

CHAPTER 3
THE TRANSFORMATION OF WAVES IN SHALLOW WATER

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The purpose of this paper is to summarize existing knowledge of the processes involved in the movement of progressive oscillatory waves through shallow water and the fundamental principles controlling these processes. Variations in wave characteristics and their physical significance will be discussed as well as agreement between theory and observation. The application of available knowledge to engineering problems is treated in the following chapters.

ENERGY CONSIDERATIONS

Development of the theory of generation of progressive oscillatory waves by wind action and the basic theory of such wave motion has made it possible to formulate a reasonable concept of the energy budget of a wave at any stage of its life. From these sources it can be shown that the total energy available in a wave at the instant it enters shallow water of depth approximately equal to one-half the wave length is

$$E = \frac{wL_0H_0^2}{8} \left(1 - 4.93 \frac{H_0^2}{L_0^2} \right) \quad (1)$$

where w is the unit weight of water, L_0 is the wave length, and H_0 is the wave height.

It can be shown further that, of this amount of energy, one-half is transmitted forward with the wave form into regions of calm. Study by Sverdrup and Munk (1947), and others (Beach Erosion Board, 1942; Rossby, 1947), has led to the conclusion that the potential energy of the wave, representing one-half the total energy, is transmitted forward with the wave velocity, leading to the following expression for the power transmitted per unit of crest width in deep water,

$$P = \frac{E}{2T} = \frac{wH_0^2}{16} \left(1 - 4.93 \frac{H_0^2}{L_0^2} \right) \sqrt{\frac{gL_0}{2\pi}} \quad (2)$$

where T is the wave period, and g is the acceleration due to gravity.

As the wave moves into shallow water the orbits of the water particles become ellipses instead of circles, with the eccentricity of the ellipses depending upon the ratio of the wave length to the depth of water. Under these conditions basic theory leads to the expression for total energy per wave length in shallow water

$$E_s = \frac{wLH^2}{8} \left\{ 1 - 19.74 \left(\frac{a'_s}{L} \right)^2 \right\} \quad (3)$$

The ratio of the semi-major axis of the water particle orbit to the wave length, a'_s/L , is a function of both H/L and d/L , and consequently equation 3 can be given the form

$$E_s = \frac{wLH^2}{8} \left(1 - M \frac{H^2}{L^2} \right) \quad (3a)$$

where M is a function of d/L , that has a limiting value when $d/L = 0.5$ of 4.93, thus agreeing with equation 1 (see equation 28, Chapter 2).

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The power transmitted per unit of crest width then is,

$$P = \frac{E}{2T} = \frac{wH^2}{16} (1 - M \frac{H^2}{L^2}) \left(\sqrt{\frac{gL}{2\pi}} \tanh \frac{2\pi d}{L} \right) \quad (4)$$

These equations are based on the assumption that waves are trochoidal and of small height.

Another approach to the energy problem, found in Lamb (1932), results in the finding that the total energy of a progressive wave system of amplitude $H_0/2$ is equal to the work required to raise a stratum of water $H_0/2$ in thickness through a height equal to $H_0/4$. In this case the total energy available per unit surface area is:

$$E' = \frac{1}{8} wH_0^2 \quad (5)$$

where H_0 is the total wave height. The power corresponding per unit of crest width is found to be:

$$P = 1/8 wH_0^2 \sqrt{\frac{gL_0}{2\pi}} \quad (6)$$

This expression is valid for deep-water waves of small steepness, and may, with sufficient accuracy, be applied to deep-water waves of great steepness. Rayleigh (1877) has shown that the expression for power in any depth of water, per unit crest width is:

$$P = n \cdot E \cdot C \quad (7)$$

where C is the wave velocity. In this equation "n" is the ratio between group velocity and wave velocity and has the value,

$$n = 1/2 \left[1 + \frac{4\pi d/L}{\sinh^4 \pi d} \right] \quad (7a)$$

The value of n approaches $1/2$ in deep water, and unity in shallow water. Thus, the mean power per unit wave crest width, in any depth of water is

$$P = \frac{n}{8} wH^2 \sqrt{\frac{gL}{2\pi}} \quad (8)$$

It must be noted that these expressions define a conservative wave system in which no energy is added to or subtracted from the wave energy; in other words, the use of these expressions in the study of shallow water wave motion involves the assumption that no energy is lost or added in divergence or convergence of the wave, no energy is lost due to bottom friction or viscous effects, and no energy is added or lost by wind action during the passage of the wave through shallow water. However, experiment and observation show that the expressions are sufficiently accurate for most purposes of an engineering nature. In the discussions to follow it will be assumed that the above limitations are admissible unless otherwise stated.

VARIABILITY OF WAVE CHARACTERISTICS

Experience in the study of waves in shallow water has shown the wave characteristics of most importance to wave action problems to be: the wave height, H ; the wave length, L ; the wave period, T ; the wave steepness, H/L ; the wave velocity, C ; and the wave energy, E , or wave power, P . The variability in shallow water of each of these characteristics will be discussed separately. However, since we are concerned primarily in this paper with the physical significance of the variability, some prior description of observed behavior of waves in shallow water may be de-

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sirable. The description will be limited to swell, ignoring local wind waves, since present theory was developed largely on the study of swell and the case may be presented somewhat more simply.

The classic description of wave behavior in shallow water states that as a depth equal to $L/2$ is reached the wave "feels" the bottom and is retarded. To an observer probably the first noticeable effect is a bending or refraction of the wave, closely followed by peaking of the wave crest. On a shoal shore partial breaking of the wave occurs at this time, the wave then reforming and partially breaking one or several times as it moves inshore, finally breaking in a single plunge or spilling over itself for an appreciable time. On a steeply sloping shore partial breaking seldom occurs and the wave breaks usually in a single plunge. In either case as the wave reaches relatively shoal water the crest appears to be an intumescence or mound, quite accented, while the trough appears attenuated and relatively flat. On the Pacific Coast, characterized by conditions of deep water close to shore, the waves appear to be long-crested, crest lengths of a mile or more being not unusual, whereas on the shoal Atlantic Coast crest lengths of several hundred feet are more the rule.

The wave changes form radically when it breaks, and apparently regardless of its prior characteristics, assumes the appearance of a miniature tidal bore, or a traveling hydraulic jump, as it rushes foaming up the beach to its limit of uprush. The recession or back-wash of the wave from its upper limit of travel on the beach resembles simple sheet flow without any characteristic wave appearance.

The theory of wave behavior in shallow water is valid only for wave travel from deep water to the point of breaking. Little is known of the transformations involved in breaking waves and much study of the feature is required.

From the physical point of view the entire transformation of a wave in shallow water must be due in the first instance to modification of the deep water flow pattern. This may occur, so far as is known, only by the imposition of boundary conditions at the bottom and the free surface. Since the free surface is not changed in moving from deep to shallow water, all the transformation must be due to bottom effects. Observation in wave tanks leads to the conclusion that the principal effect of the bottom is to reduce the vertical movements of the water particles in wave action, and since it is assumed that the wave energy is conserved, it follows that the orbits of the water particles must be flattened from circles to ellipses, as observation verifies does occur. Munk (1948) has prepared a diagram illustrative of this process (Fig. 1). The theoretical values are shown on Figs. 2 and 3 which have been extracted from a publication of the Beach Erosion Board (1942). It will be noted that the water particle motion is no longer symmetrical as it was in deep water, since the horizontal movement and therefore the velocities exceed the vertical velocities. In relatively shallow water the vertical movement at the bottom ceases altogether and the orbital motion there is simply a back and forth motion.

The influence of the modification of the particle flow pattern on wave velocity is denoted by the term $\tanh^2 \frac{\pi d}{L}$ in the wave velocity expression

$$c^2 = \frac{gL}{2\pi} \tanh^2 \frac{\pi d}{L} \quad (9)$$

This expression is subject theoretically to two limitations of possible practical importance. First, the equation is derived under the assumption of infinitely small wave steepness. Present rigorous theoretical determination indicates a small correction for wave steepness in the direction that steeper waves travel at higher velocities. Second, the equation is derived under the assumption that the depth is constant, yet it is applied to the case of sloping bottoms.

Many experiments have been made in wave tanks and some observations made in nature to determine the agreement between theory and observation. The results are summarized on Fig. 4, showing the effects of initial steepness based on observations at Scripps Pier (Scripps Institution of Oceanography, 1944), and Fig. 5, showing effects of steepness and bottom slope as determined from laboratory studies (Wiegell, 1950). The variability of wave velocity with depth, expressed as the

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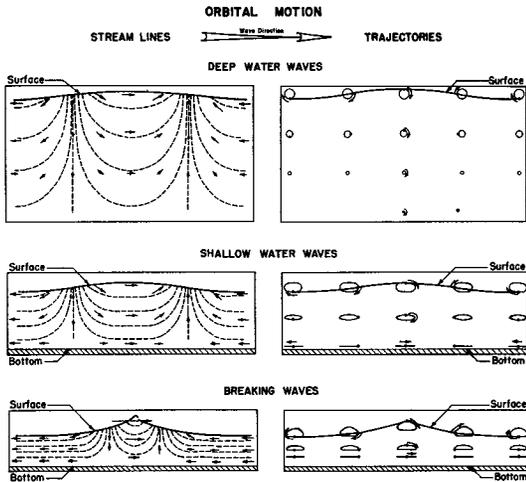


Fig. 1

Orbital motion in progressive surface waves

The dashed lines in the figures to the left are the streamlines, i.e., lines everywhere parallel to the flow. The direction and length of the arrows indicate direction and velocity of orbital motion. The figures to the right show the particle trajectories, i.e., the paths described by fixed particles of water.

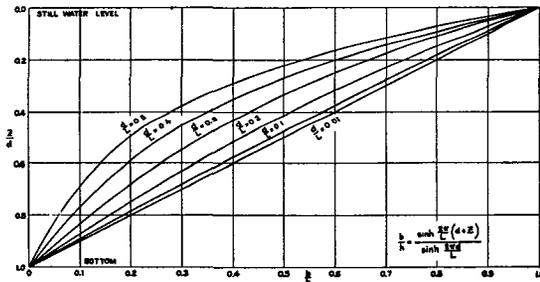


Fig. 3. Vertical amplitude of oscillation for proportional depth related to fraction of wave height.

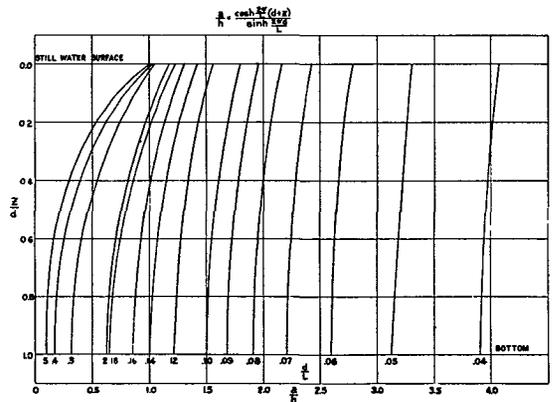


Fig. 2. Horizontal amplitude of oscillation for proportional depth related to fraction of wave height.

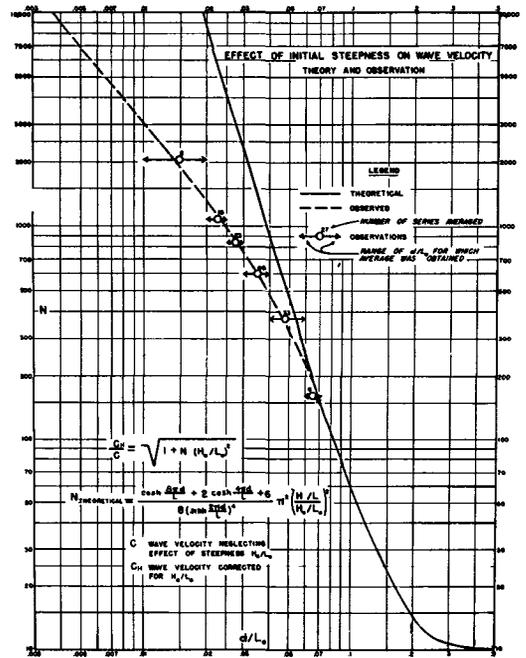


Fig. 4.

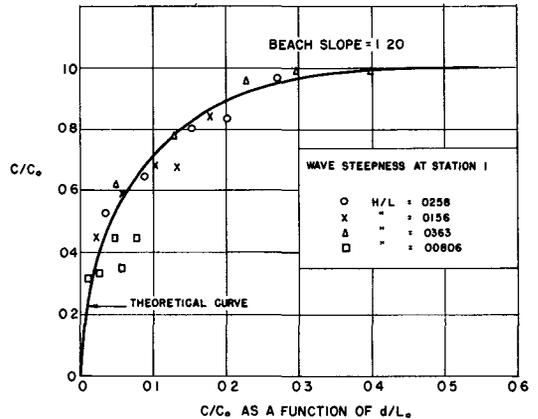
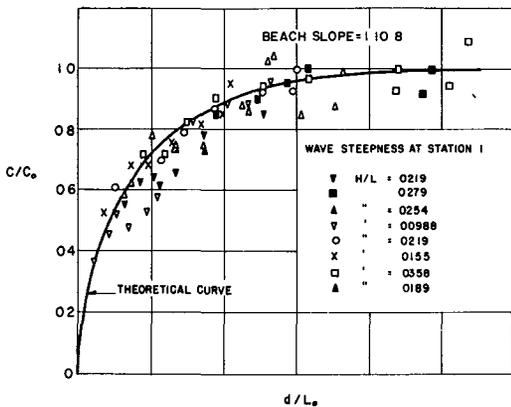


Fig. 5. Variability of wave velocity with water depth.

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ratio of the velocity in shallow water to the velocity in deep water is shown as curve C/C_0 in Fig. 6 (Hydrographic Office, 1944).

It will be remembered that all wave action satisfies the basic requirement that the wave length is equal to the product of the wave velocity and the wave period. Observation confirms the assumption made in the development of theory that the wave period is constant for the wave travel distances and times concerned in wave travel through shallow water. In these circumstances it is apparent that,

$$T = L_0/C_0 = L/C$$

and
$$L/L_0 = C/C_0 \tag{10}$$

It follows therefore that the wave length varies in shallow water to the same extent as the wave velocity. Since the wave length and velocity both decrease with decrease in depth, conservation of energy of the wave requires that the wave height increase. It is assumed as a basic premise that the power transmitted per unit width of wave crest remains constant at all points along the path of travel of the wave from deep water to the point of breaking. Physically this means that the wave energy entering a given zone equals the energy leaving the zone; therefore, no energy is either accumulated or destroyed. Referring to prior discussion of energy considerations it will be recalled that in deep water one-half the total energy, the potential energy, advances with the wave form at the wave velocity; whereas, in shallow water a larger portion of the energy advances, the increase being derived from the wave kinetic energy. The fractional addition is measured by the term n in Equation 7. The physical significance of this behavior lies in the fact that, whereas the potential energy is periodic and advances in phase with the wave form, the kinetic energy in a deep-water wave is evenly distributed, non-periodic, and thus independent of the position or velocity of the wave. However, in shallow water the particle orbits are deformed and the kinetic energy is no longer evenly distributed; in the ultimate it becomes periodic and, like the potential energy, advances in phase with the surface deformation.

If we denote deep-water conditions by the subscript 0 , we may write

$$P_0 = P = n_0 E_0 C_0 = nEC$$

and, by substitution of appropriate values it can be shown that

$$H/H_0 = \sqrt{\frac{1}{2n} C_0/C} \tag{11}$$

The variability in height of a wave traveling through shoal water without refraction can be computed from this relation and is shown as curve H/H'_0 on Fig. 6. It will be noted that the wave height decreases to about 91% of its deep water value, then increases rapidly as the wave moves into lesser and lesser depths. The reason for this behavior lies in the relative values of n and C_0/C , n increases with decreasing depth thus reducing wave height, while C_0/C increases with decreasing depth and increases wave height. As the depth becomes less than about 0.06 times the wave length, the increase in height due to velocity variation overbalances the decrease due to the variation in n and the wave height increases beyond its deep-water value (Suquet, 1949).

Theoretical studies at the University of California (Putnam, 1949; Putnam and Johnson, 1949) indicate that the assumption of constant power transmission may be inadmissible for shallow areas of flat slope. Fig. 7 shows the results of some of these studies, and indicates that reductions of as much as 30% in wave heights predicted on the basis of frictionless theory may occur.

In the usual case waves will not travel through shoal water without refraction, and correction of wave height must be made to take account of refraction effects (Groen and Weenik, 1950). These effects will be discussed in detail in Chapter 4 and, therefore, will not be treated here. For the purpose of completeness Fig. 8 is presented to indicate the magnitude of the corrections required (Hydrographic Office, 1944). In this figure the angle α_0 is the angle between the wave crest and the shore line when the wave is in deep water, and K_d is the cor-

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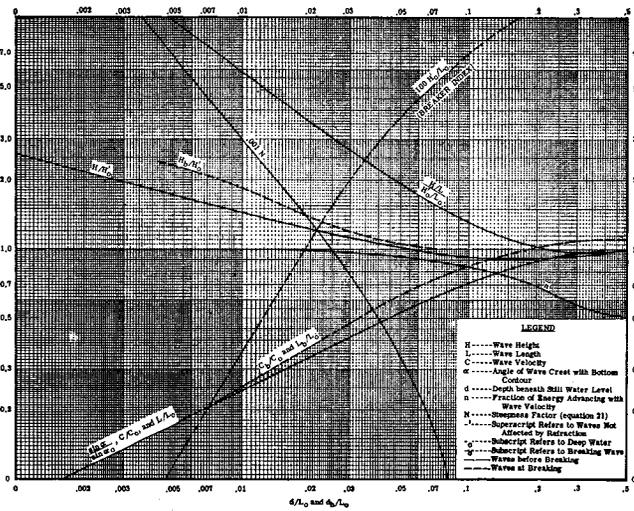


Fig. 6. Waves in shallow water. Change in height and length from deep water to point of breaking.

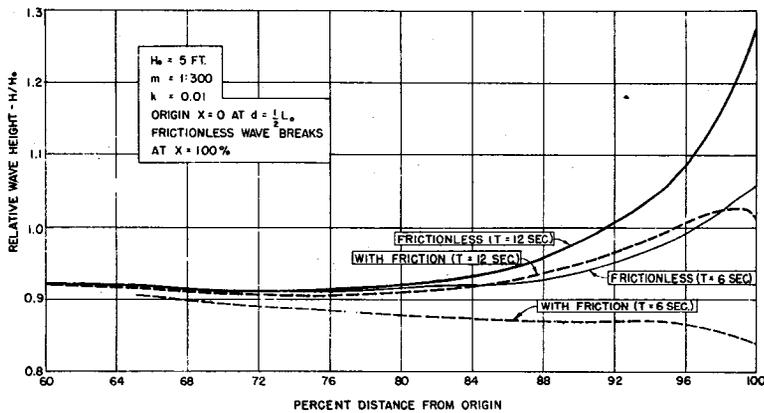


Fig. 7. Effect of bottom friction on wave height.

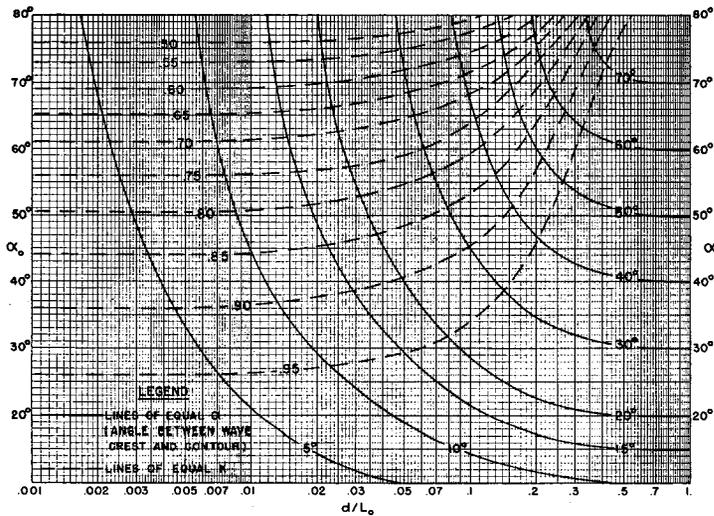


Fig. 8. Change in wave direction and height due to refraction on beaches with straight, parallel depth contours.

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rection factor to be applied to values of H obtained from Fig. 6. Since the wave height and length vary differently with wave travel through shallow water, then the wave steepness must vary. Denoting steepness by $H/L = \delta$ we can from previous relations set

$$\delta/\delta_0 = H/L/H_0/L_0 = H/H_0 \cdot C_0/C \tag{12}$$

This relation is shown on Fig. 6 as curve $(H/L)/(H_0/L_0)$. It will be noted that the steepness first decreases slightly, then increases very rapidly. The sudden, sharp increase in steepness is one of the most easily noticed features of wave travel in shallow water and is striking to even a casual observer noticing long swell on a beach. Steepness increases until the wave breaks. Studies by Michell (1893), Stokes (1847), Havelock (1918), and others on the problem of the greatest height attainable by an oscillatory wave of permanent form lead to the conclusion that the minimum included angle at the crest is 120° , and that the maximum height has a value of 0.1418 of the wave length. No experimental confirmation of this theory is yet available, however, large numbers of wave observations, made under a wide variety of conditions, have not included any reliable data showing heights exceeding the $L/7$ limit predicted by theory.

An additional feature of wave motion in shallow water that is usually of secondary importance is the variability in the percentage of wave height above and below still water. Experimental results obtained by Wiegler (1950) are shown on Fig. 9. These data show reasonably good agreement with theory and it may be considered that confirmation has been established insofar as most applications are concerned.

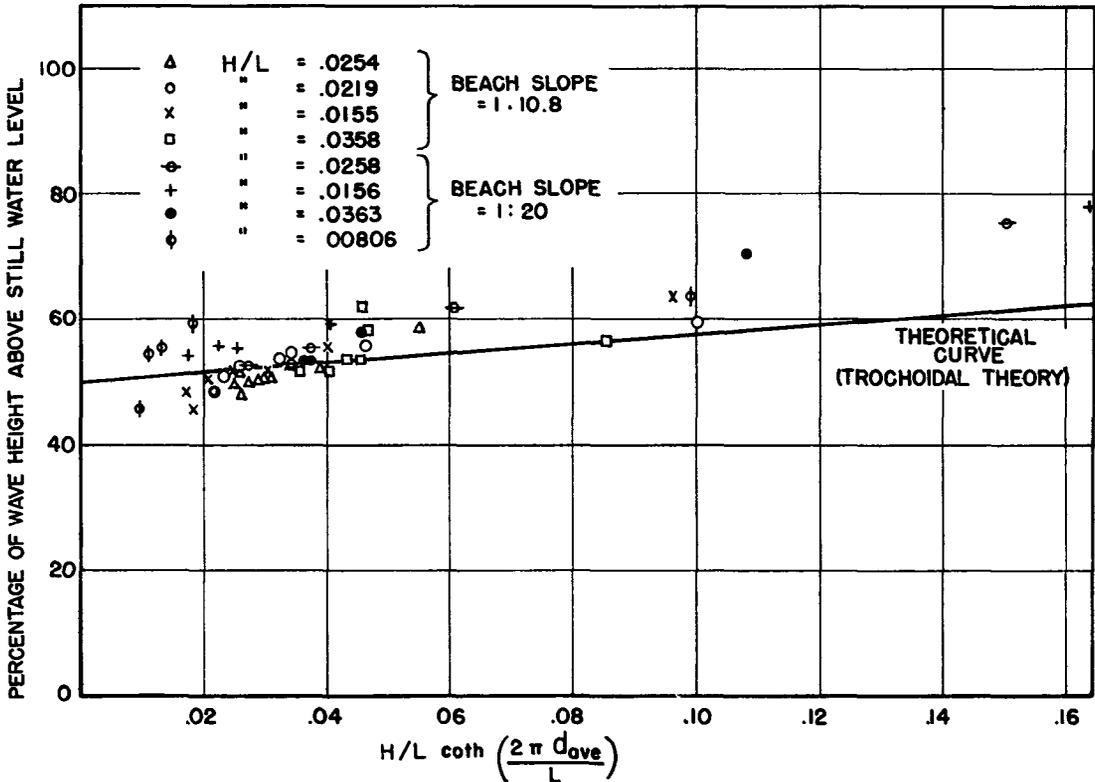


Fig. 9. Comparison between experimental and theoretical percentages of wave height above still water level.

BREAKERS AND SURF

At the outset it might be stated that there is no satisfactory theory dealing with breakers, i.e., individual waves that are breaking, or with surf, which is

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the collection of individually breaking waves. Theory indicates and observation confirms that a wave will break, i.e., lose its form, when the crest angle approximates 120° , or the steepness reaches about $1/7$. However, waves break, in the sense of losing their form, under conditions other than these critical conditions. It is well known, for example, that waves traveling over a flat slope will partially break at the crest, reform as apparently stable waves, then break again, and so on, finally breaking completely. It is equally well known that wind frequently causes waves to break partially, or perhaps simply blows off the crests of the waves.

The preceding discussion of the variability of wave characteristics shows that as a wave approaches very shallow water the crests tend to "hump" and become separated by long, flat troughs. In appearance this pattern resembles a series of solitary waves and this resemblance suggested the possibility of application of solitary wave theory to the problem of breaking of oscillatory waves.

Present theoretical knowledge of breaking oscillatory waves is based on solitary wave theory; however, a somewhat different approach was suggested recently by Stoker (1949), based on the theory of compressible flow of a gas and the analogy between a discontinuous shock wave in a compressible gas to a breaker in water.

In theory, a solitary wave consists of a single crest lying in its entirety above the still-water level, and extending infinitely to still water before and behind the crest (Keller, 1949). Application of the solitary wave theory to breakers and surf represents, therefore, a departure from the phenomenon described by theory; however, since the major portion of the energy of a solitary wave is confined to a narrow zone about the crest, the application may be admissible. Further, the character of the motion of the water particles is quite different.

No attempt will be made here to develop the solitary wave theory as applied to the breaker and surf problem, this having been done by Munk (1949a) and Keller (1949). Only the developed theory will be discussed, the reader being referred to the published works for details.

The extreme crest angle at breaking appears to be one of several elements entering the breaker problem. Solitary wave theory as well as oscillatory wave theory are in agreement that a wave must break when the extreme crest angle is 120° . For the solitary wave it can be shown that the corresponding ratio of wave height to depth has the critical value of 0.7813. This means that when the wave height reaches a value of 0.7813 times the depth the wave will break. The frequently useful inverse ratio is 1.28; i.e., waves will break in water of depths equal to 1.28 times the wave height.

A second criterion for breaking is that the velocity of the water particles at the very crest is equal to the wave velocity. This is the a priori condition assumed to exist when developing the theory with regard to extreme crest angle. It should be regarded as a distinct criterion, however, since, for example, the action of wind on the wave crest may cause equality of particle and wave velocity regardless of crest angle, or there may be rapid increases in wave steepness caused by sudden changes in depth.

It will be obvious that the theoretical treatment is based on the concept that breaking of a wave is a kinematic problem, involving only the relation between particle velocities and wave velocities. The simple criterion for breaking is that the particle velocity at the crest must exceed the wave velocity. However, it is possible to show mathematically that another criterion exists; namely, both the equations of continuity and of equal pressure must be satisfied if oscillatory motion of the water particles is to exist. This criterion controls the breaking of waves in areas of appreciable depth variation, and probably is the criterion controlling intermittent breaking and re-forming of the wave.

Munk (1949a) has established several relationships useful in a practical way. From energy considerations he shows that the ratio of wave height at breaking to deep-water wave height is

$$H_b/H_0 = \frac{1}{3.3\sqrt{H_0/L_0}} \quad (13)$$

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The agreement of theory and observation is shown by Fig. 10.

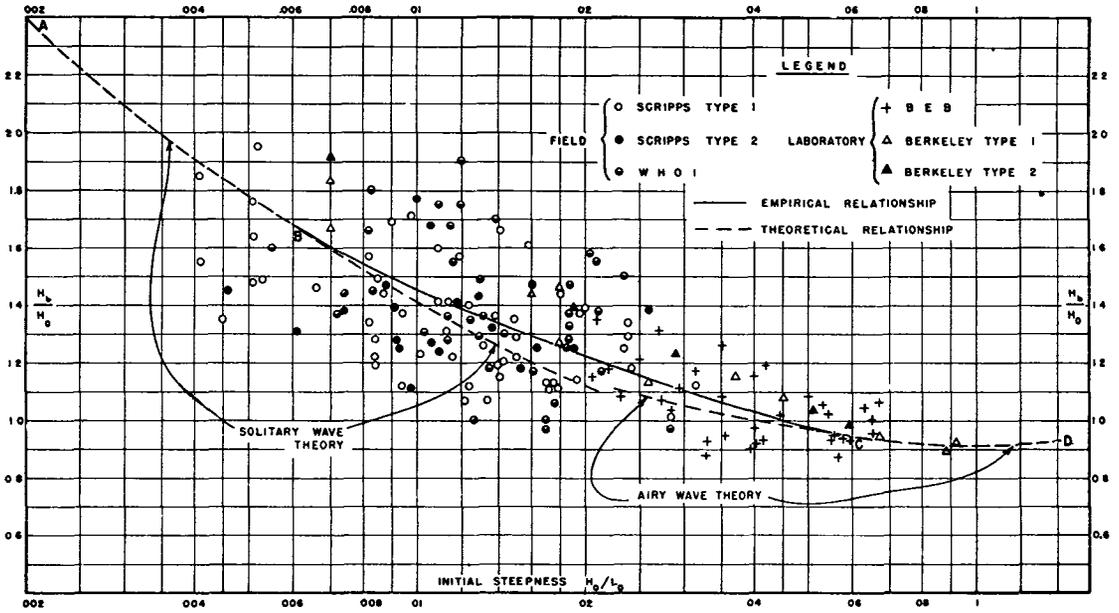


Fig. 10. Comparison between predicted and observed breaker heights.

The relationship between water depth at the breaking point and the wave height at breaking has been mentioned earlier, the relationship is

$$d_b = 1.28 H_b \quad (14)$$

Observations show a wide scatter of values of the ratio d_b/H_b , probably due in large part to variation of actual conditions from the conditions assumed in development of the theory. The observations are considered sufficiently reliable to lead to the conclusion that although theory shows fair agreement with observation on the average, the theory is deficient in not being able to account for the wide scatter of observation.

The form of the wave at breaking is not predicted acceptably by the solitary wave theory.

The elevation of the mean position of the water line at the point of breaking above mean sea level in deep water is given by

$$y = \frac{13.7}{g} \left(\frac{H_b}{T} \right)^2 \quad (15)$$

Observations in wave tanks are in good agreement with theory.

Fig. 6 shows curves indicating the relationships discussed herein for various values of the applicable water depth-wave length ratio. The curve marked "Breaker Index" is an empirical curve showing the relative depth at which a wave of a given steepness will break (Hydrographic Office, 1944).

EFFECT OF CURRENTS

The effect of currents on wave motion in shallow water has been studied by Arthur (1950) in connection with the determination of the refraction of shallow water waves moving in any given distribution of current and depth. The method evolved is applicable for the case of shallow water waves whose velocity is given to a satisfactory degree of approximation by the expression

$$c^2 = gd$$

The solution is based on Fermat's Principle in optical refraction by which the rays

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representing least time of travel are obtained, and is analogous to the problem of minimal flight path in aerial navigation.

If θ be the azimuth between the relative wave velocity and an arbitrary direction, V_c is the component in the direction of wave velocity of the horizontal current, C is the wave velocity, and n is the tangent to the wave crest, then the solution is given by:

$$\partial\theta/\partial t = - \frac{\partial(V_c + C)}{\partial n} \quad (16)$$

The physical significance of this expression is that the rate of change of direction of wave travel is numerically equal to the variation of shear along the crest in a scalar field of the relative wave speed and the current component in the direction of wave speed. The negative sign shows that the direction of wave advance turns toward the direction of negative shear along the crest.

The application of this solution to various problems involves considerable cumbersome computation, however, graphical solution probably is possible. Experimental confirmation of the theory is not yet available.

REFLECTION AND SURF BEAT

The transformations undergone by waves running through shallow water discussed previously have not included consideration of possible reflection phenomena caused by the shoaling bottom (Cochrane and Arthur, 1948). There is in fact little in the physical appearance of such waves to suggest that reflection occurs until the bottom slope becomes relatively steep. Reflected waves have been observed frequently from slopes that are steep, for example, breakwater side slopes of 1 on 2, and from vertical faces where the clapotis phenomenon is striking. The author is not aware that the problem has received much attention from the theoretical point of view, however, some empirical work has been done at the Beach Erosion Board (Caldwell, 1949).

Tests made by Caldwell (1949) of the reflection of solitary waves from impermeable slopes, varying in inclination from 6° to 90°, i.e., from about 1 on 10 to vertical show that in this range the reflected energy varies widely from a minimum of about 8% of the incident energy for the 1 on 10 slope to 100% for the vertical face. Observations by Munk (1949b) at the Scripps Pier shoreface, having an average slope of about 1 on 50 from the shore to 40 foot depth, lead him to conclude that the shore acts as a radiating line source returning approximately one per cent of the incoming wave energy in the form of long period waves. Munk (1949b) attributes the returning energy to the variability of water transport into the surf zone, and terms the phenomenon "surf beat."

Gridel (1946) in discussing the application of the methods of physical optics to the study of oscillatory wave motion in shallow water, concludes, in agreement with Miche (1944), that whenever wave refraction occurs, then wave reflection must occur also, thus leading to the conclusion that the energy content of a wave traveling through shallow water is not constant, but continually decreasing.

It is not possible at present to state which of these various situations represents the truth; it can be stated that, in common with most of the problems of wave transformation in shallow water discussed previously, the need and opportunity for much additional theoretical work is great.

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