RAPID CALCULATION OF NONLINEAR WAVE-WAVE INTERACTIONS IN WAVE-ACTION BALANCE EQUATION

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This paper presents an efficient numerical algorithm for the nonlinear wave-wave interactions that can be important in the evolution of coastal waves. Random sea waves of multiple frequencies always interact with each other and with the variable wind and pressure fields. As waves propagate and transform from deep to shallow water, it is generally difficult to determine the actual amount of nonlinear energy transfer that occurs among spectral waves and the atmospheric input and wave breaking. The classical derivation of the nonlinear wave energy transfer has involved tedious numerical calculations that are impractical for engineering applications. We propose in this paper a theoretically based formulation to efficiently calculate nonlinear wave-wave interactions in the spectral wave transformation equation. The proposed formulation has performed well in both idealized and real application examples. This rapid calculation algorithm indicates that the nonlinear energy transfer is more significant in the intermediate depth than in deep and shallow water conditions.

Keywords: nonlinear wave-wave interaction; wave-action balance equation; directional wave spectrum

INTRODUCTION

Calculation of nonlinear wave-wave interactions in ocean waves has been studied since the early 1960s (Hasselmann, 1962; Hasselmann et al., 1985). Because the exact solution of the wave-wave interaction requires solving a computationally expensive six-dimensional integral (Resio and Tracy, 1982), many approximation methods have been developed in an attempt to expedite numerical computations. However, these approximations are impractical for coastal and oceanic applications as these lack either computational efficiency or accuracy. The present study extends the theoretical method of Jenkins and Phillips (2001) to explicitly express the nonlinear transfer of wave energy density in the wave-action balance equation using a computationally efficient algorithm. The proposed new formulation is a second-order diffusion operator of general isotropic form that conserves wave energy and wave action. It captures the essential feature of nonlinear wave-wave interactions, where the nonlinear energy exchange is directed mainly from high to low frequencies, and the total wave energy is conserved.

By implementing the new nonlinear energy transfer formulation in the wave-action balance equation, the nonlinear wave-wave interactions can be calculated efficiently in the same routine for solving the wave energy spectrum, without additional integration, that has been traditionally used in wave generation, transformation and dissipation in oceanic and coastal regions.

WAVE-ACTION BALANCE EQUATION

The nonlinear wave-wave interaction term can be calculated in the wave-action balance equation (Lin et al., 2008) as

$$\frac{\partial\left(C_x N\right)}{\partial x}+\frac{\partial\left(C_y N\right)}{\partial y}+\frac{\partial\left(C_\theta N\right)}{\partial \theta}=\frac{\kappa}{2\sigma}\left[\left(CC_g\cos^2\theta N_y\right)_y-\frac{CC_g}{2}\cos^2\theta N_{yy}\right]+S_{in}+S_{dp}+S_{nl} \qquad (1)$$

where $N = E/\sigma$ is the frequency and direction dependent wave-action density, defined as the wave energy-density $E = E(x, y, \sigma, \theta)$ divided by the intrinsic frequency $\sigma = \sigma(k, h)$ that k is the wave number and h is water depth. N_y and N_{yy} denote the first and second derivatives with respect to y; x and y are the horizontal coordinates; θ is the wave direction measured counterclockwise from the x-axis; C and C_g are wave celerity and group velocity; C_x, C_y, and C_θ are the characteristic velocity with respect to x, y, and θ, respectively. κ is an empirical parameter representing the intensity of wave diffraction effect.

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The right-hand side terms respectively are: S_{in} is the source (e.g., wind input), S_{dp} is the sink (e.g., bottom friction, wave breaking, whitecapping, etc.), and S_{nl} is the nonlinear wave-wave interaction.

The first term on the right side of Equation 1 is the wave diffraction term formulated from a parabolic approximation wave theory (Mase 2001). In applications, the diffraction intensity parameter κ (≥ 0) needs to be calibrated and optimized for featured structures. The model omits the diffraction effect for κ = 0 and calculates the diffraction for κ > 0. In practice, the value of κ may range from 0 (no diffraction) to 4 (strong diffraction) for calculating diffraction effects. A constant value of κ = 2.5 has been used by Mase et al. (2001, 2005a, 2005b) to simulate wave diffraction for narrow and wide gap breakwater applications. Lin et al. (2008) and Demirbilek et al. (2009) demonstrate that value of κ = 4 is appropriate for semi-inifinite long breakwaters and also in narrow gaps (inlets) with openings equal or less than one wavelength. For wider gaps with the opening greater than one wavelength, κ = 3 is recommended. The exact value of κ in an application is dependent on the structure's geometry, local bathymetry and incident wave conditions, and may need to be fine-tuned with data. The default value of κ= 4 is used in the model, corresponding to strong diffraction. Implementation of wave diffraction is approximate, and phase-resolving wave models (Holthuijsen et al., 2004) may be used to verify estimate of waves near structures, inlets and harbors.

NONLINEAR WAVE-WAVE INTERACTIONS

Jenkins and Phillips (2001) have proposed a simple formulation to represent wave-wave interactions as a second-order diffusion operator of the isotropic form that conserves both wave energy and wave action. This formulation can be directly included in the wave-action balance equation such that no additional integration is required to calculate wave-wave interactions. It is independent of the dispersion relation so it can be applied for deep as well as shallow water. More details for the derivation, theory, and specification of the simplified nonlinear wave energy transfer equation can be found in the article by Jenkins and Phillips (2001). By keeping only the first and second-order terms in the Jenkins-Phillips formulation, the wave-wave interactions in a finite water depth can be expressed as

$$S_{nl} = a\frac{\partial F}{\partial \sigma} + b\frac{\partial^2 F}{\partial \theta^2} \qquad (2)$$

where $a = \frac{1}{2n^2}[1+(2n-1)^2 \cosh 2kh]-1$ is a function of *kh*, $n = \frac{C_g}{C}$, $b = \frac{a}{n\sigma}$, and

$$F = k^3\sigma^5 \frac{n^4}{(2\pi)^2 g}\left[(\frac{\sigma_m}{\sigma})^4 E\right]^3 \qquad (3)$$

Figure 1 shows the variation of coefficients *a* and $b\sigma$ as function of *kh*, and indicates that wave-wave interactions are more important in the intermediate depth and diminish in the shallow water. According to Equations 2 and 3, wave-wave interactions initially take place in the deep water, continue to evolve in the intermediate depth, and gradually diminish in shallow water. Therefore, the effect of interactions on wave evolution is greater over long fetches in a large ocean body and less in a local coastal region. Figure 2 shows the comparison of directionally integrated S_{nl} from Equation 2 and exact computations in the examples of Hasselmann et al. (1985). These calculations are for the JONSWAP spectra (Hasselmann et al., 1973) with the spectral peak enhancement parameter γ = 2 and γ = 5. Figure 3 shows the calculated nonlinear wave energy transfer rate S_{nl} in the frequency and direction domain for a JONSWAP spectrum with γ = 5. These calculated results are consistent with the observed and theoretical results that the nonlinear wave-wave interactions cause wave energy to transfer from high to low frequencies (downshifting). The simplified formulation did not produce well the high frequencies in the more dynamic and unstable range involving wave breaking and energy exchange with the atmospheric forcing. The spreading of energy transfer in direction in Figure 3 is the result of the conservation of wave momentum that is not investigated in the present study.

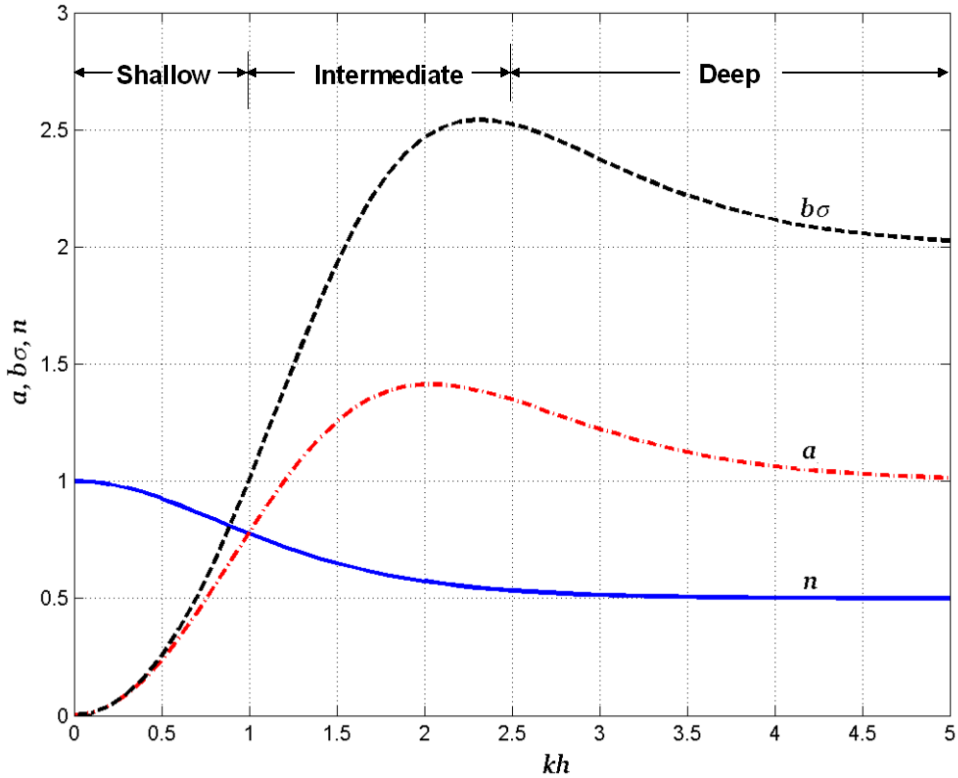

Figure 1. Nonlinear wave-wave interaction coefficients a and $b\sigma$ as functions of kh.

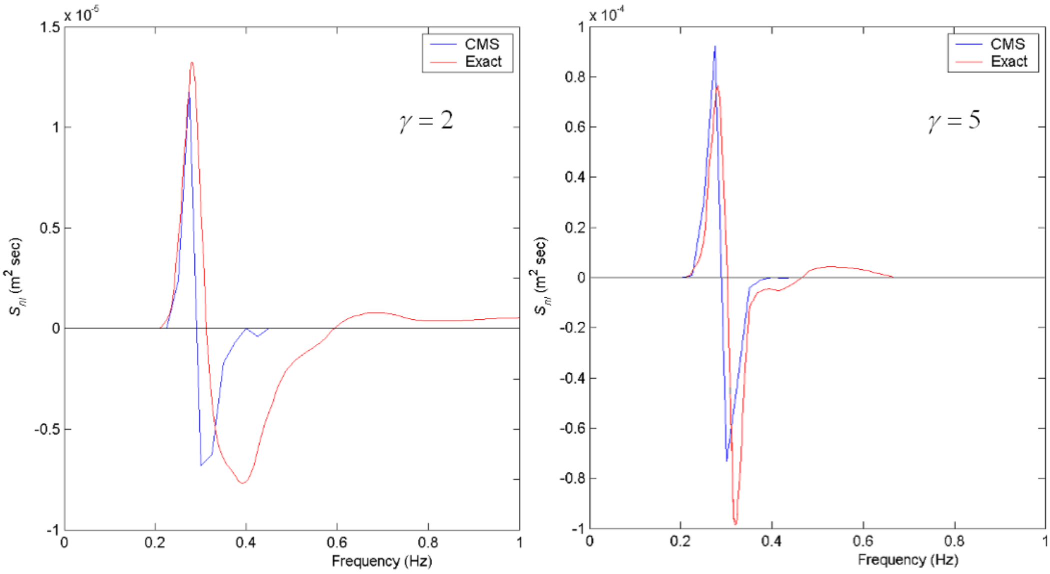

Figure 2. Comparison of directionally integrated S_{nl} for JONSWAP spectrum with γ = 2 and 5.

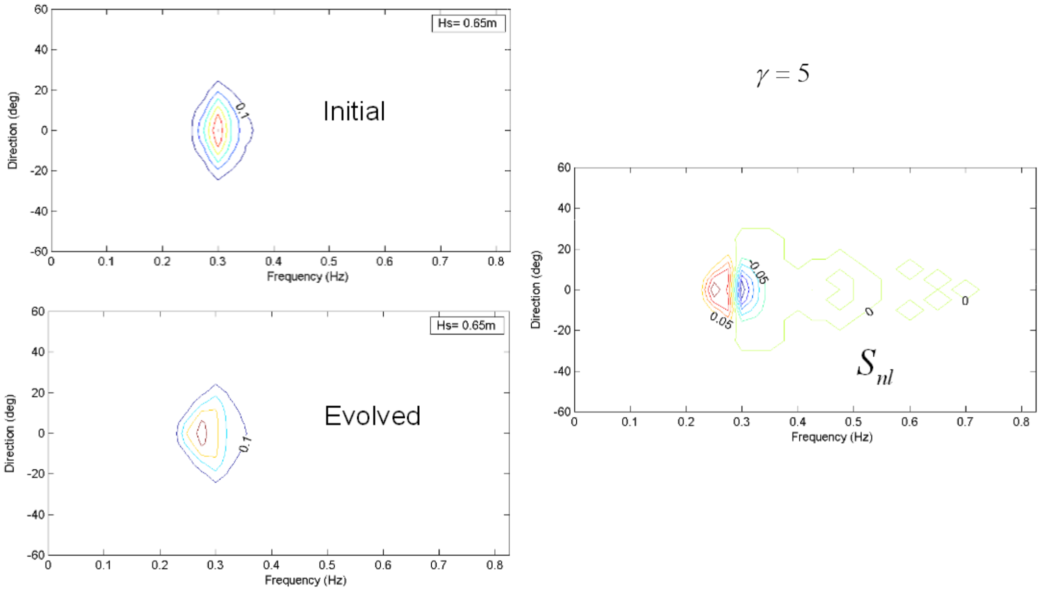

Figure 3. Calculated nonlinear energy transfer for a directional JONSWAP spectrum with γ = 5.

PROTOTYPE APPLICATIONS

Figures 4 and 5 show an example of the computational domain and calculated waves with and without wave-wave interaction at four different locations (HMB1 to HMB4), respectively, at Half-Moon Bay in Grays Harbor entrance, Washington, USA. Osborne and Davies (2004) described the field data collection at Grays Harbor. Numerical simulation is conducted for December 10 through 31, 2003. The input wind and wave data were obtained from the National Data Buoy Center (NDBC - http://www.ndbc.noaa.gov) Station 46029 and Coastal Data Information Program (http://cdip.ucsd.edu) Buoy 036 (NDBC 46211), respectively. The wave simulation was run in the coupled mode with calculated flow field in a Coastal Modeling System (CMS) forced by water level and atmospheric wind input (Lin and Demirbilek, 2010). The effect of current on waves (H_s shown in Figure 5 is the significant height defined as the mean of the highest 1/3 wave height) is pronounced at gauges HMB1 and HMB2 located closer to the navigation channel in relatively deep water. Comparison of simulation results with the field data shows that shallow water effects on wave diffraction, refraction, and breaking are evident at HMB3 and HMB4. The wave transformations near inlet jetties and in the wave diffraction zone can be improved with the nonlinear wave-wave interactions. The calculated wave height with the nonlinear energy transfer is approximately 5 percent higher than without the nonlinear energy transfer. This change is small and in practical applications can be neglected. More importantly, we recommend that the proposed new formulation that extends the Jenkins-Phillips original formulation from deep water to finite depth should be extensively tested in different depths with different wave and wind conditions. The robustness and consistency of our proposed formula must be validated with laboratory and field data and confirmed against other formulations.

Figure 6 shows the calculated wave height fields with and without nonlinear wave-wave interactions at the Louisiana coast in the north central Gulf of Mexico. The input wind and incident wave data are obtained from NDBC Buoy 42041 located approximately in the middle of offshore boundary. Figure 7 shows the corresponding wave period fields with and without the nonlinear wave energy transfer. The simulation is for the growth of an incident deepwater southeast wave of 0.3 m and 4 sec under a moderate 15-kt wind from southeast. With the nonlinear wave-wave interactions, the calculated wave height along the coast is 10 percent higher than without the nonlinear energy transfer effect. The nonlinear wave-wave interactions also increase wave periods over a longer fetch distance, which is a consequence of wave energy transfer from higher to lower frequencies. Wind input was triggered in these simulations. It should be noted that wind input can significantly improve wave-wave interaction result because more wave energy from wind input can transfer from higher to lower frequencies. Therefore, the wave-wave interactions can be rather small and insignificant in magnitude without the atmospheric wind input.

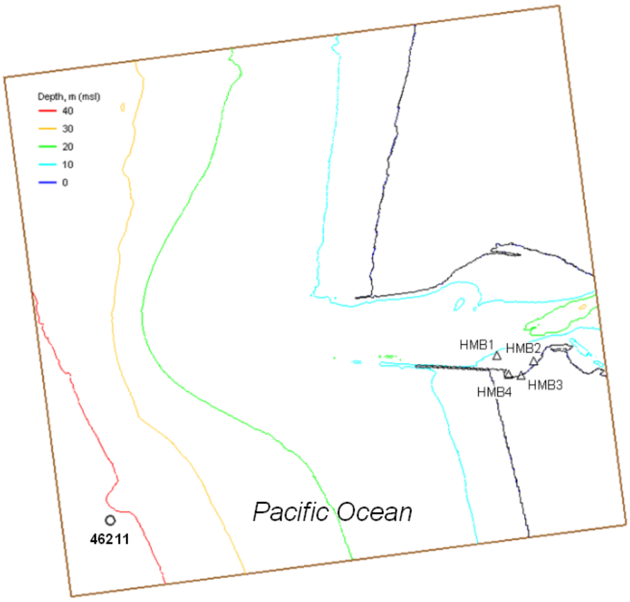

Figure 4. CMS-Wave grid and data-collection stations at Grays Harbor, WA. USA.

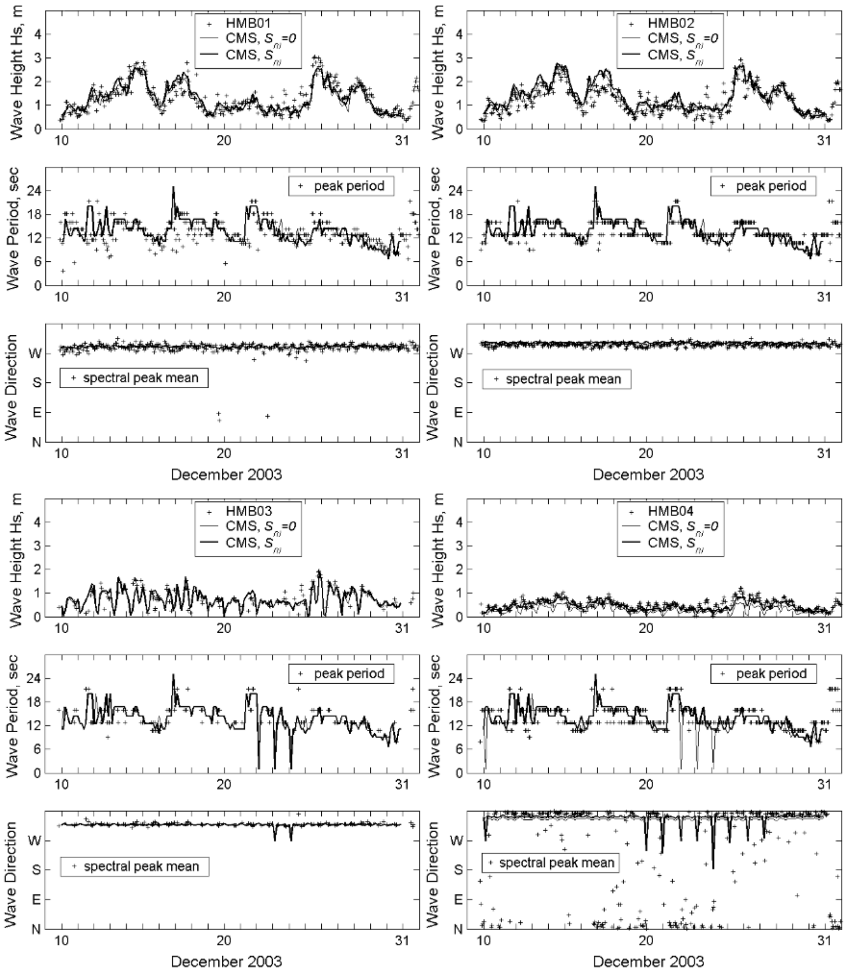

Figure 5. Measured and calculated waves at HMB1 to 4, 10-31 December 2003.

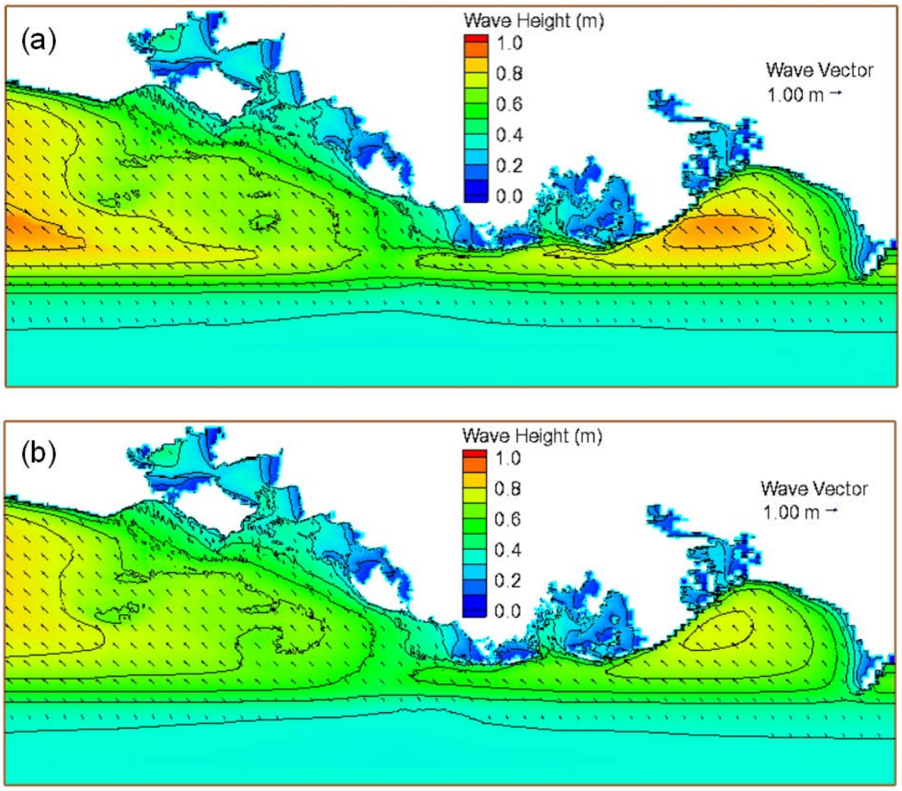

Figure 6. Calculated wave height field (a) with and (b) without nonlinear wave energy transfer.

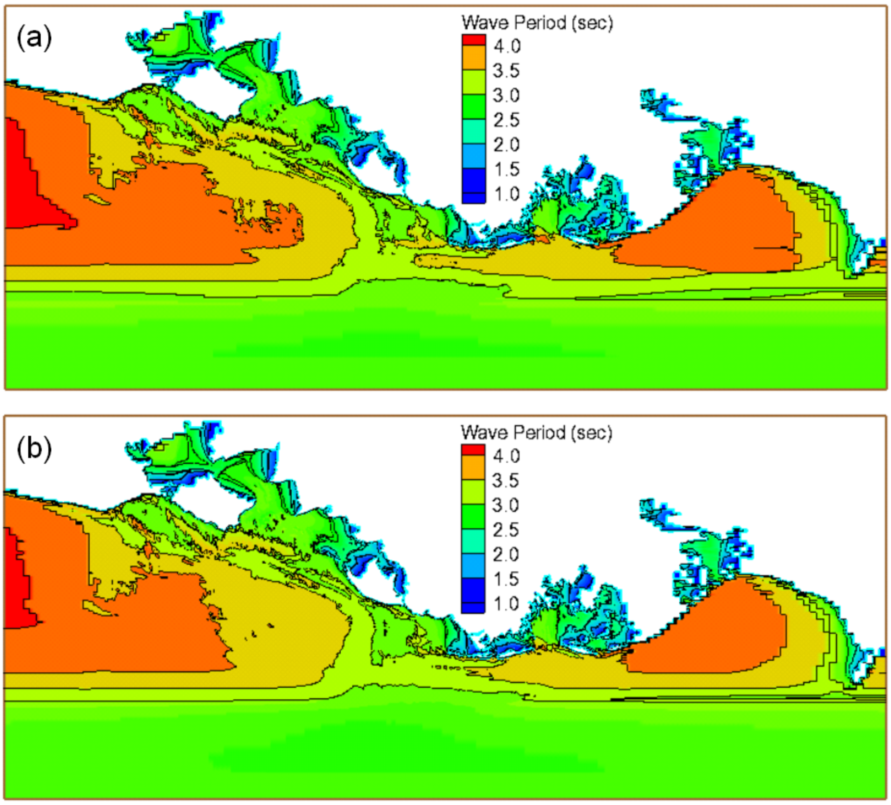

Figure 7. Calculated wave period field (a) with and (b) without nonlinear wave energy transfer.

CONCLUSIONS

A new formulation and numerical algorithm for nonlinear wave-wave interactions is provided in this paper based on the previous work of Jenkins and Phillips (2001). It has been incorporated directly into the wave-action balance equation, and because it is so efficient for application to coastal spectral wave transformation models. The new formulation conserves the wave energy and wave action by omitting the atmospheric input and wave breaking dissipation. It is applied to example applications which show the nonlinear wave energy transfer is more significant in the intermediate depth than in deep and shallow water conditions. The nonlinear wave-wave interactions become more important for long fetches which allow both time and space for efficient nonlinear wave energy transfer to occur. Atmospheric input to waves in the coastal region is demonstrated to feed more energy into higher frequencies and increases the nonlinear energy transfer to the lower frequencies.

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