J. Astrophys. Astr. (2010) **31,** 135–145

The Effect of Screening Factors and Thermonuclear Reaction Rates in the Pre-main Sequence Evolution of Low Mass Stars

İ. Küçük[1,*] & Ş. Çalışkan[2]
[1]*Erciyes University, Science Faculty, Department of Astronomy and Space Sciences, 38039 Kayseri, Turkey.*
[2]*Ankara University, Science Faculty, Department of Astronomy and Space Sciences, 06100 Ankara, Turkey.*
[*]*e-mail: kucuk@erciyes.edu.tr*

Received 2009 July 30; accepted 2010 June 7

Abstract. In understanding the nucleosynthesis of the elements in stars, one of the most important quantities is the reaction rate and it must be evaluated in terms of the stellar temperature T, and its determination involves the knowledge of the excitation function $\sigma(E)$ of the specific nuclear reaction leading to the final nucleus. In this paper, the effect of thermonuclear reaction rates to the pre-main sequence evolution of low mass stars having masses 0.7, 0.8, 0.9 and 1 $M_\odot$ are studied by using our modified Stellar Evolutionary Program.

Key words. Low mass stars—thermonuclear reaction rates—electron screening factors—composition changes.

1. Introduction

It is well known that in the calculation of thermonuclear reaction rates in dense and cool plasma, the screening effects must be taken into acoount. The internal structure and evolution of stars can be obtained with the calculations of equation of state of gas structure, opacity and thermonuclear reaction rates. For high density it is necessary to calculate the equation of state by inserting deviations from ideal gas under high density and temperature. For low mass high density stars, the effect of degenerate non-ideal interactions must be taken into account in equation of state. The rate of fusion nuclear reactions in stellar matter is important for the evolution of the star. In dense ionized matter, the rate of nuclear reactions is enhanced by screening effects (Alastuey & Jancovici 1978). One measure of nonideality in plasmas is the so-called coupling parameter Γ. In a plasma where particles have average distance $\langle r \rangle$ from each other, we can define Γ as the ratio of average potential binding energy over mean kinetic energy kT,

$$\Gamma = \left(\frac{e^2/\langle r \rangle}{kT} \right). \tag{1}$$

Plasmas with $\Gamma \gg 1$ are strongly coupled, and those with $\Gamma \ll 1$ are weakly coupled (Basu *et al.* 1999). The input physics of our published theoretical evolutionary models which is based on The Evolutionary Code (Yıldız & Kızıloğlu 1997) is as follows: OPAL opacity has been used and the ratio of mixing length to the scale height $\alpha = 1.50$. The accepted chemical composition is $X = 0.699$, and $Z = 0.019$ (Küçük *et al.* 1998).

2. Thermonuclear reaction rates

The reaction rate, r_{12}, between two nuclei, 1 and 2, is given by:

$$r_{12} = \frac{N_1 N_2 \langle \sigma \upsilon \rangle}{(1 + \delta_{12})} \, \mathrm{cm}^{-3}/\mathrm{sn}, \tag{2}$$

where N_1 and N_2 are the number densities of 1 and 2. For a gas of mass density (ρ), the number density (N_i) of the nucleide (i) is often expressed in terms of its mass fraction (X_i) by the relation

$$N_i = \rho N_A \left(\frac{X_i}{A_i} \right) \mathrm{cm}^{-3}, \tag{3}$$

where N_A is the Avogadro's number and A_i is the atomic mass of i in atomic mass units. In a stellar environment, the reaction rate per particle pair is calculated as:

$$\langle \sigma \upsilon \rangle = \frac{(8/\pi)^{1/2}}{M^{1/2} (kT)^{3/2}} \int \sigma E \exp(-E/kT) dE, \tag{4}$$

where σ is the energy-dependent reaction cross section.

At low energies, nonresonant charged particle interactions are dominated by the Coulomb-barrier penetration factors. It is therefore convenient to factor out this energy dependence and express the cross section $\sigma(E)$, by

$$\sigma = \frac{S(E)}{E} \exp[-(E_G/E)^{1/2}], \tag{5}$$

where E is the kinetic energy of the center-of-mass system, $S(E)$ is the cross section factor. The Gamow energy, E_G, is given by:

$$E_G = (2\pi\alpha \; Z_0 Z_1)^2 \left(\frac{Mc^2}{2} \right) = (9.8948 Z_1 Z_2 A^{1/2})^2 \,\mathrm{keV}, \tag{6}$$

(Lang 1999). Far from a nuclear resonance, the cross section factor $S(E)$ is a slowly varying function of E, and can be conveniently expressed as the first three terms of a Maclaurin Series in the center-of-momentum energy E. Thus,

$$S(E) = S(0) \left[1 + \frac{S'(0)}{S(0)} E + \frac{1}{2} \frac{S''(0)}{S(0)} E^2 \right], \tag{7}$$

where the prime indicates differentiation with respect to E. The values of S and associated derivates are quoted at zero energy. Substitution of equations (5) and (7) into equation (4) yields:

$$\begin{aligned} \langle \sigma \upsilon \rangle &= \frac{(8/\pi)^{1/2}}{M^{1/2} (kT)^{3/2}} \int S(E) \exp(-E_G^{1/2}/E^{1/2} - E/kT) \, dE \\ &= \left(\frac{2}{M} \right)^{1/2} \frac{\Delta E_0}{(kT)^{3/2}} S_{\mathrm{eff}} \exp(-\tau), \end{aligned} \tag{8}$$

(Fowler *et al.* 1967) where,

$$\Delta E_0 = 4(E_0 kT/3)^{1/2} \tag{9}$$

$$E_0 = [\pi\alpha Z_0 Z_1 kT (Mc^2/2)^{1/2}]^{2/3}$$
$$= 1.2204[Z_1^2 Z_2^2 A T_6^2]^{1/3}\,\text{keV} \tag{10}$$

$$\tau = \frac{3E_0}{kT} = 3\lfloor \pi\alpha\ Z_0 Z_1 (Mc^2/2kT)^{1/2} \rfloor^{2/3}$$
$$= 42.487(Z_1^2 Z_2^2 A)^{1/3} T_6^{-1/3} \tag{11}$$

$$S_{\text{eff}} = S(0)\left[1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)}\left(E_0 + \frac{35}{36}kT\right) + \frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0^2 + \frac{89}{36}E_0 kT\right)\right]\text{keV}. \tag{12}$$

Putting these equations into equation (8) yields:

$$\langle\sigma\upsilon\rangle = \{1.3006 \times 10^{-15}(Z_0 Z_1/A)^{1/3} S_{\text{eff}}\} T_6^{-2/3} \exp(-\tau)\,\text{cm}^3\text{s}^{-1}. \tag{13}$$

This is the obtained relation of cross section ($T_6 = T/10^6$). The mean lifetime, $\tau_2(1)$, of nucleus 1 for destruction by nucleus 2 is given by the relation:

$$\lambda_2(1) = \frac{1}{\tau_2(1)} = N_2\langle\sigma\upsilon\rangle = \rho N_A \frac{X_2}{A_2}\langle\sigma\upsilon\rangle\,\text{sn}^{-1}, \tag{14}$$

where $\lambda_2(1)$, is the decay rate of 1 for interaction with 2 (Fowler *et al.* 1967). Putting equations (12), (13) and (5) into equation (2) yields for unscreened reaction rate:

$$r_{\text{unscreened}} = K X_1 X_2 g\rho \exp(-\tau) T_6^{-2/3}, \tag{15}$$

where g is:

$$g = \left[1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)}\left(E_0 + \frac{35}{kT}\right) + \frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0 + \frac{89}{36}E_0 kT\right)\right], \tag{16}$$

and K is given as:

$$K = 7.83 \times 10^8 \left(\frac{N_A}{A_1 A_2}\right)\left(\frac{Z_1 Z_2}{A}\right)^{1/3} S(0). \tag{17}$$

3. The electron screening effect and enhancement factors

At high temperatures in stellar interiors all the atoms are ionized and the gas density ρ is high. The average distance between nucleus and neighbouring electrons is small. Each nucleus is then completely screened by a spherically symmetric negative charge cloud. The radius of this charge cloud is of the same order as the interparticle

Table 1. Calculated screening strength parameter and enhancement factors.

Mass ($M_\odot$)	$T_c \times 10^7$ (K)	ρ_c (gr/cm^3)	$\Lambda_{1,1}$	$\Lambda_{3,3}$	$\Lambda_{3,4}$	$f_{1,1}$	$f_{3,3}$	$f_{3,4}$
0.7	10.26	76.99	0.083	0.265	0.265	1.087	1.304	1.304
0.8	10.85	68.59	0.072	0.231	0.231	1.075	1.260	1.260
0.9	12.17	78.03	0.065	0.208	0.208	1.067	1.231	1.231
1.0	13.02	76.79	0.058	0.186	0.186	1.060	1.205	1.205

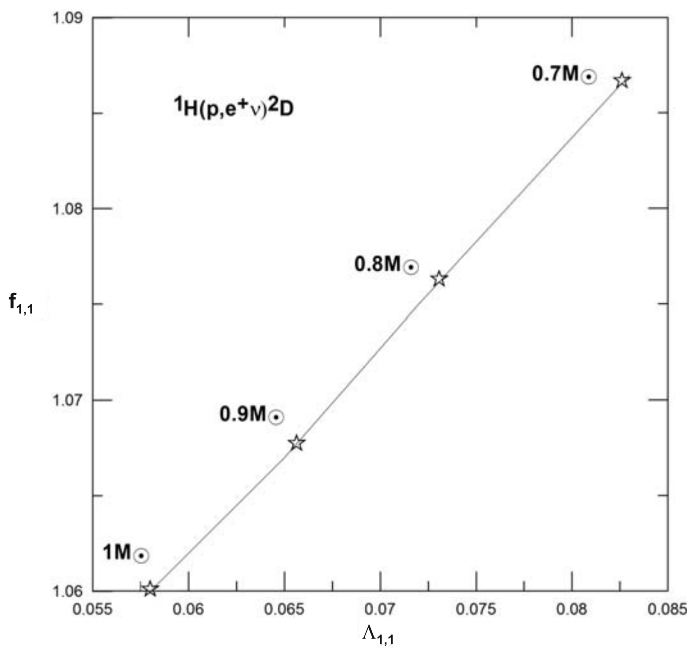

Figure 1. Variation of the screening factors (S54) for $^1\mathrm{H}(p, e^+\upsilon)^2\mathrm{D}$ reaction with the screening strength parameter Λ.

distance or larger, depending on the ratio of Coulomb repulsion between neighbouring charges and the mean thermal energy. The nucleus is surrounded in its immediate vicinity by electrons only the nuclei staying outside a sphere containing nearly electrons which effectively screen the nucleus (Salpeter 1954; hereafter S54). As explained by S54, the rate of a fusion of two nuclei charges Z_1 and Z_2 is increased by the factor

$$f = \exp\left[-\frac{U(0)}{kT}\right] = \exp\Lambda, \qquad (18)$$

where $\Lambda = U(0)/kT$ is the natural strength screening parameter.

$$\left(-\frac{U(0)}{kT}\right)_{ws} = 0.188 Z_1 Z_2 \frac{\rho^{1/2}}{T_6^{3/2}}\zeta. \qquad (19)$$

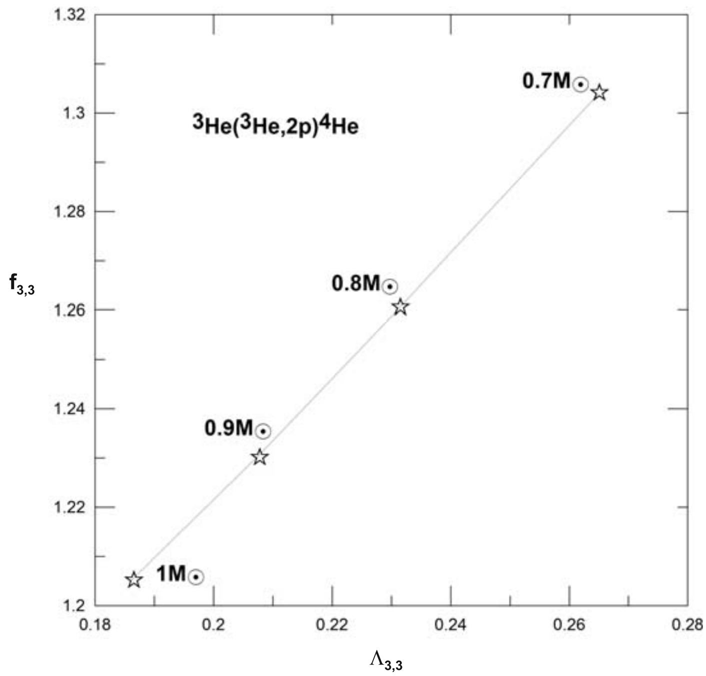

Figure 2. Variation of the screening factors (S54) for ^{3}He(^{3}He, $2p$)^{4}He reaction with the screening strength parameter.

Table 2. Enhancement factors ($T_c \sim 15.54 \times 10^6$ K, $\rho_c \sim 160.8$ gr/cm^3, $X_c = 0.35$ and $Y_c = 0.62$) (Dzitko *et al.* 1995).

	$f_{1,1}$	$f_{3,3}$	$f_{3,4}$
This paper	1.046	1.166	1.166
S54	1.050	1.215	1.215
SVH	1.045	1.186	1.186
GDGC	1.050	1.115	1.115
ML	1.045	1.176	1.176
TS	0.950	0.830	0.827

This is the so-called standard assumption or weak screening which was orginally presented by S54 (Dzitko *et al.* 1995). In Table 1, the results of screening strength parameter and enhancement factors calculated by using our Stellar Evolutionary Program are listed.

These data are used for ^{1}H($p, e^+\upsilon$)^{2}D and ^{3}He(^{3}He, $2p$)^{4}He reactions and calculated for screening strength parameters according to enhancement factors variation and displayed in Figs. 1 and 2.

Table 2 gives enhancement factors calculated with the various prescriptions for the main reactions involved in hydrogen burning: S54, Salpeter & Van Horn (1969) (hereafter SVH), Graboske *et al.* (1973) (hereafter GDGC), Mitler (1977) (hereafter ML) and Tsytovich (2000) (hereafter TS).

Thermonuclear reaction rates in stars are calculated by multiplying the screened reaction rate enhancement factor, $f_{i,j}(i, j = 1, 2, 3, \dots)$, with unscreened reaction:

$$r_{\text{screened}} = f_{i,j} \times r_{\text{unscreened}}. \tag{20}$$

Then energy generation rate in stars is given by:

$$\varepsilon_{\text{nuc}} = Q \times r_{\text{screened}}. \tag{21}$$

In this study, various data (Bahcall 1989; Morel *et al.* 1999; Weiss *et al.* 2001) are used for the calculation of thermonuclear reaction rates. The results for some reaction rates are as follows:

For $^{1}\mathrm{H}(p, e^{+}\upsilon)^{2}\mathrm{D}$ reaction:

$$\begin{aligned} r_{1,1} &= 1.15 \times 10^{11} X_1^2 f_{1,1} g_{1,1} \rho \exp\left(-\frac{33.81}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{1,1} &= 1 + 0.0123 T_6^{1/3} + 0.00114 T_6^{2/3} + 9.8 \times 10^{-4} T_6 \\ Q_{1,1} &= 1.442\,\text{MeV}, \end{aligned} \tag{22}$$

for $^{2}\mathrm{D}(p, \gamma)\ ^{3}\mathrm{He}$ reaction:

$$\begin{aligned} r_{2,1} &= 5.305 \times 10^{28} X_1 X_2 f_{2,1} g_{2,1} \rho \exp\left(-\frac{37.21}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{2,1} &= 1 + 0.0112 T_6^{1/3} + 0.299 T_6^{2/3} + 0.00234 T_6 \\ Q_{2,1} &= 5.494\,\text{MeV}, \end{aligned} \tag{23}$$

for $^{2}\mathrm{D}(d, n)^{3}\mathrm{He}$ reaction:

$$\begin{aligned} r_{2,2} &= 3.154 \times 10^{33} X_2^2 f_{2,2} g_{2,2} \rho \exp\left(-\frac{42.58}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{2,2} &= 1 \\ Q_{2,2} &= 3.269\,\text{MeV}, \end{aligned} \tag{24}$$

for $^{3}\mathrm{He}(^{3}\mathrm{He}, 2p)^{4}\mathrm{He}$ reaction:

$$\begin{aligned} r_{3,3} &= 1.859 \times 10^{35} X_3^2 f_{3,3} g_{3,3} \rho \exp\left(-\frac{122.76}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{3,3} &= 1 + 3.39 \times 10^{-3} T_6^{1/3} \\ Q_{3,3} &= 12.860\,\text{MeV}, \end{aligned} \tag{25}$$

for $^{3}\mathrm{He}(\alpha, \gamma)^{7}\mathrm{Be}$ reaction:

$$\begin{aligned} r_{3,4} &= 2.795 \times 10^{31} X_3 X_4 f_{3,4} g_{3,4} \rho \exp\left(-\frac{128.26}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{3,4} &= 1 + 3.25 \times 10^{-3} T_6^{1/3} - 3.547 \times 10^{-3} T_6^{2/3} - 8.07 \times 10^{-5} T_6 \\ Q_{3,4} &= 1.588\,\mathrm{MeV}, \end{aligned} \tag{26}$$

for $^{7}\mathrm{Be}(p, \gamma)^{8}\mathrm{B}$ reaction:

$$\begin{aligned} r_{7,1} &= 2.32 \times 10^{30} X_7 X_1 f_{7,1} g_{7,1} \rho \exp\left(-\frac{102.62}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{7,1} &= 1 + 4.06 \times 10^{-3} T_6^{1/3} \\ Q_{7,1} &= 0.137\,\mathrm{MeV}, \end{aligned} \tag{27}$$

for $^{12}\mathrm{C}(p, \gamma)^{13}\mathrm{N}$ reaction:

$$\begin{aligned} r_{1,12} &= 1.089 \times 10^{33} X_1 X_{12} f_{1,12} g_{1,12} \rho \exp\left(-\frac{136.9}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{1,12} &= 1 + 3.04 \times 10^{-3} T_6^{1/3} + 6.41 \times 10^{-3} T_6^{2/3} + 1.36 \times 10^{-4} T_6 \\ Q_{1,12} &= 1.944\,\mathrm{MeV}, \end{aligned} \tag{28}$$

for $^{14}\mathrm{N}(p, \gamma)^{15}\mathrm{O}$ reaction:

$$\begin{aligned} r_{1,14} &= 2.088 \times 10^{32} X_1 X_{14} f_{1,14} g_{1,14} \rho \exp\left(-\frac{152.28}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{1,14} &= 1 + 2.74 \times 10^{-3} T_6^{1/3} - 8.08 \times 10^{-3} T_6^{2/3} - 1.547 \times 10^{-4} T_6 \\ Q_{1,14} &= 7.297\,\mathrm{MeV}, \end{aligned} \tag{29}$$

for $^{16}\mathrm{O}(p, \gamma)^{17}\mathrm{F}$ reaction:

$$\begin{aligned} r_{1,16} &= 5.54 \times 10^{33} X_1 X_{16} f_{1,16} g_{1,16} \rho \exp\left(-\frac{166.92}{T_6^{1/3}}\right) T_6^{-2/3} \\ g_{1,16} &= 1 + 2.491 \times 10^{-3} T_6^{1/3} - 1.18 \times 10^{-2} T_6^{2/3} - 2.07 \times 10^{-4} T_6 \\ Q_{1,16} &= 0.60\,\mathrm{MeV}. \end{aligned} \tag{30}$$

The star's thermonuclear reaction rate and energy generation can be obtained from these relations. For example; $^{1}\mathrm{H}(p, e^{+}\upsilon)^{2}\mathrm{D}$ and $^{3}\mathrm{He}(^{3}\mathrm{He}, 2p)^{4}\mathrm{He}$ reactions occured

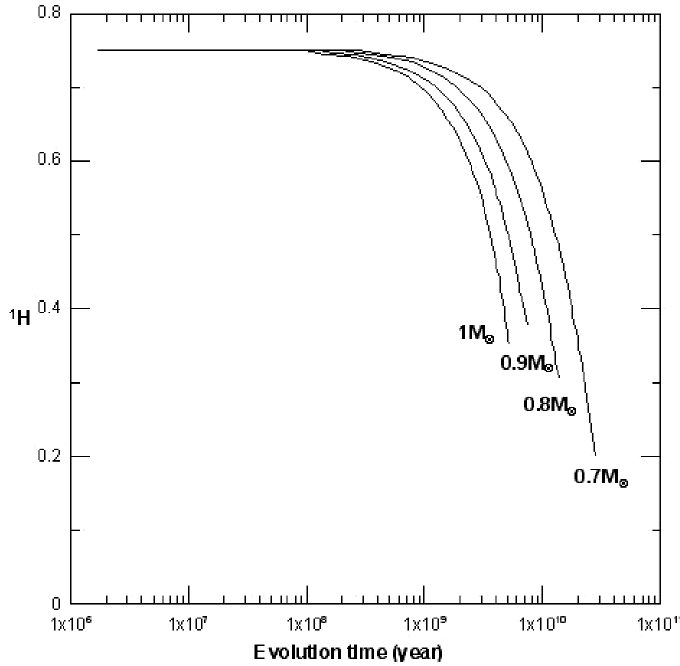

Figure 3. Variation of X1 for models of 0.7, 0.8, 0.9 and 1 $M_\odot$.

at the center, when 0.7 $M_\odot$ model reaches its ZAMS. The reaction rates and energy generations are found to be $r_{1,1} = 2.154 \times 10^5$, $E_{1,1} = 2.305$ and $r_{3,3} = 2.013 \times 10^5$, $E_{3,3} = 4.147$, respectively. Similiarly, for 0.8 $M_\odot$ the results are $r_{1,1} = 3.262 \times 10^5$, $E_{1,1} = 3.490$ and $r_{3,3} = 2.671 \times 10^5$, $E_{3,3} = 5.503$.

4. Composition change and effects to the evolution

During the lifetime of a star its composition, the relative abundances of different elements and isotopes, change basically because of nuclear reactions in the deep interior. In the evolution of 0.7, 0.8, 0.9 and 1 $M_\odot$ stars, the hydrogen burning begins and gets equlibrium with increasing central pressure and temperature. Equlibrium time increases when going lower masses. These are $\sim 2 \times 10^{10}$ and $\sim 5 \times 10^9$ years for 0.7 $M_\odot$, and 1 $M_\odot$, respectively. The variation of X1 for masses 0.7, 0.8, 0.9 is displayed in Fig. 3.

The evolutionary calculations for stars of masses 0.7, 0.8, 0.9 and 1 $M_\odot$ show that the contribution of ^{3}He reaction to the total energy generation, increases with increase of temperature and pressure. This contribution is compared with the one calculated by Siess *et al.* (2000); Siess (2007); (hereafter Siess) and displayed in Figs. 4 and 5, for 0.7 and 1 $M_\odot$, respectively.

As seen in these figures, the compatibility gets larger for high mass as compared with Siess models. The reason is that, they used different input physics (Grenoble Stellar Evolution Code) which concerns mostly changes in the equation of state and in the treatment of the boundary conditions. Comparisons of our models with other grids demonstrate the validity of this EOS in the domain of very low-mass stars.

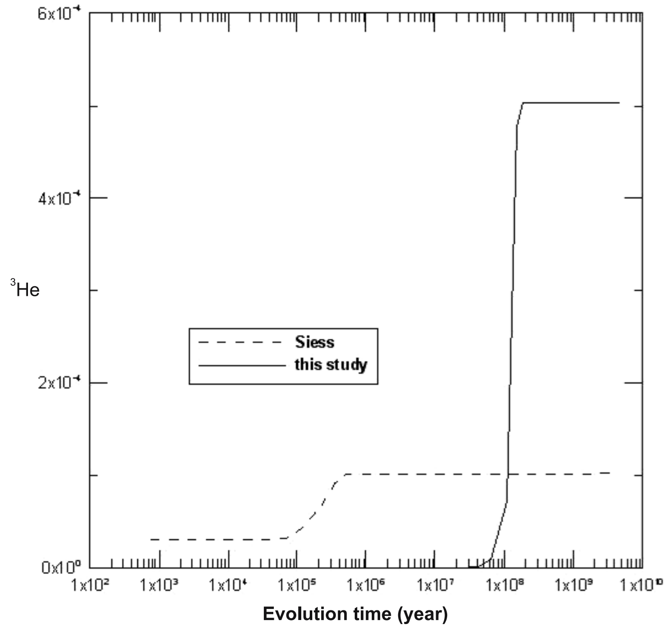

Figure 4. Comparison of variation of ^{3}He(X3) in our models with Siess models for 0.7 $M_\odot$.

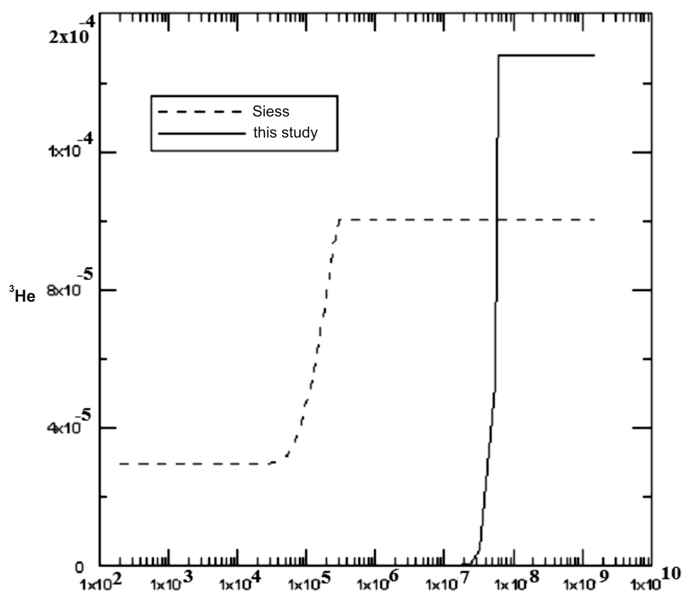

Figure 5. Comparison of variation of ^{3}He(X3) in our models with Siess models for 1 $M_\odot$.

5. Energy generation

Figure 6 shows the energy produced by the $^1\mathrm{H}(p, e^+\upsilon)^2\mathrm{D}(p, \gamma)^3\mathrm{He}$ (E_{11}) and $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ (E_{33}).

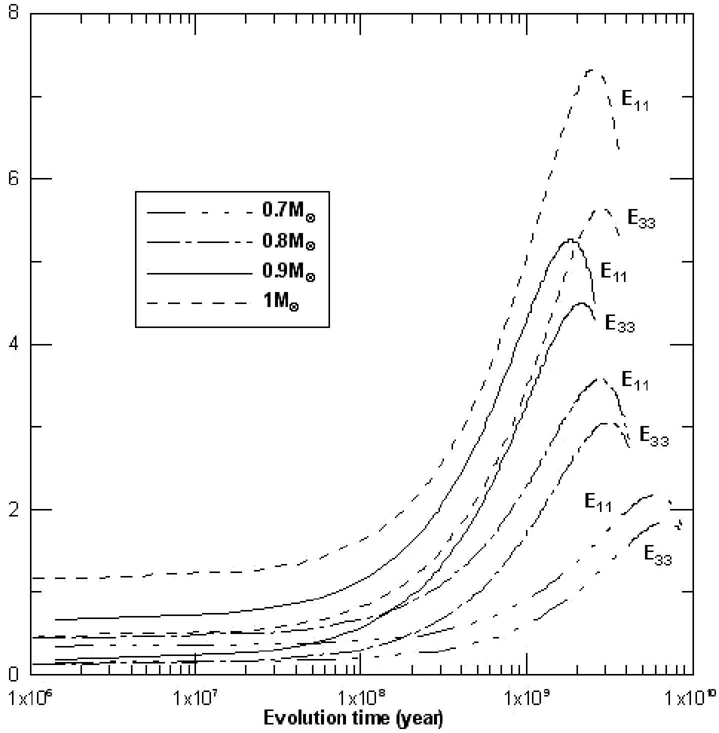

Figure 6. Energy generation with ^1H–^1H and ^3He–^3He reactions for stellar models of 0.7, 0.8, 0.9 and 1 $M_\odot$.

^3He begins to form at the central temperature of $\sim 1.03 \times 10^7$ K and central density 78.3 g/cm^3 after 10^7 years of evolution for 0.7 $M_\odot$ model. After 4×10^9 years, ^{12}C starts to contribute to energy generation of the star when convective core reaches its maximum value.

6. Conclusion

It is well known that in the calculation of nuclear reaction rates in dense, relatively cool stellar plasma, the screening of the Coulomb interaction between the reacting nuclei by the surrounding ions and electrons must be taken into account. In this paper, thermonuclear reaction rates and electron screening factors are re-calculated and updated. The effect of thermonuclear reaction rates to the pre-main sequence evolution of low mass stars having masses 0.7, 0.8, 0.9 and 1 $M_\odot$ are studied by using our modified Stellar Evolutionary Program. Each model is started from threshold of stability; evolutionary path is continued up to the point where the hydrogen amount decreases 50% after the star reaches to the zero ages main sequence (ZAMS). The ZAMS times of 0.7, 0.8, 0.9 and 1 $M_\odot$ stars are found to be 1.9×10^8, 1.3×10^8, 1.06×10^8 and 7.64×10^7 years, respectively. The input physics are improved by calculating new thermonuclear reaction rates and electron screening factors. Since the electron screening factors in low mass high density stars play an important role for thermonuclear reaction rate calculation, we paid attention to the calculation of screening factors. For this purpose, the screening factors calculated with different methods are displayed for comparison. The energies released due to nuclear reaction rates are calculated for ^{1}H–^{1}H and ^{3}He–^{3}He reactions and for each mass, respectively. It is seen that the released energy increases with increasing mass. ^{3}He(X3) composition change is compared with different models and it is seen that the use of new values of reaction rates and electron screening factors which are added to the evolutionary program in details, causes some shifts. Our results seem to be compatible with Siess models in high mass models. But in low mass models this becomes very little. For example, for the model with mass $0.7\,M_\odot$ our results for X3 is 39.8% higher than the one given by Siess.

Acknowledgement

This work was supported by the Section of Scientific Research Projects (ERU-BAP) of the Erciyes University, Project number: FBT-05-37.

References

Alastuey, A., Jancovici, B. 1978, *ApJ*, **226**, 1034–1040.
Bahcall, J. N. 1989, *Neutrino Astrophysics* (Cambridge University Press).
Basu, S. *et al.* 1999, *ApJ*, **518**, 985–993.
Dzitko, H., Turck-Chieze, S., Delbourgo-Salvador, P., Lagrange, C. 1995, *ApJ*, **447**, 428–442.
Fowler, W. A., Caughlan, G. R., Zimmerman, B. A. 1967, *A&A*, **5**, 525.
Graboske, H. C., Dewitt, H. E., Grossman, A. S., Cooper, M. S. 1973, *ApJ*, **181**, 457–474.
Küçük, İ., Kızıloğlu, N., Civelek, R. 1998, *Ap&SS*, **3**, 279.
Lang, R. K. 1999, *Astrophysical Formulae Volume I Radiation, Gas Processes and High Energy Astrophysics*, Astronomy and Astrophysics Library (Printed in Germany).
Mitler, H. E. 1977, *ApJ*, **212**, 513–532.
Morel, P., Pichon, B., Provost, J., Berthomieu, G. 1999, *A&A*, **350**, 275–285.
Salpeter, E. E. 1954, *Australian J. Phys.*, **7**, 373.
Salpeter, E. E., van Horn, H. M. 1969, *ApJ*, **155**, 183.
Shaviv, N. J., Shaviv, G. 2001, *ApJ*, **558**, 925–942.
Siess, L., Dufour, E., Forestini, M. 2000, *A&A*, **358**, 593–599.
Siess, L. 2007, *A&A*, **476**, 893.
Tsytovich, V. N. 2000, *A&AL*, **356**, L57–L61.
Weiss, A., Flaskamp, M., Tsytovich, V. N. 2001, *A&A*, **371**, 1123–1127.
Yıldız, M., Kızıloğlu, N. 1997, *A&A*, **326**, 187.