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Black Hole Entropy Calculation in a Modified Thin Film Model

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Abstract. The thin film model is modified to calculate the black hole entropy. The difference from the original method is that the Parikh–Wilczek tunnelling framework is introduced and the self-gravitation of the emission particles is taken into account. In terms of our improvement, if the entropy is still proportional to the area, then the emission energy of the particles will satisfy $\omega = T/360$.

Key words. Black hole entropy—thin film model—tunnelling framework—self-gravitation.

1. Improvement to the thin film model

According to the Parikh–Wilczek tunnelling framework, Hawking radiation (Hawking 1975) was described as a tunnelling process triggered by vacuum fluctuations near the horizon (Parikh & Wilczek 2000). The emission rate can be expressed as:

$$\Gamma \sim \mathrm{e}^{\Delta S_{\mathrm{BH}}} \tag{1}$$

and the barrier width, $\delta = r_{\mathrm{H}}(M) - r_{\mathrm{H}}(M-\omega)$. The black hole entropy is associated with the fields in the vicinity of the horizon, and we can take a thin film which covers on the event horizon and is δ wide. We think that the Bekenstein–Hawking entropy comes from the contribution of the fields in this thin film. If the self-gravitation is taken into account, the event horizon will shrink. Therefore, if we take the thin film on the horizon, the integral of the entropy will not diverge, and there will be no cut-off. We will study the Schwarzschild and the Vaidya black hole.

2. Entropy of Schwarzschild black hole

We take the thin film on the event horizon of the Schwarzschild black hole, and take the barrier width $\delta = 2\omega$ as its thickness. If self-gravitation and $T = 1/8\pi M$ are considered, the entropy in the model will be (Li & Zhao 2000)

$$S = \frac{8\pi^3}{45\beta^3}\int_{r_{\mathrm{H}}}^{r_{\mathrm{H}}+2\omega}\frac{r^2}{f(M-\omega)^2}\mathrm{d}r = \frac{8\pi^3}{45\beta^3}\frac{r_{\mathrm{H}}^4}{4\omega} = \frac{A_{\mathrm{H}}}{4}\frac{T}{360\omega}, \tag{2}$$

where $f(M-\omega) = 1-(2(M-\omega)/r)$. Obviously, if we let $S = A_{\mathrm{H}}/4$, then we get:

$$\omega = \frac{T}{360}. \tag{3}$$

From equation (3), we find that when the black hole emits energy and transits from one quantum state to another, the emission energy varies with temperature of the event horizon. If we consider $A_{\mathrm{H}} = 16\pi M^2$, then

$$\Delta A_{\mathrm{H}} \approx 32\pi M\omega = \frac{1}{90 l_{\mathrm{p}}^2}. \tag{4}$$

3. Entropy of the Vaidya black hole

Similar to the former section, the thin film covers on the event horizon of the Vaidya black hole, and the barrier width taken as δ. When self-gravitation is taken into account, the density of quantum states will be (Li & Zhao 2000)

$$g(\nu) = \frac{1}{\pi}\int_0^{\delta} \frac{\nu^3 r^2}{f(M-\omega)^2}\mathrm{d}R \approx \frac{2\nu^3 r_{\mathrm{H}}^4}{3\pi(1-2\dot{r}_{\mathrm{H}})^2}\frac{1}{2\delta}, \tag{5}$$

where the thickness of the thin film $\delta = r_{\mathrm{H}}(M) - r_{\mathrm{H}}(M-\omega)$ equal the width of the barrier, and $f(M-\omega) = 1 - 2\dot{r}_{\mathrm{H}}(2(M-\omega)/r)$.

The free energy reads

$$F = \frac{1}{\beta}\int \mathrm{d}g(\nu)\ln(1-\mathrm{e}^{-\beta\nu}) = -\frac{2\pi^3 r_{\mathrm{H}}^4}{45\beta^4(1-2\dot{r}_{\mathrm{H}})^2}\frac{1}{2\delta}, \tag{6}$$

and the entropy is given by:

$$S = \frac{A_{\mathrm{H}}}{4}\frac{1}{90\beta(1-2\dot{r}_{\mathrm{H}})^2}\frac{1}{2\delta}. \tag{7}$$

If we let $2\delta = 1/90\beta(1-2\dot{r}_{\mathrm{H}})^2$, then $S = A_{\mathrm{H}}/4$, which is proportional to the area of the event horizon. Since

$$\delta = r_{\mathrm{H}}(M) - r_{\mathrm{H}}(M-\omega) \approx \frac{2\omega}{1-2\dot{r}_{\mathrm{H}}}, \tag{8}$$

we have:

$$\omega = \frac{T}{360(1-2\dot{r}_{\mathrm{H}})}. \tag{9}$$

Our calculation indicates that if the black hole entropy is still proportional to the area, then the emission energy of the particles satisfies $\omega = T/360$.

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