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Computational Developments for Distance Determination of Stellar Groups

M. A. Sharaf* & A. M. Sendi
Department of Astronomy, Faculty of Science, King Abdul Aziz University, Jeddah, Saudi Arabia.
**e-mail: sharaf_adel@hotmail.com*

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Abstract. In this paper, we consider a statistical method for distance determination of stellar groups. The method depends on the assumption that the members of the group scatter around a mean absolute magnitude in Gaussian distribution. The mean apparent magnitude of the members is then expressed by frequency function, so as to correct for observational incompleteness at the faint end. The problem reduces to the solution of a highly transcendental equation for a given magnitude parameter α. For the computational developments of the problem, continued fraction by the Top–Down algorithm was developed and applied for the evaluation of the error function $\mathrm{erf}(z)$. The distance equation $\Lambda(y) = 0$ was solved by an iterative method of second order of convergence using homotopy continuation technique. This technique does not need any prior knowledge of the initial guess, a property which avoids the critical situations between divergent and very slow convergent solutions, that may exist in the applications of other iterative methods depending on initial guess.

Finally, we apply the method for the nearby main sequence late type stars assuming that the stars of each group of the same spectral type scatter around a mean absolute magnitude in a Gaussian distribution. The accuracies of the numerical results are satisfactory, in that, the percentage errors between r and the mean values are respectively: (2.4%, 1.6%, 0.72%, 0.66%, 3.5%, 2.4%, 2%, 2.5%, 0.9%) for the stars of spectral types: (F5V, F6V, F7V, F8V, F9V, G0V, G2V, G5V, G8V).

Key words. Distance determination—spectral type—frequency function.

1. Introduction

One of the most crucial pieces of information needed in astronomy is the distance to stars. For example (Robinson 1985), if the distance d (in parsec) of a star is known as well as its proper motion μ (in second of arc per year) then one can calculate its tangential velocity V_t to the line of sight (in km per second), which is one of the most useful quantities that could be used for the membership problem for Hyades cluster. Also, having measured the distances to the globular cluster, we can study their distribution in the galaxy (Cassisi *et al.* 2001; Duncan *et al.* 2001). If moving stellar vertex and the distance of each member are known, then one can easily determine the velocity of the cluster and also the position of its centre (Sharaf *et al.* 2000; Sharaf *et al.* 2003) thus the distribution of the cluster's members about this centre can be obtained. On the other hand, the determination of distances within our galaxy allows us to calibrate the distance indicators (Shanks 1997; Tanvir 1997; Brochkhadze & Kogoshvili 1999) used to estimate distances outside it. Moreover, determining distances would also help astronomers in their quest to understand the size and the age of the Universe (Mazumdar & Narasimha 1999; Willick & Batra 2001), since it would provide an independent estimation of the size of the first steps on the cosmic distance ladder. Consequently, it contributes to theories about the origin of the Universe.

Modern observational astronomy has been characterized by an enormous growth in acquisition stimulated by the advent of new technologies in telescopes, detectors and computations. This new astronomical data give rise to innumerable statistical problems (Feigelson & Babu 1992).

In this paper, we consider a statistical method for distance determination of stellar groups. The method depends on the assumption that the members of the group scatter around a mean absolute magnitude in Gaussian distribution. The mean apparent magnitude of the members is then expressed by frequency function, so as to correct for observational incompleteness at the faint end. The problem reduces to the solution of a highly transcendental equation for a given magnitude parameter α. For the computational developments of the problem, continued fraction by the Top–Down algorithm was developed and applied for the evaluation of the error function $\mathrm{erf}(z)$. The distance equation $\Lambda(y) = 0$ was solved by an iterative method of second order of convergence using homotopy continuation technique. This technique does not need any prior knowledge of the initial guess, a property which avoids the critical situations between divergent and very slow convergent solutions that may exist in the applications of other iterative methods depending on initial guess.

Finally, we apply the method for the nearby main sequence late type stars assuming that the stars of each group of the same spectral type scatter around a mean absolute magnitude in a Gaussian distribution. The accuracies of the numerical results are satisfactory, in that, the percentage errors between r and the mean values are respectively: (2.4%, 1.6%, 0.72%, 0.66%, 3.5%, 2.4%, 2%, 2.5%, 0.9%) for the stars of spectral types: (F5V, F6V, F7V, F8V, F9V, G0V, G2V, G5V, G8V).

2. Basic formulations

2.1 *Linear model analysis of observational data in the sense of least-squares criterion*

Let z be represented by the general linear model of the form $\sum_{i=1}^{n} c_i \varphi_i(x)$ where φ's are linearly independent functions of x. Let $\mathbf{c}$ be the vector of the exact values of the c's coefficients and $\hat{\mathbf{c}}$ the least-squares estimators of $\hat{\mathbf{c}}$ obtained from the solution of the normal equations of the form $\mathbf{G}\hat{\mathbf{c}} = \mathbf{b}$. The coefficient matrix $\mathbf{G}(n \times n)$ is symmetric positive definite, i.e., all its eigen values $\lambda_i; i = 1, 2, \ldots, n$ are positive. Let $E(f)$ denotes the expectation of f and $\tilde{\sigma}^2$ the variance of the fit, defined as:

$$\tilde{\sigma}^2 = \frac{q_n}{N - n}, \tag{1}$$

where

$$q_n = (\mathbf{y} - \Phi^T \hat{\mathbf{c}})^T (\mathbf{y} - \Phi^T \hat{\mathbf{c}}), \tag{2}$$

N is the number of observations, $\mathbf{y}$ is the vector with elements z_k and $\Phi(n \times N)$ has elements $\Phi_{ik} = \Phi_i(x_k)$. The transpose of a vector or a matrix is indicated by the superscript 'T'. According to the least-squares criterion, it could be shown that (Kopal & Sharaf 1980):

- The estimators $\hat{\mathbf{c}}$ by the method of least-squares gives the minimum of q_n.
- The estimators $\hat{\mathbf{c}}$ of the parameters $\mathbf{c}$, obtained by the method of least-squares are unbiased; i.e., $E(\hat{\mathbf{c}}) = \mathbf{c}$.
- The variance–covariance matrix Var($\hat{\mathbf{c}}$) of the unbiased estimators $\hat{\mathbf{c}}$ is given by:

$$\mathrm{Var}(\hat{\mathbf{c}}) = \tilde{\sigma}^2 \mathbf{G}^{-1}. \tag{3}$$

- The average squared distance between $\hat{\mathbf{c}}$ and $\mathbf{c}$ is:

$$E(L^2) = \tilde{\sigma}^2 \sum_{i=1}^{n} \frac{1}{\lambda_i}. \tag{4}$$

Finally it should be noted that if the precision is measured by the probable error e, then

$$e = 0.6745\tilde{\sigma}. \tag{5}$$

2.2 *Continued fraction*

In fact, continued fraction expansions are generally far more efficient tools for evaluating the classical functions than the more familiar infinite power series. Their convergence is typically faster and more extensive than the series. Due to the importance of accurate evaluations of the space orbital maneuvers and the efficiency of continued fractions, we purpose to use them as the computational tools for evaluating the included functions.

2.2.1 *Top–Down continued fraction evaluation*

There are several methods available for the evaluation of continued fraction. Traditionally, the fraction was either computed from the bottom up, or the numerator and denominator of the nth convergent were accumulated separately with three-term recurrence formulae. The drawback of the first method is, obviously having to decide far down the fraction to being in order to ensure convergence. The drawback of the second method is that the numerator and denominator rapidly overflow numerically even though their ratio tends to a well-defined limit. Thus, it is clear that an algorithm that works from top down while avoiding numerical difficulties would be ideal from a programming stand-point.

Gautschi (1967) proposed very concise algorithm to evaluate continued fraction from the top down and may be summarized as follows. If the continued fraction is written as:

$$c = \cfrac{n_1}{d_1 + \cfrac{n_2}{d_2 + \cfrac{n_3}{d_3 + \ddots}}},$$

then, initialize the following parameters,

$$\begin{aligned} a_1 &= 1, \\ b_1 &= n_1/d_1, \\ c_1 &= n_1/d_1 \end{aligned}$$

and iterate ($k = 1, 2, \ldots$) according to:

$$\begin{aligned} a_{k+1} &= \frac{1}{1 + [(n_{k+1})/(d_k d_{k+1})]a_k} \\ b_{k+1} &= [a_{k+1} - 1]b_k, \\ c_{k+1} &= c_k + b_{k+1}. \end{aligned}$$

In the limit, the c sequence converges to the value of continued fraction.

2.3 *Homotopy continuation method for solving* $Y(x) = 0$

Usually, equations resulting in most problems of theoretical astrophysics are highly transcendental and could be solved by iterative methods which in turn need: (a) initial guess and (b) an iterative scheme. In fact, these two points are not separated from each other, but there is a full agreement that even accurate iterative schemes are extremely sensitive to initial guess. Moreover, in many cases the initial guess may lead to drastic situation between divergent and very slow convergent solutions.

In the field of numerical analysis, very powerful techniques have been devoted (Allgower & George 1993) to solve transcendental equations without any prior knowledge of the initial guess. These techniques are known as *homotopy continuation methods.* The method was first applied to the universal initial value problem of space dynamics (Sharaf & Sharaf 2003), and in stellar statistics in reference (Sharaf 2006). The homotopy method is summarized as follows:

Suppose one wishes to obtain a solution of a single non-linear equation in one variable x (say):

$$Y(x) = 0, \tag{6}$$

where $Y : \mathbf{R} \to \mathbf{R}$ is a mapping which, for our application assumed to be smooth, i.e., a map has as many continuous derivatives as requires. Let us consider the situation in which no prior knowledge concerning the zero point of Y is available. Since we assume that such *a priori* knowledge is not available, then any of the iterative methods will often fail to calculate the zero $\bar{x}$, because poor starting value is likely to be chosen. As a possible remedy, one defines a homotopy or deformation $H : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ such that:

$$H(x, 1) = Q(x); \qquad H(x, 0) = Y(x),$$

where $Q : R \to \mathbf{R}$ is a (trivial) smooth map having known zero point and H is also smooth. Typically, one may choose a convex:

$$H(x, \lambda) = \lambda \qquad Q(x) + (1 - \lambda)Y(x) \tag{7}$$

and attempt to trace an implicitly defined curve $\Phi(z) \in H^{-1}(0)$ from a starting point $(x_1, 1)$ to a solution point $(\bar{x}, 0)$. If this succeeds, then a zero point $\bar{x}$ of Y is obtained.

3. The distance equation

3.1 *Assumptions*

- All the members in a given stellar group are at the same distance, r parsecs.
- The frequency function for the absolute magnitudes of the members is

$$\Phi(M) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(M-M_0)^2/2\sigma^2}. \tag{8}$$

  That is the members scatter around a mean absolute magnitude M_0 in a Gaussian distribution with dispersion σ.
- The mean apparent magnitude $\bar{m}$ of the members of the cosmic group and the limiting apparent magnitude m_1 are related through the quantity α where

$$\alpha = \frac{m_1 - \bar{m}}{\sigma}.$$

3.2 *Expression for the distance equation*

According to the above assumptions, the distance r and the frequency functions $\Psi(m)$ of the apparent magnitude are given (Sharaf *et al.* 2003) respectively by:

$$r = 10^{1+(m_1-M_0-\sigma y-A)/5}, \tag{9}$$

$$\Psi(m) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(m+5-5\log r-M_0)^2/2\sigma^2}, \tag{10}$$

where A is the interstellar absorption, y is the solution of the transcendental equation:

$$\Lambda(y) = y + e^{-y^2/2}\left\{\sqrt{\frac{\pi}{2}}\left[1 + \operatorname{erf}\left(\frac{y}{\sqrt{2}}\right)\right]\right\}^{-1} - \alpha = 0, \tag{11}$$

and $\operatorname{erf}(z)$ is the error function defined by the integral:

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt, \tag{12}$$

we call equation (11) the *distance equation*.

Table 1. Coefficients B_j; $j = 1, \ldots, 16$.

j	B_j	j	B_j	j	B_j	j	B_j
1	0.797775	5	−0.000184904	9	0.000025662	13	-2.40909×10^{-7}
2	0.36338	6	−0.00127215	10	9.30564×10^{-6}	14	-3.49127×10^{-8}
3	0.109007	7	−0.00036512	11	8.48003×10^{-7}	15	9.2136×10^{-9}
4	0.0191284	8	0.0000129345	12	-5.148893×10^{-7}	16	6.036×10^{-9}

The relation between M_0 and σ is given by Malmquist bias (Binney & Merrifield 1998):

$$M_0 = \bar{M} + 1.382\sigma^2, \tag{13}$$

where $\bar{M}$ is the mean absolute magnitudes of the stellar group.

3.3 *Power series representation of the distance equation*

Equation (11) could be represented as power series in y as:

$$y + \exp\{-y^2/2\}\left\{\sqrt{\frac{\pi}{2}}\left[1 + \mathrm{erf}\left(\frac{y}{\sqrt{2}}\right)\right]\right\}^{-1} = \sum_{j=1}^{16} B_j y^{j-1}.$$

The coefficients B_j for $j = 1, \ldots, 16$ are listed in Table 1.

4. Computational developments

4.1 *Continued fraction of erf(z)*

It could be shown that (Sharaf 2006) the continued fraction of $\mathrm{erf}(z)$ is:

$$\mathrm{erf}(z) = \sqrt{\frac{2}{\pi}} e^{-z^2} \cfrac{\hat{z}}{1 - \cfrac{\hat{z}^2}{3 + \cfrac{2\hat{z}^2}{5 - \cfrac{3\hat{z}^2}{7 + \cfrac{4\hat{z}^2}{9 - \ddots}}}}}; \quad \hat{z} = \sqrt{2}z \tag{14}$$

4.2 *The solution of the distance equation*

As we have mentioned in section 2.3 that the most powerful method for solving transcendental equations, is the homotopy continuation method. The method does not need any prior knowledge of the initial guess, a property which avoids the critical situations between divergent and very slow convergent solutions, that may exist in the application of other numerical methods depending on initial guess.

To solve the distance equation (equation 11) by the homotopy continuation method we use $Q(y) \equiv y + 1/\alpha$ together with Gautschi's algorithm for evaluating the error function $\mathrm{erf}(z)$ (equation 14) by the continued fraction as mentioned in section 2.2.

4.3 *Analytical relation between M_V and $(B-V)_0$*

Colour indices of distant stars (objects) are usually affected by interstellar extinction, i.e., they are redder than those of closer stars. The amount of reddening is characterized by colour excess defined as the difference between the observed colour index and the normal colour index (or intrinsic colour index). For example, in the UBV photometric system we can write it for the $B-V$ colour:

$$E(B-V) = (B-V) - (B-V)_0,$$

where $(B-V)$ is the observed colour index, $(B-V)_0$ is the intrinsic colour index and $E(B-V)$ is the excess. The relation between the colour excess and interstellar reddening A is:

$$A = 3E(B-V). \tag{15}$$

To measure $(B-V)$, one observes the magnitude of an object successively through two different filters, B and V, where B is sensitive to blue light, and V is sensitive to visible (green-yellow) light. The difference in magnitudes found with these filters is $(B-V)$. The smaller the colour index, the more blue (or hotter) the object is.

That is $(B-V)$ can be determined directly from observations, it remains to compute $(B-V)_0$ so as to determine the interstellar reddening A (equation 15). Due to its important role in distance determination (e.g., equation 9) this tempted us to establish analytical formulae for the intrinsic colour index.

An empirical relation between the absolute visual magnitude M_V and $(B-V)_0$ is given in Mihalas and Binney book (1981) listed in Table 2. We established according to the least-squares method of section 2.1, new analytical formulae between $(B-V)_0$ and M_V in the form:

$$(B-V)_0 = -0.0974 + 0.0989M_V + 0.01M_V^2. \tag{16}$$

Table 2. Empirical relation between M_V and $(B-V)_0$.

$(B-V)_0$	M_V	$(B-V)_0$	M_V
−0.30	−3.50	0.30	2.80
−0.25	−2.30	0.40	3.35
−0.20	−1.30	0.50	4.05
−0.15	−0.50	0.60	4.60
−0.10	−0.30	0.70	5.20
−0.05	0.9	0.80	5.70
0.00	1.30	0.90	6.10
0.05	1.55	1.00	6.60
0.10	1.80	1.10	7.00
0.20	0.25	1.20	7.45

This relation is extremely accurate as indicated in the following error analysis report:

1. The solutions and their probable errors

$$c_1 = -0.0974322 \pm 0.00592847$$
$$c_2 = 0.0989122 \pm 0.00246673$$
$$c_3 = 0.0103065 \pm 0.00040501$$

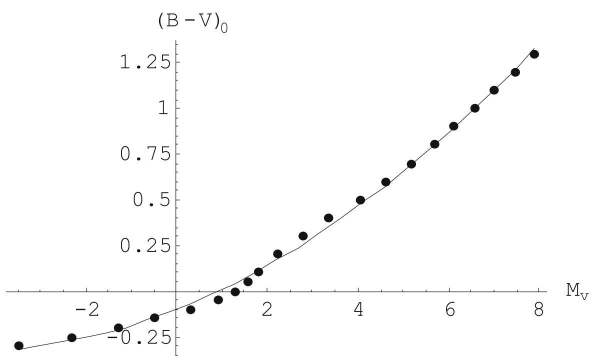
2. The probable error of the fit

$$e = 0.019427$$

3. The average squared distance between $\hat{\mathbf{c}}$ and $\mathbf{c}$

$$E(L^2) = 0.0000909892$$

4. Graph of the raw and fitted data



5. Numerical developments

There is a variety of data for stellar groups, up on the set of the available data, the computational algorithm has to be constructed accordingly. The following algorithm has been designed according to the most commonly listed data available in catalogues.

5.1 *Computational algorithm*

5.1.1 *Purpose*

- To compute C_1, C_2, and C_3 for the frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitudes (equations 8 and 10) for a stellar group assuming that the stars of the group of the same spectral type, scatter around a mean absolute magnitude in a Gaussian distribution, where

$$\Phi(M) = C_1 \text{Exp}\{-C_2(M - M_0)^2\}, \tag{17}$$

$$\Psi(m) = C_1 \text{Exp}\{-C_2(m - C_3)^2\}. \tag{18}$$

- Upon this assumption of the Gaussian distribution, the second goal is to compute the distance r of the stellar group.

5.1.2 *Input*

- V_{mag}: Apparent magnitude.
- $B - V$: Colour index.
- plx: Parallax.
- M_V: Absolute visual magnitude.

For N stars of a given stellar group.

5.1.3 *Computational sequence*

- Compute for each star of the group $(B - V)_0$ from its given value of M_V using equation (16).
- Compute the absorption A_V for each star of the group from equation (15).
- Compute the average $\bar{A}_V$ of A_V.
- Correct the apparent magnitudes for absorption from:

$$V_{\text{mag}} = V_{\text{mag}} - \bar{A}_V.$$

- Compute the average value $\bar{M}_V$ of the absolute magnitudes M_V.
- Compute the average value $\bar{r}$ of the individual distances $r_j(r_j = 1/p_j)$.
- Compute the median r_{median} of the individual distances.
- Select start and end values of σ, say σ_s and σ_e respectively. (In the present application, $\sigma_s = 0.5$ and $\sigma_e = 2$.)
- Compute the optimum value of σ as follows:

  (a) For $\sigma = \sigma_s(0.01)\sigma_e$ perform the following calculations:

  - $M_0 = \bar{M}_V + 1.382\sigma^2$.
  - From the values of V_{mag}, M_0 and σ compute the distance r (in parsec) of the stellar group as obtained from section 4.2.
  - Compute the percentage error f in the mean from $f = |(r - \bar{r})/\bar{r}| \times 100$.
  - Compute the percentage error g in the median from $g = |(r - r_{\text{median}})/r_{\text{median}}| \times 100$.

  (b) Compute the value σ_1 that gives the minimum of the f's values.
  (c) Compute the value σ_2 that gives the minimum of the g's values.
  (d) Compute the optimum value of σ as $\sigma = (\sigma_1 + \sigma_2)/2$.

- From the optimum values of M_0 and σ compute the C's coefficients of equations (17) and (18).

5.2 *Numerical examples*

We consider the data for some nearby stars of late spectral types of luminosity class V are listed. These types are: F5V, F6V, F7V, F8V, F9V, G0V, G2V, G5V, and G8V.

5.2.1 *Data*

Source of data: (V/70 A) Nearby stars, preliminary 3rd version (Gliese + 1991) of (I/239) Hipparcos and Tycho Catalogues (HTC)-ESA 1997.

Table 3. Data for F5V near by stars.

M_v (mag)	plx (mas)	$U - B$ (mag)	$B - V$ (mag)	V_{mag} (mag)	Name
3.91	56.8	−0.06	0.45	5.14	Gl 54.2
3.44	43	−0.05	0.43	5.27	GJ 1043
4.68	68.6	−0.01	0.44	5.5	Gl 105.4
3.75	49	0.1	0.46	5.3	Gl 209.1
4.27	69.9	−0.06	0.44	5.05	Gl 292
3.38	53	−0.03	0.43	4.76	Gl 297.1
3.89	36	0.04	0.5	6.11	Wo 9421
3.55	51.7	−0.06	0.45	4.98	Gl 501
3.83	39	−0.1	0.5	5.87	Gl 542.1
3.42	50	−0.02	0.43	4.93	Gl 578
3.34	51	−0.01	0.43	4.8	NN 4017
3.75	59.8	−0.03	0.47	4.87	Gl 692
3.54	50.3	−0.04	0.38	5.03	Gl 708.1
3.56	77	0.02	0.43	4.13	Gl 805
3.27	79.9	−0.04	0.44	3.76	Gl 848
3.49	39	−0.05	0.44	5.53	NN 4330
3.46	24	−0.12	0.43	6.56	Wo 9843

Table 4. The number of the used stars for different spectral types.

Spectral types	F5V	F6V	F7V	F8V	F9V	G0V	G2V	G5V	G8V
No.	17	30	24	36	18	34	34	59	43

Tables: The data used for the above of late spectral types of luminosity class V are listed in tables, to illustrate the entries of these tables, let us consider Table 3 for F5V stars as a typical example. In this table, the columns have the following meaning:

Column 1, M_v (unit mag): Absolute visual magnitude. M_v is calculated from the known relation: $M_v = V_{\mathrm{mag}} + 5 + 5\log(\mathrm{plx}/1000)$ where V_{mag} is the apparent magnitude and is listed in Column 5 and plx is the parallax (in milli-second of arc) is listed in Column 2. Column 3, $B - V$ (unit mag): colour. Column 4, $U - B$ (unit mag): colour. Column 6, Name with the following acronyms:

- G1 Gliese: Veroeff Astron Rechen – Inst. Heidelberg, 22, 1 (1969).
- GJ Gliese & Jahreiss, AA&AS, 38423 (1979).
- Wo Woolley *et al.*, Roy. Obs. Ann., No. 5 (1970).
- NN newly added stars (number added at CDS).

The number of the used stars for different spectral type: In Table 4, the number of used stars for different spectral types are listed.

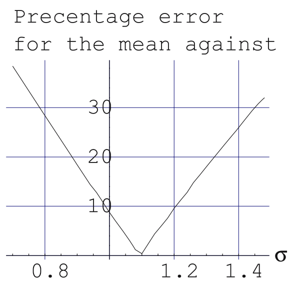
Errors in the measurements: It should be mentioned that, in the original tables of HTC (from which we take the data), there are two columns in each table for the statistical errors of V_{mag} and $(B - V)$.

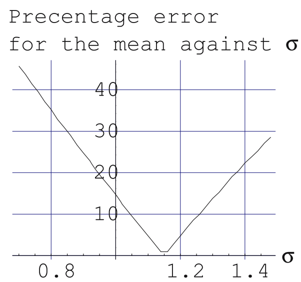
5.2.2 *Determination of the optimal values of σ*

We illustrate the determination of the optimal value of σ for F5V and G2V.

The Optimum Choice of σ for F5V:

- Graphical representations of the percentage errors for the Mean and Median

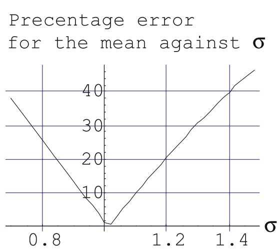


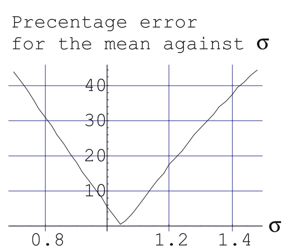


- The minimum value of the percentage error in the Mean is $= 0.54643$ occurs at $\sigma = 1.1$.
- The minimum value of the percentage error in the Median is $= 0.874761$ occurs at $\sigma = 1.14$.
- The optimum value of σ is $= 1.12$.

The Optimum Choice of σ for G2V:

- Graphical representations of the percentage errors for the Mean and Median





- The minimum value of the percentage error in the Mean is $= 0.854855$ occurs at $\sigma = 1.02$.
- The minimum value of the percentage in the Median is $= 0.887998$ occurs at $\sigma = 1.04$.
- The optimum value of σ is $= 1.03$.

5.2.3 *Numerical results of other parameters*

The numerical results of the above procedure are given for each spectral type, in Tables 5, 6 and 7. In Table 5, the solution y of the distance equation (equation 11) together with its accuracy are listed, while the other computed parameters are given in Table 6. Finally, Table 7 is devoted for the coefficients C_1, C_2 and C_3 of the frequency functions $\Phi(M)$ and $\Psi(m)$ (equations 17 and 18).

Table 5. The solution y of the distance equation and its accuracy.

Spectral type	y	$\lvert\Lambda(y)\rvert$
F5V	0.950559	1.19058×10^{-6}
F6V	3.83341	2.98319×10^{-8}
F7V	1.42494	1.02526×10^{-6}
F8V	2.49132	4.42657×10^{-7}
F9V	2.22488	5.95237×10^{-7}
G0V	2.45604	4.62193×10^{-7}
G2V	2.06513	6.89123×10^{-7}
G5V	1.73244	8.76278×10^{-7}
G8V	2.14521	6.421×10^{-7}

Table 6. Numerical values of the basic parameter.

Spectral type	m_1	$\bar{m}$	σ	M_0	r	$\bar{r}$	$f = \lvert(r - \bar{r})/\bar{r}\rvert \times 100$
F5V	7.99776	6.59011	1.12	5.41182	20.1494	20.6355	2.35549%
F6V	10.7274	6.85541	1.01	5.36311	19.8844	20.2061	1.59237%
F7V	8.08461	6.59794	0.94	5.09947	21.3372	21.1844	0.721612%
F8V	9.64125	7.10681	1.01	5.45922	21.5357	21.6796	0.663949%
F9V	9.53971	7.21304	1.03	5.76172	19.8277	20.545	3.49157%
G0V	10.0514	7.50139	1.03	6.02175	19.9518	20.4468	2.42074%
G2V	9.94592	7.76915	1.03	6.26881	20.4176	20.8369	2.01238%
G5V	9.99376	8.15025	1.01	6.59644	21.3555	21.9013	2.4917%
G8V	10.7584	8.57261	1.0	7.10177	20.0581	20.247	0.932936%

Table 7. C_1, C_2 and C_3 of the frequency functions $\Phi(M)$ and $\Psi(m)$.

Spectral type	C_1	C_2	C_3
F5V	0.356198	0.398597	6.93313
F6V	0.394992	0.490148	6.85567
F7V	0.424407	0.565867	6.74516
F8V	0.394992	0.490148	7.12501
F9V	0.387323	0.471298	7.24808
G0V	0.387323	0.471298	7.52166
G2V	0.387323	0.471298	7.81883
G5V	0.394992	0.471298	8.24399
G8V	0.398942	0.5	8.61322

6. Conclusion

In conclusion, we report that the present paper uses a statistical method for distance determination of stellar groups. The method depends on the assumption that the members of the group scatter around a mean absolute magnitude in Gaussian distribution.

The mean apparent magnitude of the members is then expressed by frequency function, so as to correct for observational incompleteness at the faint end. The problem reduces to the solution of a highly transcendental equation for a given magnitude parameter α. For the computational developments of the problem, continued fraction by the Top–Down algorithm was developed and applied for the evaluation of the error function $\mathrm{erf}(z)$. The distance equation $\Lambda(y) = 0$ was solved by an iterative method of second order of convergence using homotopy continuation technique. This technique does not need any prior knowledge of the initial guess, a property which avoids the critical situations between divergent and very slow convergent solutions, that may exist in the applications of other iterative methods depending on initial guess.

Finally, we apply the method for the nearby main sequence late type stars assuming that the stars of each group of the same spectral type scatter around a mean absolute magnitude in a Gaussian distribution. The accuracies of the numerical results are satisfactory, in that, the percentage errors between r and the mean values are respectively: (2.4%, 1.6%, 0.72%, 0.66%, 3.5%, 2.4%, 2%, 2.5%, 0.9%) for the stars of spectral types: (F5V, F6V, F7V, F8V, F9V, G0V, G2V, G5V, G8V).

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