BOUSSINESQ TYPE MODELLING OF STORM SURGES

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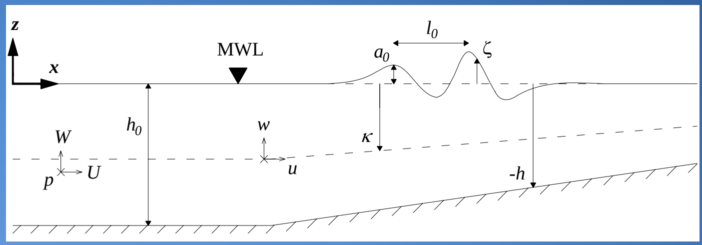
Current challenges in coastal engineering

- To be better prepared for future maritime extreme events
- To expand the understanding of the dynamics of storm surges and tsunamis
- To continue improving the accuracy and efficiency of numerical simulation tools
- To focus on the development of depth-integrated models.

This work aims to present a new numerical implementation of a depth-integrated model and an application for to the dynamics of storm surges.



Boussinesq type model

The Boussinesq type model developed uses the fully nonlinear and first order dispersive equations of Liu (1994), corresponding to the one-layer model of Lynett and Liu (2004). Within this velocity formulation, the horizontal component of the unknown arbitrary located velocity u is evaluated at the vertical position κ=-0.531h. This value ensures a less than 2% error over the range $0 < kh < \pi$ for the linear dispersion relation. The model solves the following systems of equations:

$$\frac{\partial \widetilde{\boldsymbol{U}}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S} \qquad u + \frac{\kappa^2}{2}\frac{\partial Q}{\partial x} + \kappa\frac{\partial R}{\partial x} - \frac{\partial}{\partial x}\left(\frac{\zeta^2}{2}Q\right) - \frac{\partial}{\partial x}(\zeta R) = \tilde{u}$$

$$\widetilde{\boldsymbol{U}} = \begin{pmatrix} \zeta \\ \tilde{u} \end{pmatrix} \qquad \boldsymbol{F} = \begin{pmatrix} (\zeta+h)u - \left(\frac{\zeta^3+h^3}{6} - \frac{\kappa^2(\zeta+h)}{2}\right)\frac{\partial Q}{\partial x} - \left(\frac{\zeta^2-h^2}{2} - \kappa(\zeta+h)\right)\frac{\partial R}{\partial x} \\ \frac{u^2}{2} + g\zeta + \frac{\kappa^2-\zeta^2}{2}u\frac{\partial Q}{\partial x} + (\kappa-\zeta)u\frac{\partial R}{\partial x} + \frac{1}{2}(\zeta Q+R)^2 + \frac{\partial \zeta}{\partial t}(\zeta Q+R) \end{pmatrix}$$

$$\boldsymbol{S} = \begin{pmatrix} -h_t \\ \frac{-1}{\rho}\frac{\partial p_{atm}}{\partial x} \end{pmatrix} \qquad Q = \frac{\partial u}{\partial x} \qquad R = \frac{\partial (hu)}{\partial x} + \frac{\partial h}{\partial t}$$

The discretisation scheme is based on the discontinuous Galerkin (DG) method, which consists of the calculation of the local projection of the solution onto a polynomial basis using the weak formulation of the equations, written on each cell.

The fundamental part of the DG method is the adequate design of numerical fluxes at cell interfaces to deal with the discontinuities of the solution: a local Lax-Friedrichs flux is employed for the hyperbolic part, while for parabolic and dispersive terms the alternating fluxes methodology, described in Yan and Shu (2002), is followed.

The final ODE system is advanced in time by a total variation diminishing Runge-Kutta time discretisation.

Application to storm surges

A storm surge is an offshore rise of water, driven by wind and/or a low pressure weather system. Due to its location on the Tropic of Cancer, Mexico is particularly affected every year in the hurricane season, with significant inland flooding, so it is important to be able to have an overview of the behaviour of surges.

In this study, storm surges are simulated ideally, without considering the wind effect. The propagation is forced on a uniform bed of constant depth *h*=20m, and simulations are performed at second order in space and time, with a moving Gaussian pressure forcing:

$$p_{atm} = P_0 \exp\left(-\frac{(x-ct)^2}{R^2}\right)$$

Four situations are identified and reproduced for the displacement of the low pressure system: $c = 0$, $c < \sqrt{gh}$, $c \approx \sqrt{gh}$, $c > \sqrt{gh}$

In Figures 1, 2, 3, 4, the dispersion of the two free waves travelling in opposite directions can be observed, which was not taken into account in the analytical solution of the wave equation for forced small amplitude shallow water waves (see Nielsen et al, 2008). In the second case, it is interesting to see how the superharmonics of the free wave go over the asymptotic steady forced positive surge.

It clearly appears that for both directions, a negative surge will always precede a positive one, which can be seen as a recession of the coastline followed by smaller waves or a higher positive surge, depending on the celerity of the low pressure system.

From the measured residual data shown in Figure 5 (the component of the astronomical tide has been extracted), a similar behaviour to the 2nd case can be identified.

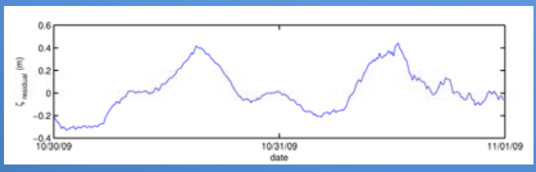


Figure 5 – measured storm, Campeche (Mexico), 2009

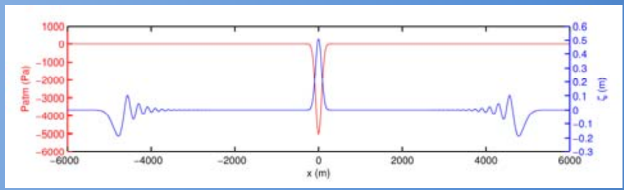


Figure 1 – free surface elevation at time t=350s for $c = 0$

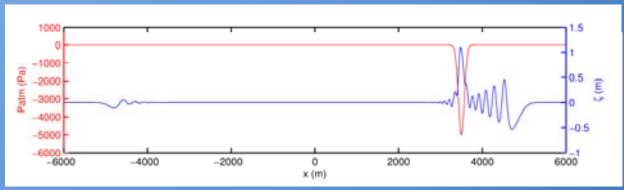


Figure 2 – free surface elevation at time t=350s for $c < \sqrt{gh}$

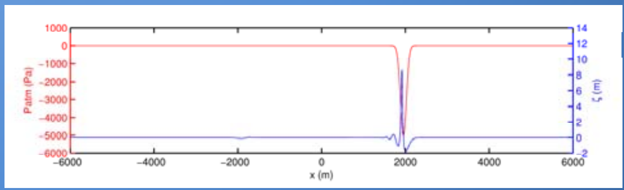


Figure 3 – free surface elevation at time t=140.5s for $c \approx \sqrt{gh}$

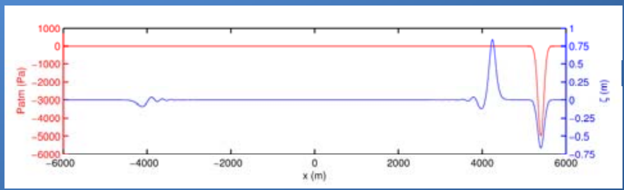


Figure 4 – free surface elevation at time t=300s for $c > \sqrt{gh}$

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