + \frac{\mu_e H_y}{4\pi}\left(\frac{\partial H_y}{\partial z}\right) = 0, \tag{17}$$

$$\frac{\partial^2 \phi_0}{\partial z^2} = 0, \tag{18}$$

$$\frac{\partial H_z}{\partial z} = 0. \tag{19}$$

Remark 1. One can observe from equation (19) that H_z is a constant and consequently equation (16) implies that H_y is also a constant.

Also, it is to be noted that the above equilibrium solution is considered to be valid under the hypothesis of Jeans (i.e. Jeans swindle).

4. Linearized perturbation equations

Let the initial (basic) state described by equations (15)–(19) be slightly perturbed so that the perturbed state is given by

$$\begin{aligned} u' &= (0 + u'_x, 0 + u'_y, 0 + u'_z), \quad H' = (0 + h'_x, H_y + h'_y, H_z + h'_z), \\ p' &= p_0 + \delta p; T' = T_0 + \delta T, \quad \phi' = \phi_0 + \delta\phi, \quad \rho' = \rho_0 + \delta\rho, \end{aligned} \tag{20}$$

where (u'_x, u'_y, u'_z), (h'_x, h'_y, h'_z), δp, $\delta\rho$ δT and $\delta\phi$ are the respective perturbations from basic state values in velocity, magnetic field, pressure, density temperature and gravitational potential.

Substituting (20) in equations (1)–(11), using equations (15)–(19) and retaining the only non-zero component $\partial/\partial z$ of the gradient, as the propagation of waves is taken along the longitudinal direction (z-direction), ignoring the terms of second and higher orders in the perturbations and treating H_y and H_z as constants as defined in Remark 1, we have the following linearized perturbed equations (in component forms):

$$\left(\frac{\partial u_x}{\partial t} - 2\Omega_z u_y + 2\Omega_y u_z\right) - \frac{\mu_e}{4\pi\rho_0}\left(H_z \frac{\partial h_x}{\partial z}\right) = 0, \tag{21}$$

$$\left(\frac{\partial u_y}{\partial t} + 2\Omega_z u_x - 2\Omega_x u_z\right) - \frac{\mu_e}{4\pi\rho_0}\left(H_z \frac{\partial h_y}{\partial z}\right) = 0, \tag{22}$$

$$\left(\frac{\partial u_y}{\partial t} - 2\Omega_y u_x + 2\Omega_x u_y\right) + \frac{\mu_e}{4\pi\rho_0}\left(H_y \frac{\partial h_y}{\partial z}\right) = \frac{\partial \delta\phi}{\partial z} - \frac{1}{\rho_0}\frac{\partial \delta p}{\partial z}, \tag{23}$$

$$\frac{\partial h_x}{\partial t} - H_z \frac{\partial u_x}{\partial z} = 0, \tag{24}$$

$$\frac{\partial h_y}{\partial t} - H_z \frac{\partial u_y}{\partial z} + H_y \frac{\partial u_z}{\partial z} = 0, \tag{25}$$

$$\frac{\partial h_z}{\partial t} = 0, \tag{26}$$

$$\frac{\partial}{\partial t}\delta\rho + \rho_0 \frac{\partial u_z}{\partial z} = 0, \tag{27}$$

$$\frac{\partial^2 \delta\phi}{\partial z^2} = -4\pi G \delta\rho, \tag{28}$$

$$\frac{\partial h_z}{\partial z} = 0, \tag{29}$$

$$\rho_0 c_\mathrm{p} \frac{\partial \delta T}{\partial t} - \frac{\partial \delta p}{\partial t} = \kappa \nabla^2 \delta T, \tag{30}$$

$$\frac{\delta p}{p_0} = \frac{\delta\rho}{\rho_0} + \frac{\delta T}{T_0}. \tag{31}$$

Using the value of δT obtained from equation (31) in equation (30), we get

$$\frac{\partial}{\partial t}(\delta p - c^2 \delta\rho) = \bar{\kappa} \frac{\partial^2}{\partial z^2}(\delta p - c'^2 \delta\rho), \tag{32}$$

where $c^2 = \gamma \frac{p_0}{\rho_0}$, $c'^2 = \frac{p_0}{\rho_0}$, $\bar{\kappa} = \frac{\kappa}{\rho_0 c_\mathrm{v}}$ and $\gamma = \frac{c_\mathrm{p}}{c_\mathrm{v}}$.

Here, c_v and c_p respectively denote specific heat capacity at constant volume and pressure and are taken as constants throughout the medium, $\bar{\kappa}$ is the thermometric conductivity, c and c' the adiabatic and thermal velocities of sound; γ is the ratio of specific heats of a gas at a constant pressure to a gas at a constant volume and is also known as the *isentropic expansion* factor and arises because a classical sound wave induces an adiabatic compression, in which the heat of the compression does not have enough time to escape the pressure pulse, and thus contributes to the pressure induced by the compression. It is important to note here that the approximate numerical values of γ ranges from 1 to 1.4. Further, in the above equations the dashes have been dropped for convenience.

5. Gravitational instability

In order to investigate the stability of the foregoing stationary state, we shall consider the dependence of the perturbations on z and t of the form

$$\psi\,(z, t) = \psi^*(z) \exp(\sigma t)\,. \tag{33}$$

Here, σ is the frequency of perturbation.

For this type of dependence of perturbations on z and t, we have

$$\frac{\partial}{\partial t} \equiv \sigma \quad \text{and} \quad \frac{\partial}{\partial z}\psi^*(r) = \frac{\mathrm{d}}{\mathrm{d}z}\psi^*(r)\,. \tag{34}$$

Using the above dependence of the perturbations u_x, u_y, u_z, h_x, h_y, h_z, δp, $\delta\rho$ δT and $\delta\phi$ in equations (21)–(29) and (32), we obtain the following equations:

$$\sigma u_x - 2\Omega_z u_y + 2\Omega_y u_z - \frac{\mu_e}{4\pi\rho_0}\left(H_z \frac{\mathrm{d}h_x}{\mathrm{d}z}\right) = 0, \tag{35}$$

$$\sigma u_y + 2\Omega_z u_x - 2\Omega_x u_z - \frac{\mu_e}{4\pi\rho_0}\left(H_z \frac{\mathrm{d}h_y}{\mathrm{d}z}\right) = 0, \tag{36}$$

$$\sigma u_z - 2\Omega_y u_x + 2\Omega_x u_y + \frac{\mu_e H_y}{4\pi\rho_0}\frac{\mathrm{d}h_y}{\mathrm{d}z} - \frac{\mathrm{d}\delta\phi}{\mathrm{d}z} + \frac{1}{\rho_0}\frac{\mathrm{d}\delta\rho}{\mathrm{d}z} = 0, \tag{37}$$

$$\sigma h_x - H_z \frac{\mathrm{d}u_x}{\mathrm{d}z} = 0, \tag{38}$$

$$\sigma h_y - H_z \frac{\mathrm{d}u_y}{\mathrm{d}z} + H_y \frac{\mathrm{d}u_z}{\mathrm{d}z} = 0, \tag{39}$$

$$\sigma h_z = 0, \tag{40}$$

$$\sigma\delta\rho + \rho_0 \frac{\mathrm{d}u_z}{\mathrm{d}z} = 0, \tag{41}$$

$$\frac{\mathrm{d}^2\delta\phi}{\mathrm{d}z^2} = -4\pi G\delta\rho, \tag{42}$$

$$\frac{\mathrm{d}h_z}{\mathrm{d}z} = 0, \tag{43}$$

$$\sigma(\delta p - c^2\delta\rho) = \bar{\kappa}\frac{\mathrm{d}^2}{\mathrm{d}z^2}(\delta p - c'^2\delta\rho). \tag{44}$$

In the above equations the asterisks have been dropped for convenience.

Remark 2. In the above equations, Ω_x, Ω_y and Ω_z are functions of z, so the conditions for global instability cannot be obtained for this system of equations. Following Anand and Khushwaha (1962) and Bel and Schatzman (1958), we shall investigate the local stability of the above system in the neighborhood of $z = z_0$.

In such a situation, the coefficients of u_x, u_y, u_z in equations (35)–(44) are to be evaluated at $z = z_0$. For this, let us assume that the perturbations have a periodic form in the neighborhood of $z = z_0$, as

$$\bar{f}\exp(-ikz), \tag{45}$$

where k is the wave number.

For this type of dependence, we have

$$\frac{\mathrm{d}}{\mathrm{d}z} = -ik.$$

Using (45) in equations (35)–(44), we obtain the following system of algebraic equations for amplitudes (marked with bars):

$$\sigma\bar{u}_x - 2\Omega_z\bar{u}_y + 2\Omega_y\bar{u}_z + ik\frac{\mu_e H_z}{4\pi\rho_0}\bar{h}_x = 0, \tag{46}$$

$$\sigma\bar{u}_y + 2\Omega_z\bar{u}_x - 2\Omega_x\bar{u}_z + ik\frac{\mu_e H_z}{4\pi\rho_0}\bar{h}_y = 0, \tag{47}$$

$$\sigma\bar{u}_z - 2\Omega_y\bar{u}_x + 2\Omega_x\bar{u}_y - ik\frac{\mu_e H_y}{4\pi\rho_0}\bar{h}_y + ik\delta\bar{\phi} - \frac{ik}{\rho_0}\delta\bar{p} = 0, \tag{48}$$

$$\sigma\bar{h}_x + ikH_z\bar{u}_x = 0, \tag{49}$$

$$\sigma\bar{h}_y + ikH_z\bar{u}_y - ikH_y\bar{u}_z = 0, \tag{50}$$

$$\sigma\bar{h}_z = 0, \tag{51}$$

$$\sigma\delta\bar{\rho} - ik\rho_0\bar{u}_z = 0, \tag{52}$$

$$k^2\delta\bar{\phi} - 4\pi G\delta\bar{\rho} = 0, \tag{53}$$

$$-ik\bar{h}_z = 0, \tag{54}$$

$$(\sigma + \bar{k}k^2)\delta\bar{p} - \frac{ik\rho_0}{\sigma}(c^2\sigma + \bar{k}k^2c'^2)\delta\bar{\rho} = 0. \tag{55}$$

Equations (51) and (54), and equations (52) and (53) respectively yield

$$\bar{h}_z = 0, \quad \delta\bar{\rho} = \frac{ik\rho_0}{\sigma}\bar{u}_z, \quad \delta\bar{\phi} = \frac{4\pi G\rho_0}{\sigma k}i\bar{u}_z. \tag{56}$$

Eliminating $\delta\bar{\phi}$, $\delta\bar{\rho}$ and $\bar{h}_z$ from the above equations, we have the following system of homogeneous equations:

$$\sigma\bar{u}_x - 2\Omega_z\bar{u}_y + 2\Omega_y\bar{u}_z + ik\frac{\mu_e H_z}{4\pi\rho_0}\bar{h}_x = 0, \tag{57}$$

$$\sigma\bar{u}_y + 2\Omega_z\bar{u}_x - 2\Omega_x\bar{u}_z + ik\frac{\mu_e H_z}{4\pi\rho_0}\bar{h}_y = 0, \tag{58}$$

$$-2\Omega_y\bar{u}_x + 2\Omega_x\bar{u}_y + \frac{1}{\sigma}\left(\sigma^2 - 4\pi G\rho_0\right)\bar{u}_z - ik\frac{\mu_e H_y}{4\pi\rho_0}\bar{h}_y - \frac{ik}{\rho_0}\delta\bar{p} = 0, \tag{59}$$

$$\sigma\bar{h}_x + ikH_z\bar{u}_x = 0, \tag{60}$$

$$\sigma\bar{h}_y + ikH_z\bar{u}_y - ikH_y\bar{u}_z = 0, \tag{61}$$

$$(\sigma + \bar{k}k^2)\delta \bar{p} - \frac{ik\rho_0}{\sigma}(c^2\sigma + \bar{k}k^2c'^2)\bar{u}_z = 0. \tag{62}$$

Since equations (57)–(62) possess constant coefficients in the vicinity of $z = z_0$, therefore the above system of homogenous equations can be put in the following matrix notation:

$$\begin{bmatrix} \sigma & -2\Omega_z & 2\Omega_y & ik\frac{\mu_e H_z}{4\pi\rho_0} & 0 & 0 \\ 2\Omega_z & \sigma & -2\Omega_x & 0 & ik\frac{\mu_e H_z}{4\pi\rho_0} & 0 \\ -2\Omega_y & 2\Omega_x & \frac{1}{\sigma}(\sigma^2 - 4\pi G\rho_0) & 0 & -ik\frac{\mu_e H_y}{4\pi\rho_0} & -\frac{ik}{\rho_0} \\ ikH_z & 0 & 0 & \sigma & 0 & 0 \\ 0 & ikH_z & -ikH_y & 0 & \sigma & 0 \\ 0 & 0 & -\frac{ik\rho_0}{\sigma}(\sigma c'^2 + \bar{k}k^2c'^2) & 0 & 0 & (\sigma + \bar{k}k^2) \end{bmatrix} \begin{bmatrix} \bar{u}_x \\ \bar{u}_y \\ \bar{u}_z \\ \bar{h}_x \\ \bar{h}_y \\ \delta\bar{p} \end{bmatrix}. \tag{63}$$

For a non-trivial solution of equation (63), the determinant of the matrix on the left-hand side should vanish; and thus on expanding the determinant, we obtain the following dispersion relation:

$$\sigma^7 + (\bar{k}k^2)\sigma^6 + V_I\sigma^5 + (\bar{k}k^2)V_I'\sigma^4 + V_J\sigma^3 + (\bar{k}k^2)V_J'\sigma^2 + V_K\sigma + (\bar{k}k^2)V_K' = 0, \tag{64}$$

where

$$V_I = (c^2k^2 + 4|\Omega|^2 + 2k^2V_A^2 + k^2V_B^2 - 4\pi G\rho_0), \tag{65}$$

$$V_I' = (c'^2k^2 + 4\,|\Omega|^2 + 2k^2V_A^2 + k^2V_B^2 - 4\pi G\rho_0), \tag{66}$$

$$\begin{aligned} V_J \;=\;& k^4V_A^2\,|H|^2 + 2c^2k^2(2\Omega_z^2 + k^2V_A^2) + 4k^2(V_A^2\Omega_x^2 + V_B^2\Omega_z^2 + V_C^2\Omega_y^2) \\ & - 8k^2V_C^2\Omega_z\Omega_y - 2\,(4\pi G\rho_0)\,(k^2V_A^2 + 2\Omega_z^2), \end{aligned} \tag{67}$$

$$\begin{aligned} V_J' \;=\;& k^4V_A^2\,|H|^2 + 2c'^2k^2(2\Omega_z^2 + k^2V_A^2) + 4k^2(V_A^2\Omega_x^2 + V_B^2\Omega_z^2 + V_C^2\Omega_y^2) \\ & - 8k^2V_C^2\Omega_z\Omega_y - 2\,(4\pi G\rho_0)\,(k^2V_A^2 + 2\Omega_z^2), \end{aligned} \tag{68}$$

$$V_K = k^4V_A^4(k^2c^2 - 4\pi G\rho_0), \tag{69}$$

$$V_K' = k^4V_A^4(k^2c'^2 - 4\pi G\rho_0). \tag{70}$$

Here, $V_A^2 = \frac{\mu_e H_z^2}{4\pi\rho_0}$, $V_B^2 = \frac{\mu_e H_y^2}{4\pi\rho_0}$ and $V_C^2 = \frac{\mu_e H_y H_z}{4\pi\rho_0}$ are the *Alfven's wave* velocities. Also, $|\Omega|^2 = (\Omega_x^2 + \Omega_y^2 + \Omega_z^2)$ and $|H|^2 = (H_x^2 + H_y^2)$.

Equation (64) is of seventh degree in σ with all the coefficients real in the neighbourhood of $z = z_0$. If the seventh degree polynomial is a *Hurwitz polynomial*, then the real part of σ will be negative and consequently the system under study will be stable. Guillemin (1950) gave the necessary and sufficient conditions under which any polynomial behaves like a *Hurwitz polynomial*. One of the necessary conditions for this is that, all coefficients must be positive. If one or more coefficients are negative, then the real part of σ will be positive and the system will be unstable. Now, in equation (64) the constant term (V'_K) is negative and we have a sufficient condition of instability.

Thus, the condition for gravitational instability for the present problem is given by

$$V'_K < 0 \tag{71}$$

which implies that

$$k^2 c'^2 < 4\pi G\rho_0. \tag{72}$$

6. Conclusions

A study of gravitational instability of a homogenous, self-gravitating, conducting stratified gaseous medium has been carried out to study the simultaneous effect of non-uniform rotation and magnetic field. We obtain from inequality (72) that self-gravitation leads to instability for all wave numbers

$$k < k_j = \sqrt{\frac{4\pi G\rho_0}{c'^2}}, \tag{73}$$

which is a similar criterion as obtained by Chandrasekhar (1961) for the gravitational instability of an isothermal medium when both rotation and magnetic field were uniform with the only difference that c here is replaced by c' for the case of a heat conducting medium.

It is evident from the above result that when the medium is considered to be heat conducting, the thermal conductivity plays its role by replacing c with c' in the instability criteria.

From the definitions of c and c' it is clear that $c^2 > c'^2$ and hence from expression (73), we conclude that thermal conductivity has a stabilizing effect on the onset of *gravitational instability.*

Further, it can be easily seen from inequality (73) that non-uniform rotation and magnetic field have no effect on the onset of *gravitational instability* of a heat conducting medium, which also validates the results of Kumar (1960, 1961) and Nayyar (1961) for a heat conducting medium, and confirms the view point of Brüggen (2003) and others; that the effect of thermal conductivity cannot be neglected in comparison to the magnetic field as it has some effect on the stability of the self-gravitating medium.

Thus, the present analysis extends the results of Chandrasekhar (1961) for the case of heat conducting medium and for non-uniform rotation and magnetic field.

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