

Constraining the Relative Inclinations of the Planets B and C of the Millisecond Pulsar PSR B1257+12

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Abstract. We investigate on the relative inclination of the planets B and C orbiting the pulsar PSR B1257+12. First, we show that the third Kepler's law does represent an adequate model for the orbital periods P of the planets, because other Newtonian and Einsteinian corrections are orders of magnitude smaller than the accuracy in measuring $P_{B/C}$. Then, on the basis of available timing data, we determine the ratio $\sin i_C / \sin i_B = 0.92 \pm 0.05$ of the orbital inclinations i_B and i_C independently of the pulsar's mass M . It turns out that coplanarity of the orbits of B and C would imply a violation of the equivalence principle. Adopting a pulsar mass range $1 \lesssim M \lesssim 3$, in solar masses (supported by present-day theoretical and observational bounds for pulsar's masses), both face-on and edge-on orbital configurations for the orbits of the two planets are ruled out; the acceptable inclinations for B span the range $36 \text{ deg} \lesssim i_B \lesssim 66 \text{ deg}$, with a corresponding relative inclination range $6 \text{ deg} \lesssim (i_C - i_B) \lesssim 13 \text{ deg}$.

Key words. Planetary systems—pulsars: general, individual, (PSR B1257+12)—extrasolar planets.

The 6.2-ms PSR B1257+12 pulsar was discovered in 1990 during a high galactic latitude search for millisecond pulsars with the Arecibo radiotelescope at 430 Hz (Wolszczan 1990). Two years later, PSR B1257+12 turned out to be orbited by at least two earth-sized planets (B and C) along almost circular paths (Wolszczan & Frail 1992). In 1994, it was announced the discovery of a third, moon-sized planet (A) in an inner, circular orbit (Wolszczan 1994). Its presence, questioned by Scherer *et al.* (1997), was subsequently confirmed in Konacki *et al.* (1999) and Wolszczan *et al.* (2000). The relevant orbital parameters of the PSR B1257+12 system are listed in Table 1. Note that the ratios $\gamma_{B,C}$ of the masses of B and C to M were measured from timing data exploiting their mutual gravitational perturbations (Konacki *et al.* 2000), without using the standard reference value $M = \bar{M} \doteq 1.4 M_\odot$ for the pulsar's mass.

Given the peculiarity of the PSR B1257+12 system, it is certainly important to deepen the knowledge of the orbital configuration of its planets in order to gain insights about the evolutionary dynamics of such a rare system. Here, without making any *a priori* assumptions about the inclinations of B and C, we wish to constrain them from

Table 1. Relevant parameters (Konacki & Wolszczan 2003) of the three planets (Wolszczan & Frail 1992; Wolszczan 1994) A, B and C, hosted in the PSR B1257+12 system (Wolszczan 1990), derived from analysis of timing data ranging 12 years (1990–2003) collected at the 305-m Arecibo telescope. P is the orbital period, x is the projected barycentric semimajor axis of pulsar’s motion and $\gamma \doteq m/M$ (Konacki *et al.* 2000) is the ratio of the planet’s mass to the pulsar’s mass. Figures in parentheses are the formal 1σ uncertainties in the last digits quoted.

Planet	$P(\text{d})$	$x(\text{ms})$	$\gamma(10^{-6})$
A	25.262(3)	0.0030(1)	–
B	66.5419(1)	1.3106(1)	9.2(4)
C	98.2114(2)	1.4134(2)	8.3(4)

the available timing data by assuming a reasonable interval of masses for the hosting pulsar.

To this aim, we will exploit the third Kepler’s law whose use is justified in detail below. First of all, let us note that, by defining $s \doteq \sin i$, it is possible to write the planetary relative (i.e., pulsar-to-planet) semimajor axis a in terms of the measured quantities γ and x as:

$$a = \left(1 + \frac{M}{m}\right) \frac{xc}{s} = \left(\frac{1 + \gamma}{\gamma}\right) \frac{xc}{s}, \quad (1)$$

where c is the speed of light in vacuum. Note that both x and γ were phenomenologically determined in Konacki & Wolszczan (2003) independently of the third Kepler’s law itself and of the pulsar’s mass. The third Kepler’s law is:

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM(1 + \gamma)}, \quad (2)$$

from which it is possible to express the pulsar’s mass in terms of the phenomenologically determined quantities x , γ , P , apart from s which will be considered as unknown.

Note that a purely Keplerian model for the orbital period is quite adequate because non-Keplerian corrections like those due to the oblateness of the pulsar (if any), to the planet–planet interaction and to the 1PN $\mathcal{O}(c^{-2})$ terms are negligible given the present-day accuracy in determining $P_{\text{B/C}}$. Indeed, concerning the oblateness of the central mass, its contribution $\Delta P^{(\text{obl})}$ to the orbital period of an orbiting test particle can be written as (Iorio & Lichtenegger 2005):

$$\Delta P^{(\text{obl})} = -\frac{6\pi R^2 J_2}{\sqrt{GMa}}, \quad (3)$$

where $|J_2| < 1$ represents the first even zonal harmonic coefficient of the multipolar expansion of the gravitational potential of the pulsar¹ and R is the pulsar’s radius; by

¹In this rough order-of-magnitude estimate there is no need to take into account contributions due to the peculiarity of the matter state in the neutron star.

assuming typical values² $R = \bar{R} \doteq 10$ km and $M = \bar{M} = 1.4 M_\odot$ one has

$$\Delta P_{C/B}^{(\text{obl})} \approx -J_2 \times 10^{-12} \text{ d.} \quad (4)$$

The corrections $\Delta P^{(\text{3rd body})}$ to the orbital period of a planet of mass m induced by another planet of mass m' can be written as (Iorio & Lichtenegger 2005):

$$\Delta P^{(\text{3rd body})} = -\frac{4\pi G m'}{n a'^3}, \quad (5)$$

where $n = \sqrt{Gm/a^3}$ is the Keplerian mean motion of the perturbed planet and a' is the perturber's semimajor axis. In the case of the planets B and C it turns out that

$$\Delta P^{(\text{3rd body})} \approx 10^{-15} \text{ d.} \quad (6)$$

The fact that the planets B and C are in a 3:2 resonance does not affect their orbital periods. Indeed, according to the Lagrange's perturbation equation for the variation of a (Brouwer *et al.* 1961),

$$\dot{a} \propto \frac{\partial H_1}{\partial \sigma}, \quad (7)$$

where $\sigma = -nT_p$ is related to the time of pericentre's passage T_p and H_1 is the interacting Hamiltonian of equation (23) in Konacki *et al.* (2000). Since H_1 does not explicitly contain σ there is no secular change in the semimajor axis of B and C. The mutual perturbing effects employed in Konacki *et al.* (2000) to estimate γ_B and γ_C are not the corrections to the Keplerian orbital periods.

The 1PN post-Newtonian correction $\Delta P^{(\text{1PN})}$ to the orbital period of order $\mathcal{O}(c^{-2})$ can be written as (Damour & Deruelle 1986):

$$\Delta P^{(\text{1PN})} = \frac{3\pi}{c^2} \sqrt{GMa}; \quad (8)$$

it turns out that

$$\Delta P_{C/B}^{(\text{1PN})} \approx 10^{-6} \text{ d.} \quad (9)$$

The latter contribution is the most important post-Keplerian correction to P , but it is two orders of magnitude smaller than the 1σ formal errors in the phenomenologically determined orbital periods quoted in Table 1.

Thus, from equations (1, 2) we have:

$$GM = \left[\frac{2\pi(1+\gamma)}{P} \right]^2 \left(\frac{xc}{\gamma s} \right)^3. \quad (10)$$

Taking the ratio of equation (10) for both B and C allows us to get information about the relative orbit inclination independently of M itself: indeed, we have, from Table 1

$$S \doteq \frac{s_C}{s_B} = \left(\frac{P_B}{P_C} \right)^{2/3} \left(\frac{x_C \gamma_B}{x_B \gamma_C} \right) \left(\frac{1+\gamma_C}{1+\gamma_B} \right)^{2/3} = 0.92 \pm 0.05. \quad (11)$$

²As a consequence, we also used $i_B = 53$ deg and $i_C = 47$ deg obtained in (Konacki & Wolszczan 2003) with \bar{M} .

The 1σ error was conservatively assessed by propagating through equation (11) the uncertainties in P_B , P_C , x_B , x_C , γ_B , γ_C quoted in Table 1 and linearly adding the resulting biased terms. It turns out that the most important sources of errors are γ_B and γ_C yielding $\delta S_{\gamma_B} = 0.02$ and $\delta S_{\gamma_C} = 0.03$.

It may be interesting to note that, by assuming $\sin i_B = \sin i_C$, the quantity S in equation (11) may be interpreted as a measure of a violation of the equivalence principle. Indeed, according to Nobili *et al.* (2008), by putting $m_g = m_i(1 + \eta)$ for the gravitational and inertial masses of a test particle orbiting a central body of mass M , the 2-body problem encompassing a violation of the equivalence principle can be described by the same formulas derived in the classical 2-body problem, provided that whenever they contain the product GM , we substitute it with $GM(1 + \eta)$. Applying it to equation (10) for the planets B and C, it turns out that S can be interpreted as:

$$S \doteq \left(\frac{1 + \eta_C}{1 + \eta_B} \right)^{1/3}. \quad (12)$$

Thus, coplanarity and the result of equation (11) would yield a violation of the equivalence principle in the PSR B1257+12 system at 1.6σ level. However, it must be noted that such a test would be stronger if one had evidence that the composition of the two planets was very different.

The constrain of equation (11) is a consistent, genuine dynamical one which does not make use of any assumption about M ; Konacki & Wolszczan (2003) did not obtain it. However, we note that the values of $\gamma_{B/C}$, entering equation (11), were measured in Konacki & Wolszczan (2003) by using the model of Konacki *et al.* (2000) which

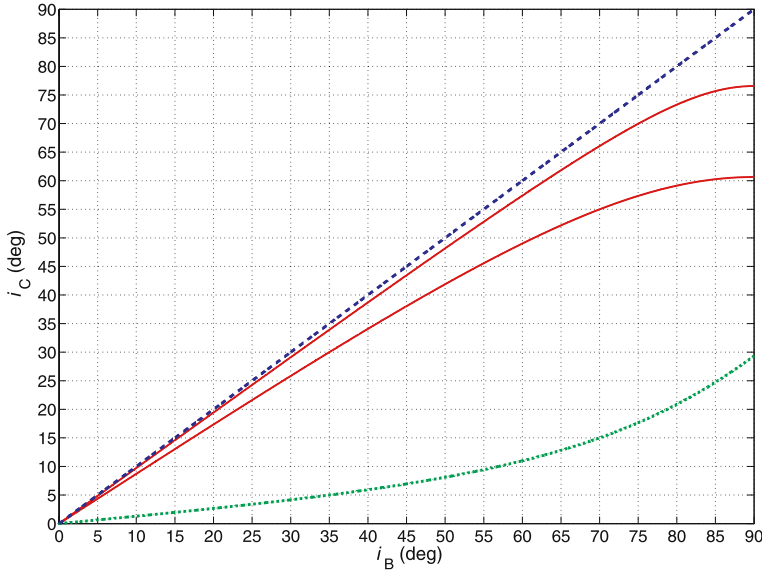


Figure 1. Plot of $i_C = \arcsin(S \sin i_B)$ for the minimum (lower red curve) and maximum (upper red curve) values of S according to equation (11). Such constraints are independent of the pulsar's mass. The blue dashed line represents the coplanarity case. The distance between the blue coplanarity line and the lower red curve yields the maximum value of the relative inclination I allowed by the system's parameters along with their uncertainties; it is depicted in green and shows that $I \gtrsim 10$ deg for $i_B \gtrsim 56$ deg.

neglects terms in $\sin^2(I/2)$, where $I = |i_B - i_C|$ is the relative inclination of the orbital planes of B and C assumed to be $I \lesssim 10$ deg. In Fig. 1, it is plotted the allowed region in the plane $\{i_B, i_C\}$, according to equation (11), delimited by the minimum and maximum values of the ratio S . The blue dashed line represents the coplanarity condition, while the green line is the maximum value of I allowed by the system's parameters. It can be noted that for face-on ($i \rightarrow 0$ deg) geometries the orbital planes tend to be coplanar. Most remarkable deviations from coplanarity (more than 10 deg with a maximum of about 32 deg for $i_B = 90$ deg) occur for edge-on ($i \rightarrow 90$ deg) geometries, but in such cases caution is required since we would fall outside the $I \lesssim 10$ deg condition on which our analysis relies upon; Fig. 1 shows that this occurs for $i_B \gtrsim 56$ deg. In the following, we will explore the viability of various possible orbital inclinations in terms of physically plausible values of M .

We will study the behaviour of equation (10) for B and C as a function of i_B (because of equation 11) in order to constrain the inclinations. Indeed, since we have no independent information at all on $i_{B/C}$, we will use a reasonable interval of masses for the pulsar to constrain them (and their relative inclination I through equation (11) which is independent of M). We wish to preliminarily notice that, in principle, PSR B1257+12, as a member of the rare class of the planetary pulsars, may have had a different formation and evolution with respect to the other neutron stars. However, in absence of any other indication on the details for the evolutionary history of PSR B1257+12, we will rely the following analysis upon standard mass intervals, commonly adopted for other kinds of neutron stars. This is a notable difference with respect to Konacki & Wolszczan (2003) in which the pulsar's mass was kept fixed to $1.4 M_\odot$. Theoretically speaking, different Equations-of-State for nuclear matter inside neutron star yield different pulsar's mass ranges; we will adopt $1-3 M_\odot$ (Lattimer & Prakash 2004). Let us note that present-day observations are all compatible with that range. As to the lower bound, all the best determinations of the mass of a neutron star fall well above $1 M_\odot$, approaching that value only for the most uncertain cases (see Stairs 2006 for an overview of measured pulsar masses). As to the upper bound, the highest securely measured value of the mass of a pulsar is that recently obtained for PSR J04374715 (about $1.76 \pm 0.20 M_\odot$ (Verbiest *et al.* 2008)). Anyway, our adopted mass range also includes the cases of more massive neutron stars (e.g., Freire *et al.* 2008a), whose claimed high mass must still be confirmed by additional investigations.

The 1σ error in the value of M calculated using equation (10) can be conservatively evaluated by propagating the uncertainties in P , x , γ of Table 1, and linearly summing the resulting individual biased terms. It is of the order of 13% and the major contribution to it turns out to be due to γ . By considering the allowed regions for M determined by the curves $M \pm \delta M$ by means of equation (10) applied to both B and C, it turns out that the tightest constraints come from B whose allowed region is entirely enclosed in that due to C. In Fig. 2, we depict the constraints on i_B for $1 \lesssim M \lesssim 3$ in solar masses. As can be noted, it turns out that $36 \lesssim i_B \lesssim 66$ deg. From an inspection of Fig. 1 it turns out that the relative inclination I is different from zero being $6 \lesssim I \lesssim 13$ deg. It is interesting to note that larger values of I , which, at least to a certain extent, may still be compatible with the analysis presented here,³ are ruled out by the lower bound on the pulsar's mass.

³After all, $\sin^2 I/2 = 0.007$ for $I = 10$ deg; a relative inclination of, say, $I = 15$ deg would yield $\sin^2 I/2 = 0.02$.

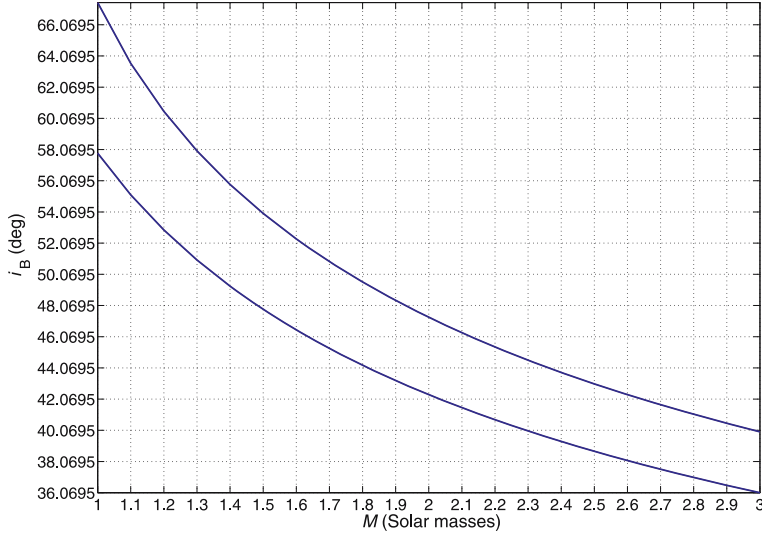


Figure 2. Allowed region for i_B as a function of M . The 1–3 M_\odot interval of Lattimer & Prakash (2004) has been chosen.

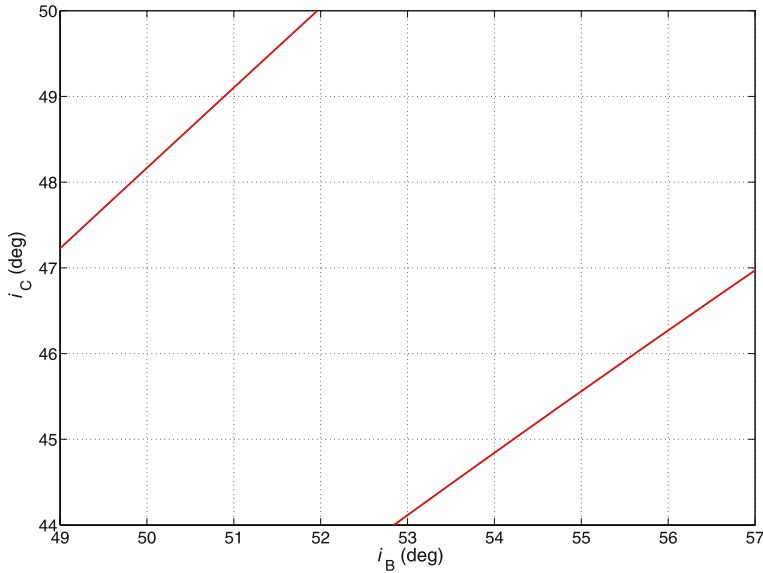


Figure 3. Detail of Fig. 1. Zoomed section of Fig. 1, focusing on the most likely region for i_B and i_C ($49 \text{ deg} \leq i_B \leq 57 \text{ deg}$) \times ($44 \text{ deg} \leq i_C \leq 50 \text{ deg}$) obtained in Konacki & Wolszczan (2003) under the assumption $M = 1.4 M_\odot$. It is possible to see that only a portion of the region, delimited by the red lines, is compatible with the mass-independent constraint of equation (11).

Konacki & Wolszczan (2003) by using $M = \overline{M}$, obtain $49 \text{ deg} \leq i_B \leq 57 \text{ deg}$ and $44 \text{ deg} \leq i_C \leq 50 \text{ deg}$. On one hand, our analysis confirms, as expected, that a larger range of values for i_B and i_C is allowed when a suitable interval of values for the mass of the pulsar is taken into account. On the other hand, thanks to the new constraint

implied by equation (10), it is now possible to show that not all the combinations of orbital inclinations i_B and i_C indicated in Konacki & Wolszczan (2003) are acceptable (even when adopting the same uncertainty level of Konacki & Wolszczan 2003). In fact, as depicted in Fig. 3, the rectangle of acceptable values ($49 \text{ deg} \leq i_B \leq 57 \text{ deg}$) \times ($44 \text{ deg} \leq i_C \leq 50 \text{ deg}$) indicated in Konacki & Wolszczan (2003) is not entirely included in the region allowed by the mass-independent constraint of equation (11).

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