Mechanical sources in orthotropic micropolar continua

Rajneesh Kumar and Suman Choudhary

Mathematics Department, Kurukshetra University, Kurukshetra 136 119, Haryana, India

The present paper is concerned with the problem of an orthotropic micropolar half-space subjected to concentrated and distributed loads. The disturbance due to normal and tangential loads are investigated by employing eigen-value approach. The integral transforms have been inverted by using a numerical technique to obtain the normal displacement, normal force stress and tangential couple stress in the physical domain. The results of these quantities are given and illustrated graphically.

1. Introduction

In many engineering phenomena, including the response of soils, geological materials and composites, the assumptions of an isotropic behaviour may not capture some significant features of the continuum response. The formulation and solution of anisotropic problems are far more difficult and cumbersome than its isotropic counterpart. In the last years the elastodynamic response of anisotropic continuum has received the attention of several reseachers. In particular, transversely isotropic and orthotropic materials, which may not be distinguished from each other in plane strain and plane stress cases, have been more regularly studied.

The theory of micropolar elasticity introduced and developed by Eringen (1966) aroused much interest because of its possible utility in investigating the deformation properties of solids for which the classical theory is inadequate. The micropolar theory is believed to be particularly useful in investigating materials consisting of bar-like molecules which exhibit microrotation effects and which can support body and surface couples. A review of literature on micropolar orthotropic continua shows that Iesan (1973, 1974a, 1974b) analyzed the static problems of plane micropolar strain of a homogeneous and orthotropic elastic solid, torsion problem of homogeneous and orthotropic cylinders in the linear theory of micropolar elasticity and bending of orthotropic micropolar elastic beams by terminals couples. Nakamura *et al* (1984) derived the finite element method for orthotropic micropolar elasticity.

Most of the problems studied so far, in micropolar elasticity, involve the use of potential functions. However, the use of the eigen-value approach has the advantage of finding the solutions of equations in the coupled form directly, in the matrix notations, whereas the potential function approach requires decoupling of equations. Yet, the eigen-value approach has not been applied in micropolar orthotropic medium. Mahalanabis and Manna (1989, 1997) applied the eigen-value approach to linear micropolar elasticity by arranging basic equations of linear micropolar elasticity in the form of matrix differential equations. Recently, Kumar *et al* (2001) applied the eigen-value approach to micropolar elastic medium due to impulsive force at origin.

2. Problem formulation

We consider a homogeneous and orthotropic micropolar half-space. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface $y = 0$ with y axis vertical into the medium is introduced. A normal or tangential source is assumed to be acting at the origin of the rectangular cartesian co-ordinates.

Keywords. Micropolar; orthotropic; eigen-value; Fourier and Laplace transforms.

Proc. Indian Acad. Sci. (Earth Planet. Sci.), **111**, No. 2, June 2002, pp. 133–141

If we restrict our analysis parallel to xy-plane with displacement vector $\vec{u}=(u_1,u_2,0)$ and microrotation vector $\vec{\phi}=(0,0,\phi_3)$, the basic equations in the dynamic theory of the plane strain of homogeneous and orthotropic micropolar solids in the absence of body forces and body couples, given by Eringen (1968), can be recalled as:

$$t_{ji,j}=\rho\frac{\partial^2 u_i}{\partial t^2}, \tag{1}$$

$$m_{i3,i}+\epsilon_{ij3}t_{ij}=\rho j\frac{\partial^2\phi_3}{\partial t^2}. \tag{2}$$

The constitutive relations, given by Iesan (1973), can be written as:

$$t_{11}=A_{11}\epsilon_{11}+A_{12}\epsilon_{22},\qquad t_{12}=A_{77}\epsilon_{12}+A_{78}\epsilon_{21},$$

$$t_{21}=A_{78}\epsilon_{12}+A_{88}\epsilon_{21},\qquad t_{22}=A_{12}\epsilon_{11}+A_{22}\epsilon_{22},$$

$$m_{13}=B_{66}\phi_{3,1},\qquad m_{23}=B_{44}\phi_{3,2}, \tag{3}$$

where

$$\epsilon_{ij}=u_{j,i}+\epsilon_{ji3}\phi_3. \tag{4}$$

In these relations, we have used the following notations: t_{ij} – components of the force stress tensor, m_{ij} – component of the couple stress tensor, ϵ_{ij} – component of micropolar strain tensor, u_i – components of displacement vector, ϕ_3 – component of microrotation vector, ϵ_{ijk} – permutation symbol, $A_{11}, A_{12}, A_{22}, A_{77}, A_{78}, A_{88},\ B_{44}, B_{66}$ – characteristic constants of the material, ρ – the density and j – the microinertia.

Introducing the dimensionless quantities

$$x^*=\frac{\omega}{c_1}x,\ y^*=\frac{\omega}{c_1}y,\ u_i^*=\frac{\omega}{c_1}u_i,\ \phi_3^*=\frac{A_{11}}{K_1}\phi_3,$$

$$t_{ij}^*=\frac{t_{ij}}{A_{11}},\ m_{ij}^*=\frac{c_1}{B_{44}\omega}m_{ij},\ t^*=\omega t, \tag{5}$$

where, $c_1^2=A_{11}/\rho$ and $\omega^2=\chi/\rho j$, in equations (1)–(4) (dropping the asterisks for convenience) and applying Laplace transform w.r.t 't' defined by

$$\{\overline{u}_i(x,y,p),\overline{\phi}_3(x,y,p)\}$$

$$=\int_0^\infty\{u_i(x,y,t),\phi_3(x,y,t)\}e^{-pt}dt,\ i=1,2 \tag{6}$$

and then Fourier transform w.r.t 'x' defined by

$$\{\tilde{u}_i(\xi,y,p),\tilde{\phi}_3(\xi,y,p)\}$$

$$=\int_{-\infty}^{\infty}\{\overline{u}_i(x,y,p),\overline{\phi}_3(x,y,p)\}e^{\iota\xi x}dx,\ i=1,2 \tag{7}$$

on the resulting expressions, we obtain

$$\tilde{u''}_1=Q_{11}\tilde{u}_1+Q_{15}\tilde{u'}_2+Q_{16}\tilde{\phi'}_3, \tag{8}$$

$$\tilde{u''}_2=Q_{22}\tilde{u}_2+Q_{23}\tilde{\phi}_3+Q_{24}\tilde{u'}_1, \tag{9}$$

$$\tilde{\phi''}_3=Q_{32}\tilde{u}_2+Q_{33}\tilde{\phi}_3+Q_{34}\tilde{u'}_1, \tag{10}$$

where, primes in equations (8)–(10) represent the first and second order differentiation w.r.t y, respectively and

$$Q_{11}=\frac{A_{11}(\xi^2+p^2)}{A_{88}},$$

$$Q_{15}=\frac{\iota\xi(A_{12}+A_{78})}{A_{88}},$$

$$Q_{16}=\frac{K_1^2}{A_{11}A_{88}},$$

$$Q_{22}=\frac{(\xi^2A_{77}+p^2A_{11})}{A_{22}},$$

$$Q_{23}=-\frac{\iota\xi K_1K_2}{A_{11}A_{22}},$$

$$Q_{24}=\frac{\iota\xi(A_{12}+A_{78})}{A_{22}},$$

$$Q_{32}=\frac{\iota\xi K_2A_{11}^2}{\omega^2\rho B_{44}K_1},$$

$$Q_{33}=\frac{(\xi^2B_{66}\omega^2+c_1^2\chi)+jp^2\omega^2A_{11}}{B_{44}},$$

$$Q_{34}=-\frac{A_{11}^2}{\rho\omega^2B_{44}},$$

$$K_1=A_{78}-A_{88},\ K_2=A_{77}-A_{78},\ \chi=K_2-K_1. \tag{11}$$

The system of equations (8)–(10) can be written as

$$\frac{d}{dy}W(\xi,y,p)=A(\xi,p)W(\xi,y,p), \tag{12}$$

where

$$W = \begin{bmatrix} U \\ U' \end{bmatrix}, \quad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, \quad U = \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{\phi}_3 \end{bmatrix},$$

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & Q_{15} & Q_{16} \\ Q_{24} & 0 & 0 \\ Q_{34} & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} Q_{11} & 0 & 0 \\ 0 & Q_{22} & Q_{23} \\ 0 & Q_{32} & Q_{33} \end{bmatrix}. \tag{13}$$

To solve equation (12), we take

$$W(\xi, y, p) = X(\xi, p)e^{qy} \tag{14}$$

so that

$$A(\xi, p)W(\xi, y, p) = qW(\xi, y, p) \tag{15}$$

which leads to the eigen-value problem. The characteristic equation corresponding to the matrix A is given by

$$\det[A - qI] = 0, \tag{16}$$

which on expansion provides us

$$q^6 - \lambda_1 q^4 + \lambda_2 q^2 - \lambda_3 = 0, \tag{17}$$

where

$$\begin{aligned}
\lambda_1 &= Q_{15}Q_{24} + Q_{16}Q_{34} + Q_{11} + Q_{22} + Q_{33}, \\
\lambda_2 &= Q_{15}(Q_{24}Q_{33} - Q_{23}Q_{34}) + Q_{16}(Q_{22}Q_{34} \\
&\quad - Q_{24}Q_{32}) + Q_{11}Q_{22} + Q_{22}Q_{33} + Q_{11}Q_{33} \\
&\quad - Q_{23}Q_{32}, \\
\lambda_3 &= Q_{11}(Q_{22}Q_{33} - Q_{23}Q_{32}).
\end{aligned} \tag{18}$$

The roots of equation (17) are $\pm q_i, \ \ i = 1, 2, 3$.

The eigen values of the matrix A are the roots of equation (17). We assume that real parts of q_i are positive. The vector $X(\xi)$ corresponding to the eigen values q_i can be determined by solving the homogeneous equation

$$[A - qI]X(\xi, p) = 0. \tag{19}$$

The set of eigen vectors $X_i(\xi, p), \ (i = 1, 2, 3, 4, 5, 6)$ may be obtained as

$$X_i(\xi, p) = \begin{bmatrix} X_{i1}(\xi, p) \\ X_{i2}(\xi, p) \end{bmatrix}, \tag{20}$$

where

$$X_{i1}(\xi, p) = \begin{bmatrix} a_i q_i \\ b_i \\ 1 \end{bmatrix}, \quad X_{i2}(\xi, p) = \begin{bmatrix} a_i q_i^2 \\ b_i q_i \\ q_i \end{bmatrix},$$

$$q = q_i; \ i = 1, 2, 3, \tag{21}$$

$$X_{j1}(\xi, p) = \begin{bmatrix} -a_i q_i \\ b_i \\ 1 \end{bmatrix}, \quad X_{j2}(\xi, p) = \begin{bmatrix} a_i q_i^2 \\ -b_i q_i \\ -q_i \end{bmatrix},$$

$$j = i + 3, q = -q_i; \ i = 1, 2, 3, \tag{22}$$

$$\begin{aligned}
a_i &= (q_i^2 Q_{15} + Q_{16}Q_{32} - Q_{15}Q_{33})/\Delta_i, \\
b_i &= [q_i^4 - q_i^2(Q_{16}Q_{34} + Q_{11} + Q_{33}) + Q_{11}Q_{33}]/\Delta_i, \\
\Delta_i &= q_i^2(Q_{15}Q_{34} + Q_{32}) - Q_{32}Q_{11}, \quad i = 1, 2, 3.
\end{aligned} \tag{23}$$

The solution of equation (12) is given by

$$\begin{aligned}
W(\xi, y, p) = \sum_{i=1}^{3} &[B_i X_i(\xi, p) \exp(q_i y) \\
&+ B_{i+3} X_{i+3}(\xi, p) \exp(-q_i y)],
\end{aligned} \tag{24}$$

where, $B_i (i = 1, 2, 3, 4, 5, 6)$ are arbitrary constants.

The equation (24) represents the solution of the general problem in the plane strain case of micropolar orthotropic elasticity by employing the eigenvalue approach and therefore can be applied to a broad class of problems in the domains of Laplace and Fourier transforms.

3. Application

In this section the general solutions presented for displacement and stresses in equation (24) will be particularized to yield the response of a half-space subjected to a uniform traction distribution and to a point load. The constants B_i will be determined by imposing the proper boundary conditions. These constants when substituted in equation (24) deliver the displacement and stress solutions in the Fourier and Laplace transformed (ξ, y, p) domain. The final solution in the original domain (x, y, t) is obtained by a numerical inversion of both transforms.

Case I: Load in normal direction: On the half-space the load $F(x)$ is applied in normal direction at the origin of the co-ordinate system. For this loading case the boundary conditions are:

$$t_{22}(x,0,t) = -F(x)H(t), \quad t_{21}(x,0,t) = 0,$$
$$m_{23}(x,0,t) = 0. \tag{25}$$

where $H(\ )$ is the Heaviside Function.

Case II: Load in tangential direction: On the half-space the load $F(x)$ is applied in tangential direction at the origin of the co-ordinate system. For this loading case the boundary conditions are:

$$t_{22}(x,0,t) = 0, \quad t_{21}(x,0,t) = -F(x)H(t),$$
$$m_{23}(x,0,t) = 0. \tag{26}$$

It can be seen that six unknowns are to be determined in equation (24) and only three boundary conditions are in each case. For the half-space the radiation conditions implies outgoing waves with decreasing amplitudes in the positive y-direction. Therefore the radiation condition impose that $B_1 = B_2 = B_3 = 0$.

3.1 *Influence functions*

The way to obtain the half-space influence function, i.e., the solutions due to distributed loads applied at the half-space surface, is to set directly the distributed loads $F(x)$ in equations (25) and (26). The Fourier transform w.r.t the pair (x,ξ) for the case of uniform strip load of amplitude F_o and width $2a$ applied at the origin of the co-ordinate system is:

$$\tilde{F}(\xi) = F_o \frac{2\sin(\xi a)}{\xi}. \tag{27}$$

Case I: Load in normal direction: The solutions for this case due to the uniformly distributed load are obtained as

$$\tilde{u}_2(\xi,y,p) = b_1 B_4 e^{-q_1 y} + b_2 B_5 e^{-q_2 y} + b_3 B_6 e^{-q_3 y}, \tag{28}$$

$$\tilde{m}_{23}(\xi,y,p) = -\frac{K_1}{A_{11}}[q_1 B_4 e^{-q_1 y} + q_2 B_5 e^{-q_2 y} + q_3 B_6 e^{-q_3 y}], \tag{29}$$

$$\tilde{t}_{22}(\xi,y,p) = -[N_1 B_4 e^{-q_1 y} + N_2 B_5 e^{-q_2 y} + N_3 B_6 e^{-q_3 y}], \tag{30}$$

where

$$B_i = 2F_0(M_j q_k - M_k q_j)\sin(\xi a)/\xi p\Delta;$$
$$i=4, j=2, k=3; \quad i=5, j=3, k=1;$$
$$i=6, j=1, k=2. \tag{31}$$

and

$$M_i = [(-\iota\xi A_{78} b_i + A_{88} a_i q_i^2)A_{11} + K_1(A_{88} - A_{78})]/A_{11}^2,$$
$$N_i = (A_{22} b_i - \iota\xi A_{12} a_i)q_i/A_{11}\ ;\ i=1,2,3, \tag{32}$$

$$\Delta = M_1(q_2N_3 - q_3N_2) + M_2(q_3N_1 - q_1N_3) + M_3(q_1N_2 - q_2N_1). \tag{33}$$

Case II: Load in tangential direction: The solutions for this case are obtained as in equation (28)–(30) by changing the values of constant with

$$B_i = 2F_0(N_j q_k - N_k q_j)\sin(\xi a)/\xi p\Delta;$$
$$i=4, j=2, k=3; \quad i=5, j=3, k=1;$$
$$i=6, j=1, k=2. \tag{34}$$

3.2 *Green's functions*

To synthesize the Green functions, i.e., the displacement and stress solutions due to a point load described as a Dirac's Delta $F(x) = F_0\delta(x)$, its Fourier transform with respect to the pair (x,ξ)

$$\tilde{F}(\xi) = F_o \tag{35}$$

must be used. The expressions for displacement and stresses may be obtained as in equations (28)–(30) by replacing constants for the corresponding case.

Case I: Load in normal direction:

$$B_i = F_0(M_j q_k - M_k q_j)/p\Delta;$$
$$i=4, j=2, k=3; \quad i=5, j=3, k=1;$$
$$i=6, j=1, k=2. \tag{36}$$

Case II: Load in tangential direction:

$$B_i = F_0(N_j q_k - N_k q_j)/p\Delta;$$
$$i=4, j=2, k=3; \quad i=5, j=3, k=1;$$
$$i=6, j=1, k=2, \tag{37}$$

Particular cases: Taking

$$A_{11} = A_{22} = \lambda + 2\mu + K, \quad A_{77} = A_{88} = \mu + K,$$
$$A_{12} = \lambda, A_{78} = \mu, \quad B_{44} = B_{66} = \gamma,$$
$$-K_1 = K_2 = \chi/2 = K,$$

we obtain the corresponding expressions for the micropolar isotropic elastic medium.

4. Inversion of transforms

The transformed displacements and stresses (28)–(30) are functions of y, the parameters of Laplace and Fourier transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi, y, p)$. To get the function $f(x, y, t)$ in the physical domain, we invert the Fourier and Laplace transforms by using the inversion technique as used in Kumar and Choudhary (2001).

5. Numerical results and discussion

For numerical computations, we take the following values of relevent parameters for orthotropic micropolar solid:

$$A_{11} = 13.97 \times 10^{10}\text{dyne/cm}^2,$$
$$A_{22} = 13.75 \times 10^{10}\text{dyne/cm}^2,$$
$$A_{77} = 3.0 \times 10^{10}\text{dyne/cm}^2,$$
$$A_{88} = 3.2 \times 10^{10}\text{dyne/cm}^2,$$
$$A_{12} = 8.13 \times 10^{10}\text{dyne/cm}^2,$$
$$A_{78} = 2.2 \times 10^{10}\text{dyne/cm}^2,$$
$$B_{44} = 0.056 \times 10^{10}\text{dyne},$$
$$B_{66} = 0.057 \times 10^{10}\text{dyne}.$$

For comparison with micropolar isotropic solid, following Gauthier (1982), we take the following values of relevent parameters for the case of aluminum epoxy composite as

$$\rho = 2.19\text{gm/cm}^3,$$
$$\lambda = 7.59 \times 10^{10}\text{dyne/cm}^2,$$
$$\mu = 1.89 \times 10^{10}\text{dyne/cm}^2,$$
$$K = 0.0149 \times 10^{10}\text{dyne/cm}^2,$$
$$\gamma = 0.0268 \times 10^{10}\text{dyne},$$
$$j = 0.00196\text{cm}^2.$$

The comparison of dimensionless normal displacement $U_2[= u_2/F_o]$, normal force stress $T_{22}[= t_{22}/F_o]$ and couple stress $M_{23}[= m_{23}/F_o]$, for micropolar orthotropic solid (MOS) and micropolar isotropic solid (MIS) due to normal and tangential uniform strip load (USL) have been studied and shown in figures 1 to 6. The computations were carried out for three values of dimensionless time $t = 0.10, 0.20$ and $t = 0.50$ at $y = 1.0$ in the

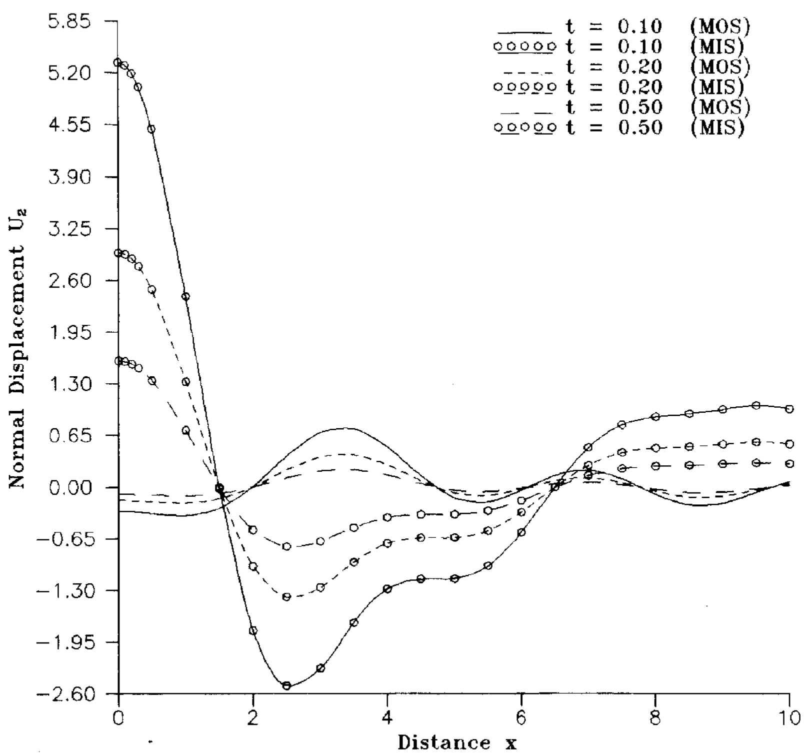

Figure 1. Variations of normal displacement $U_2(x, 1, t)(= u_2/F_0)$ due to normal USL with distance x.

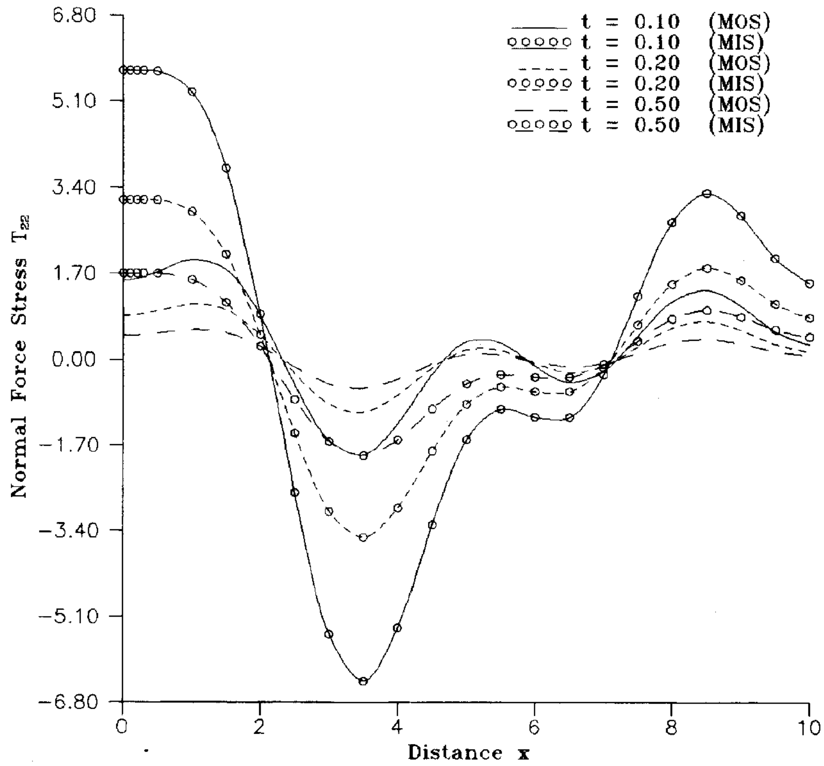

Figure 2. Variations of normal force stress $T_{22}(x,1,t)(=t_{22}/F_0)$ due to normal USL with distance x.

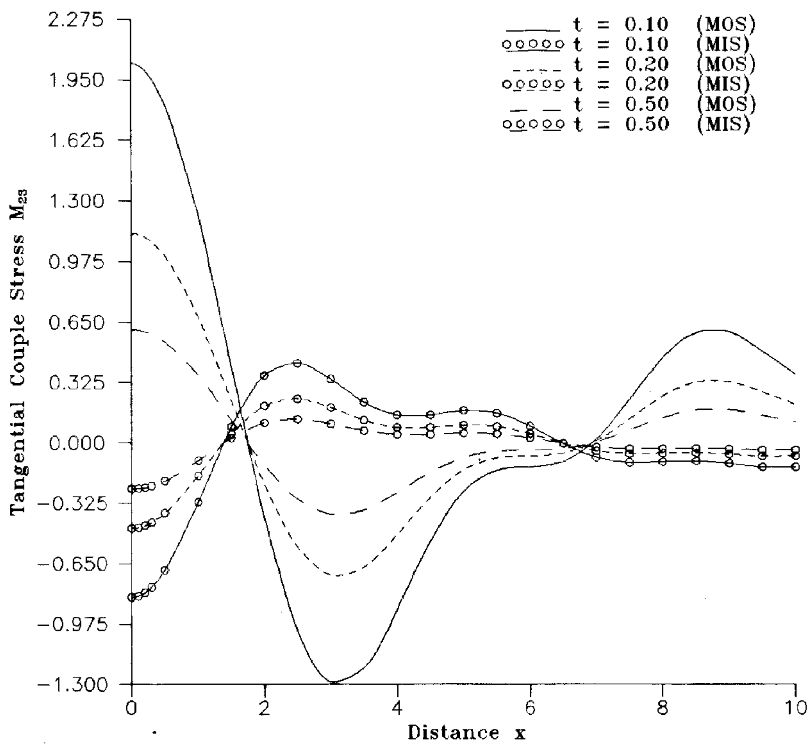

Figure 3. Variations of tangential couple stress $M_{23}(x,1,t)(=m_{23}/F_0)$ due to normal USL with distance x.

range $0 \leq x \leq 10$. The solid lines either without center symbol or with center symbol represent the variations for $t = 0.1$ whereas the small dashed lines with or without center symbol represent the variations for $t = 0.2$ and large dashed lines with or without center symbol represent variations for $t = 0.5$. The curves without center symbol correspond to the case of MOS whereas those with

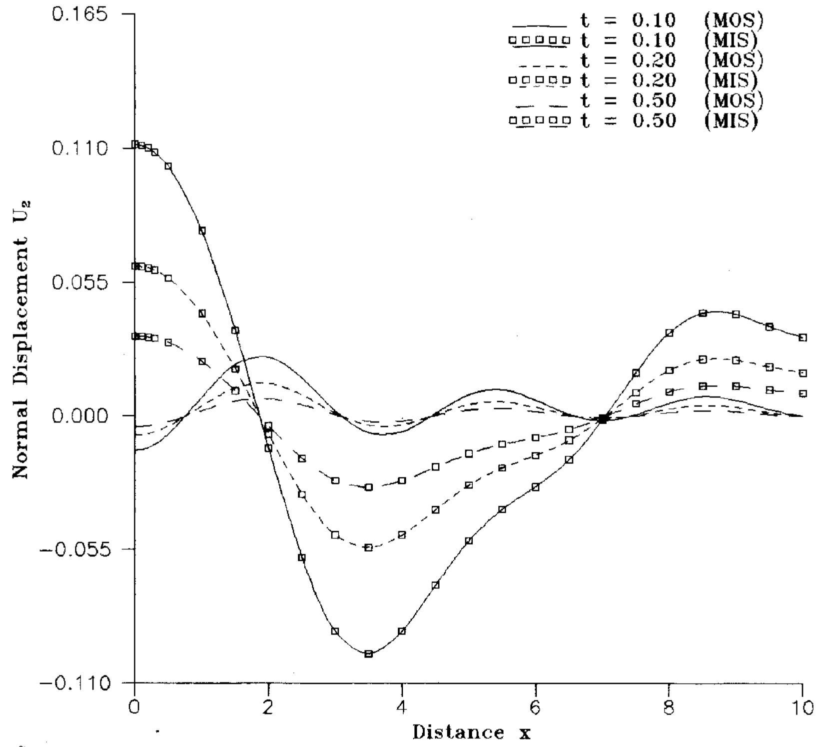

Figure 4. Variations of normal displacement $U_2(x, 1, t)(= u_2/F_0)$ due to tangential USL with distance x.

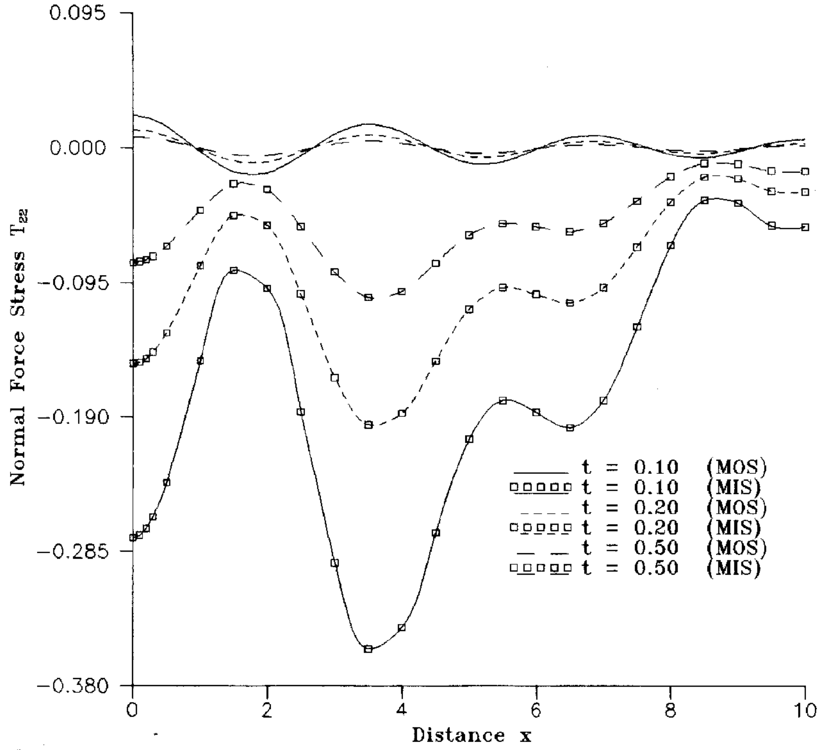

Figure 5. Variations of normal force stress $T_{22}(x, 1, t)(= t_{22}/F_0)$ due to tangential USL with distance x.

center symbol correspond to the case of MIS. All results are for one value of dimensionless width $a_o = \omega a/c_1 = 1$.

Case I: Normal source: The comparison of normal displacement $U_2[= u_2/F_o]$, normal force stress $T_{22}[= t_{22}/F_o]$ and couple stress $M_{23}[= m_{23}/F_o]$, for micropolar orthotropic solid (MOS) and micropolar isotopic solid (MIS) have been studied due to a normal USL and have been shown in figures 1, 2 and 3.

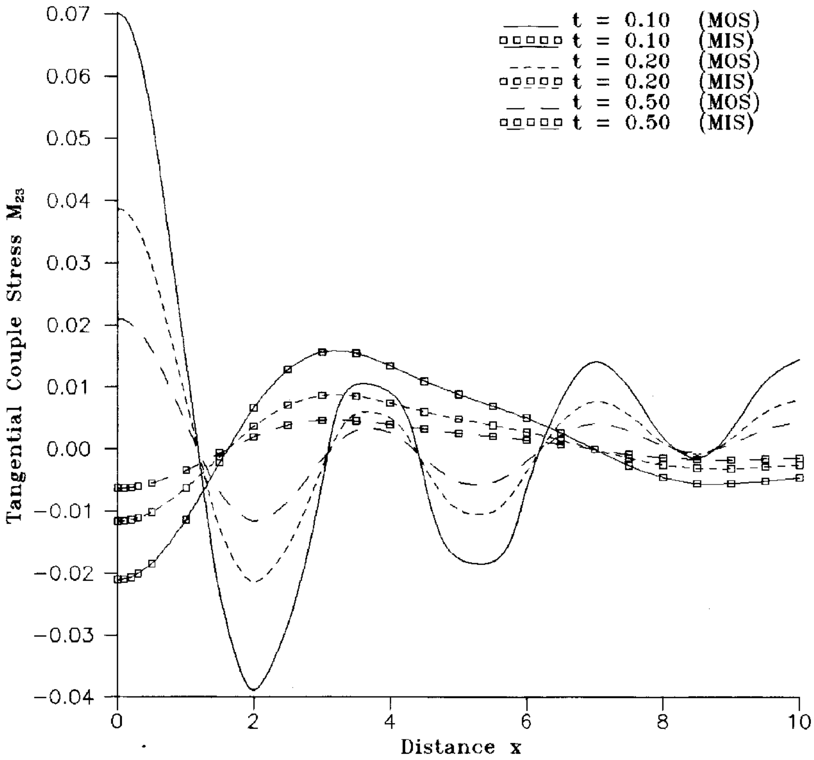

Figure 6. Variations of tangential couple stress $M_{23}(x,1,t)(=m_{23}/F_0)$ due to tangential USL with distance x.

Figure 1 shows the variations of normal displacement U_2 with x due to normal USL. The value of displacement U_2 for MOS have been magnified by multiplying with 10, for all three values of time. For the case of MIS as the time t increases from 0.1 to 0.5, the values of U_2 decrease at initial range of x whereas for the cases MOS the response of displacement with respect to time is reverse. At the point of application of source the values for MOS are lesser than those for MIS due to USL. The behaviour of variation is oscillatory in the whole range for both the cases.

Figure 2 shows the variations of normal force stress T_{22} with x due to normal USL. The values of T_{22} for MOS have been multiplied by 10 for comparison for all three times. For all three times the values of T_{22} for MIS are more than the corresponding values for MOS at the point of application of source. For MOS, initially, values of T_{22} start with a small decrease and then oscillate in further range whereas for MIS values of stress initially decrease smoothly. For both MIS and MOS at the initial stage for the maximum value of time value of stress is maximum.

Figure 3 shows the variations of tangential couple stress M_{23} with x due to normal USL. For all three times and for the case of MOS the value of M_{23} starts with a sharp decrease and than start oscillating in the range $3 \leq x \leq 10$. Whereas for the case of MIS the behaviour of variation of couple stress with reference to times is reverse to that for MOS. As the range of x increases the values of couple stress goes towards zero.

Case II: Tangential source: The comparison of normal displacement $U_2[= u_2/F_o]$, normal force stress $T_{22}[= t_{22}/F_o]$ and couple stress $M_{23}[= m_{23}/F_o]$, for micropolar orthotropic solid (MOS) and micropolar isotopic solid (MIS) have been studied due to a tangential uniform strip load (USL) and have been shown in figures 4, 5 and 6. The values of U_2 and T_{22} for MOS have magnified by multiplying with 10 for all three times.

Figure 4 shows the variations of normal displacement U_2 with x due to tangential USL. The behaviour of variation of displacement is similar to that due to normal USL as in figure 1. However their corresponding values are different.

Figure 5 shows the variations of normal force stress T_{22} with x due to tangential USL. The behaviour of variation of stress for MOS is oscillating with smooth change whereas behaviour of variation for MIS is oscillating with more changes. As the value of x increases the value for both the cases as well as for all three approaches to zero.

Figure 6 shows the variations of tangential couple stress M_{23} The behaviour of variation of couple stress is similar to that due to normal USL as in figure 3. However their corresponding values are different.

Conclusion

Significant anisotropy effect is obtained on normal displacement, force stress and couple stress, for all different times. Due to impulsive force the character of solution is transient. It is also notable that as x diverse from the point of application the components of displacement and stresses approach to zero.

References

Eringen A C 1966 Linear theory of micropolar elasticity; *J. Math. Mech.* **15** 909–924

Eringen A C 1968 Theory of micropolar elasticity; In: *Fracture* vol II, (Academic Press) 621–729

Gauthier R D 1982 Mechanics of Micropolar media; In: Experimental investigations on micropolar media, (ed) O Brulin and R K T Hsieh. (Singapore: World Scientific)

Iesan D 1973 The plane micropolar strain of orthotropic elastic solids; *Archives of Mechanics* **25** 547–561

Iesan D 1974a Torsion of anisotropic elastic cylinders; *ZAMM* **54** 773–779

Iesan D 1974b Bending of orthotropic micropolar elastic beams by terminal couples; *An. St. Uni. Iasi.* **XX** 411–418

Kumar R and Choudhary S 2001 Dynamical problem of micropolar viscoelasticity; *Proc. Indian Acad. Sci. (Earth Planet Sci.)* **110** 215–223

Kumar R, Singh R and Chadha T K 2001 Eigen value approach to micropolar medium due to impulsive force at the origin; *Indian J. Pure Appl. Math.* **32** 1127–1144

Mahalabanabis R K and Manna J 1989 Eigenvalue approach to linear micropolar elasticity; *Indian J. Pure Appl. Math.* **20** 1237–1250

Mahalabanabis R K and Manna J 1997 Eigenvalue approach to the problem of linear micropolar thermoelasticity; *Indian Acad. Math. Sci.* **19** 69–86

Nakamura S, Benedict R and Lakes R 1984 Finite element method for orthotropic micropolar elasticity; *Int. J. Engng. Sci.* **22** 319–330

MS received 5 November 2001; revised 14 March 2002