SIMPLE MODELS FOR EQUILIBRIUM PROFILES UNDER BREAKING AND NON-BREAKING WAVES

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ABSTRACT: *Simple theoretical models are presented to calculate the equilibrium profile shape under breaking and non-breaking waves. For the case of breaking waves the seaward transport in the undertow at equilibrium is locally balanced by a net vertical sedimentation so that no bottom changes occur. The parameterization of the water and sediment flux in the surf zone yields a power curve for the equilibrium profile with a power of 2/3. Three different models are developed to derive the profile shape under non-breaking waves, namely (1) a variational formulation where the wave energy dissipation in the bottom boundary layer is minimized over the part of the profile affected by non-breaking waves, (2) an integration of a small-scale sediment transport formula over a wave period where the slope conditions that yield zero net transport determine equilibrium, and (3) a conceptual formulation of mechanisms for onshore and offshore sediment transport where a balance between the mechanisms defines equilibrium conditions. All three models produce equilibrium profile shapes of power-type with the power typically in the range 0.15-0.30. Comparisons with laboratory and field data support the results obtained indicating different powers for the equilibrium profile shape in the surf zone and offshore zone.*

INTRODUCTION

The concept of an equilibrium profile (EP) is of central importance to coastal engineers because it provides a basis for assessing a characteristic shape to a beach in design and analysis situations. A beach of a specific grain size, if exposed to constant forcing conditions (monochromatic waves or random waves with constant statistical properties), normally assumed to be short-period breaking waves, will develop a profile shape that displays no net change in time, although sediment will be in motion. The validity of this concept has been verified through a large number of laboratory

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experiments on beach-profile change (e.g., Saville 1957, Kajima et al. 1982, Kraus and Smith 1994, Dette et al. 1998). On a natural beach, however, the forcing conditions are never constant and changes in the beach topography occur at all times. In spite of this, the beach profile in the field exhibits a remarkably persistent concave shape (Bruun 1954, Dean 1977), where changes may be taken as perturbations upon the main profile configuration. Such changes in beach profile shape can be regarded as adjustment of the profile from the course of one equilibrium state to another as the forcing conditions change (for example, during a storm). With this view the equilibrium concept is valid not only for the long-term average forcing conditions but for varying forcing conditions over different time scales.

The conditions for equilibrium on a beach and associated slopes and profile shapes have been a topic of research since the 1950's. Bruun (1954) proposed a power law to describe the profile depth as a function of distance from the shoreline based on field data from the Danish West coast and from California (a power of 2/3 provided the best fit). Dean (1977) analyzed an extensive data set consisting of beach profiles measured along the Atlantic and Gulf coasts of United States. He also found that a power law with a power of 2/3 provided the best overall fit to the measured profile shapes. Furthermore, Dean (1977) theoretically motivated this power law by assuming that equilibrium occurred for constant wave energy dissipation per unit water volume along the profile. This constant is known as the equilibrium energy dissipation and has been shown to be a function of grain size (Moore 1982) or fall speed (Dean 1987).

Bowen (1980) derived EP shapes by analysis of the sediment transport formulas proposed by Bagnold (1963). Analytical expressions for the profile shape were obtained for the cases of a steady drift due to wave mass transport in the boundary layer and wave asymmetry, producing powers of 2/3 and 2/5, respectively. Larson and Kraus (1989) generalized the derivation by Dean (1977) to include the effect of gravity leading to a planar beach slope at the shoreline. Inman et al. (1993) divided the profile into two portions, an inner and outer region, which corresponded to the regions with breaking and non-breaking waves, respectively. Both portions were successfully approximated with power curves matched at the break point and the optimum values of the power was 0.4 for both curves. In fitting the power curve to the inner portion of the profile the height of the berm was employed as the base elevation; this differs from previous studies where typically the mean sea level was used as vertical datum.

Although a number of studies have been carried out regarding EPs, as indicated above, there are presently no general, physically based theories to derive the EP shape under both breaking and non-breaking waves that produce a realistic EP shape over the entire active profile. Thus, the main objective of this study is to develop theories for the EP shape under breaking and non-breaking waves. It is assumed that the region where wave breaking prevails may be treated separately from the offshore zone where mainly non-breaking waves control the profile shape. This separation is conceptually justified because intense turbulence exists in the surf zone, making both bedload and suspended load significant, whereas bottom-boundary layer processes and bedload transport are expected to be dominant in the deeper and less turbulent water offshore under non-breaking waves. First, the EP shape in the surf zone will be discussed, where wave breaking controls the profile development. Then, the EP under non-breaking waves is treated where oscillatory waves determine the profile shape. The derived EP formulas were validated through comparisons with laboratory and field data.

THEORETICAL CONSIDERATIONS

EP Shape Under Breaking Waves (Surf Zone)

Although the theoretical model proposed by Dean (1977) for EPs produces a shape that is in agreement with field data, the physical justification for the equilibrium condition is not clear and the assumptions made are rather *ad hoc.* Also, a profile that is close to equilibrium may still produce a significant net sediment transport in the undertow that is difficult to explain within the framework of the Dean model. Here, an alternative model is derived that relies on certain assumptions about the circulation of water and sediment in the surf zone. In this sense the model is non-local as opposed to the Dean model where the equilibrium conditions are established from a local criterion of zero transport. A beach subject to waves experiences a return flow across the profile that carries sediment stirred up by the waves offshore in the undertow. Even at equilibrium conditions, when no net change in the profile shape occurs, this transport should take place implying that material has to come onshore above the undertow layer to compensate for the offshore transport. When the undertow reaches the break point the transported sediment has to be resuspended up into the water column and pushed onshore to ensure an equilibrium situation where no material goes offshore. Thus, such a simplified picture yields a surf zone with sediment moving, but with no net changes in time of the profile depths, and the break point acts almost as a singularity. Fig. 1 schematically illustrates the water and sediment flow in the surf zone at equilibrium conditions (note that the size of the arrows is exaggerated in the figure).

To arrive at an EP shape it is assumed that the change in the sediment transport in the undertow is balanced by sedimentation through the water column. This sedimentation represents the net effect from sediment being resuspended locally by the waves and the settling of the material. If it is assumed that the transport in the undertow is the product between the flow q and a characteristic bottom concentration c, and that the gradient in this transport is balanced by a net sedimentation (μwc), the following equation for the EP profile may be derived (Larson et al. 1998),

$$\frac{d}{dx}\left(\lambda_u \sqrt{gh}\gamma h^2\right) = \mu wh \qquad (1)$$

where h is the water depth, w the sediment fall speed, g the acceleration of gravity, and λ_u, γ, and μ constants (referring to the undertow flow, wave height-depth ratio, and net sedimentation, respectively). The undertow flow was set proportional to the mass transport in the breaking waves, c was derived from a balance between the work needed to keep grains in suspension in the bottom layer and the energy dissipation in the bottom boundary layer (Larson et al. 1998), and the vertical sediment transport was parameterized in terms of c. Integrating the above equation with $h=0$ when $x=0$ yields:

$$h=\left(\frac{3}{5}\frac{\mu w}{\lambda_u\sqrt{g\gamma}}\right)^{2/3} x^{2/3} \tag{2}$$

This is the same EP shape as Dean derived with a similar functional dependence on w for the constant in front of $x^{2/3}$ as what Kriebel et al. (1991) obtained (c.f., Bowen 1980).

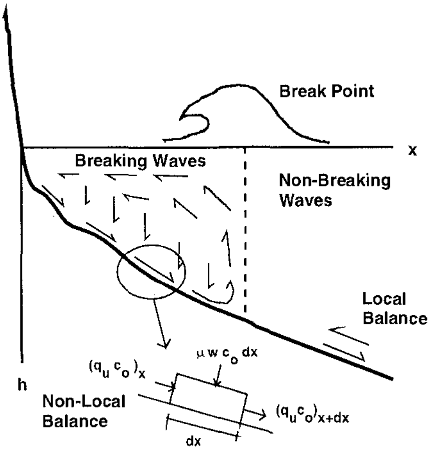

Figure 1. Schematic picture of assumed sediment transport pattern in the surf zone for deriving an EP shape.

EP Shape Under Non-Breaking Waves (Offshore Zone)

Three different approaches were employed to derive the EP shape under non-breaking waves. The first approach was based on the heuristic assumption that the profile shape seaward of the break point at equilibrium is such that the waves dissipate a minimum of energy when traveling across the profile. In the second approach a detailed sediment transport formula proposed by Madsen (1991) was integrated over a wave period, and an equilibrium slope is determined that produces zero net transport. Finally, a conceptual model was formulated that assumed a balance at equilibrium between onshore transport due to wave asymmetry and offshore transport due to gravity.

Variational Formulation

Many systems in nature strive towards equilibrium in such a way that some energy quantity attains a minimum. Thus, it seems reasonable to assume that a beach profile is in equilibrium with the forcing conditions when an appropriate energy quantity reaches its minimum value. The basic assumption employed in the present formulation is that the total wave energy dissipation per unit beach width and time along the profile exposed to non-breaking waves attains a minimum at equilibrium. This implies that the waves loose minimum energy as they propagate across the profile, experiencing the least reduction in wave height for all possible profile shapes. Using the Euler-Lagrange approach from variational calculus to determine the optimal profile shape that minimizes the integral expressing the total wave energy dissipation gives the following equation to solve for the EP shape,

$$C_1 h^{9/4}\sqrt{1+(dh/dx)^2} = -1 \tag{3}$$

where C_1 is a constant. However, this equation does not have a general solution that satisfies the boundary conditions $h=h_b$ for $x=x_b$ and $h=h_c$ for $x=x_c$, where subscripts b and c denote break point and closure depth, respectively (dissipation is assumed to occur between x_b and x_c). Instead, by assuming a general power shape for the EP, that is, $h=(h_b^{1/n}+(h_c^{1/n}-h_b^{1/n})(x-x_b)/(x_c-x_b))^n$, the following integral should be minimized to obtain the optimal power n,

$$I = \int_0^1 \frac{\sqrt{1+\left(\dfrac{Z^{1/n}-1}{W}\right)^2 n^2\left(1+(Z^2-1)\xi\right)^{2(n-1)}}}{\left(1+(Z^{1/n}-1)\xi\right)^{9n/4}}d\xi \tag{4}$$

where $Z=h_c/h_b$, $W=(x_c-x_b)/h_b$, and $\xi=(x-x_b)/(x_c-x_b)$. The optimal value of n (n_o) is a function of h_c/h_b and $(x_c-x_b)/h_b$ and may be found from solving the equation $dI/dn=0$. Thus, the EP shape depends on the location of the boundaries and these have to be specified before the shape can be predicted. The breakpoint location can be calculated from the incident wave height, whereas the seaward limit of the profile experiencing significant dissipation is more difficult to estimate. Fig. 2 displays how n_o varies with h_c/h_b and the mean offshore slope $\tan\beta=(h_c-h_b)/(x_c-x_b)$. In all cases with realistic values on these parameters, n_o is markedly lower than the value 2/3 that is often found for the surf zone. Thus, the predicted EP shape under non-breaking waves has a lower curvature than the EP shape under breaking waves.

Microscale Formulation

Madsen (1991) derived a formula for the instantaneous bedload sediment transport rate based on a detailed description of the physical processes controlling the transport in the offshore zone. Under certain conditions this formula will produce zero net transport over a wave period which implies that the beach does not experience any changes because of the transport. Only the wave motion will be considered here and there will be no attempt to include a steady current, although it is possible if the current speed can be specified. The Madsen formula always produces offshore transport for a sloping bed under purely sinusoidal waves (gravity promotes down-slope transport); thus, it is necessary to include the asymmetry of the velocity field so that a realistic balance is obtained between the tendency of onshore transport in shoaling waves and offshore transport due to gravity. The local condition for equilibrium may be expressed as (Larson et al. 1998),

$$\int_{h_b}^{h} \frac{I_p + I_n}{I_p - I_n} dh = \tan\phi_m (x - x_b) \qquad (5)$$

where I is an integral over the bottom shear stress, ϕ_m the friction angle for a moving grain, and subscripts p and n denote the periods when the shear stress is positive and negative, respectively. The EP shape can only be obtained if the shear stress integrals I_p and I_n are calculated which require a detailed solution of the flow in the bottom boundary layer. However, using a number of approximations a simple analytical solution can be derived from the above exact equation.

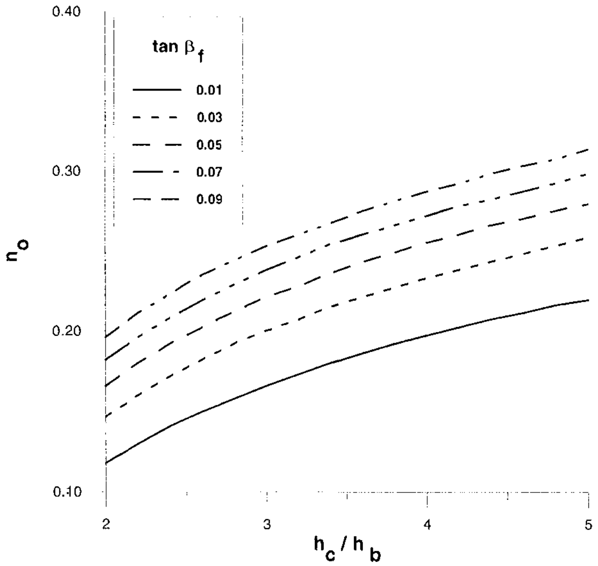

Figure 2. Dependence of the optimal power in the EP equation for the offshore on the geometric parameters.

Cnoidal wave theory was used to describe the asymmetric properties of the waves and an empirical equation was fitted to the theory to allow straight-forward calculation of these properties. Furthermore, the effect of the initiation of motion was neglected and the asymmetric waves were schematized using two sinusoidals to describe the periods when the shear stress is positive and negative. These assumptions together with a nonlinear shoaling law for the wave height yield the following EP shape for larger values of h/h_b (Larson et al. 1998),

$$\frac{h}{h_b} = \left(1 + \frac{1}{K_1} + \frac{\tan\theta_m}{K_1} \frac{x - x_b}{h_b} \right)^{\frac{1}{1+(2+p)m}} \tag{6}$$

where K_1 is a constant, p the power in the nonlinear shoaling law, and m an empirical power (about 7/5) originating from the empirical fit to cnoidal wave theory. Realistic values on p and m yields a power in the EP equation of about 0.25.

Conceptual Formulation

Instead of starting from a detailed sediment transport formula (e.g., Madsen) a conceptual approach was taken to derive the EP shape. It is assumed that the two main mechanisms that govern the profile shape in the offshore are the onshore transport due to the shoaling waves and the offshore transport due to gravity (compare Niedoroda et al. 1995). At equilibrium these two mechanisms produce equal amounts of transport and there will be no local net transport. The mean cross-shore transport rate q_c is often related to the bottom Shields stress ψ_m to a power k. To include the effect of wave asymmetry on the transport rate, a dependence on the Ursell number U_r was introduced,

$$\frac{q_c}{wd} = C\psi_m^k U_r^m \tag{7}$$

where d is the grain size and C a constant. The U_r-dependence (including the power m) is the same as was used in the previous section. The transport q_c should be balanced by the offshore transport due to gravity, which is estimated from a sediment layer with a characteristic concentration c moving offshore at the speed $w dh/dx$ (c is estimated in a similar manner to what was done in the surf zone; see Larson et al. 1998). Equating the two types of transport and introducing shallow-water approximations yields the following EP shape,

$$\frac{h}{h_b} = \left(1 + rK_2 \frac{x - x_b}{h_b} \right)^{1/r} \tag{8}$$

where $r=1+3/2k+9/4(m-1)$ and K_2 is a constant. Again, an EP shape is derived that follows a power law. The coefficient k is typically in the range 3/2 to 3, and $m=7/5$

provided a good fit to cnoidal wave theory. Thus, for $k=3/2$ the equation gives the power a value of 0.24, whereas $k=3$ produces a value of 0.15.

COMPARISONS WITH LABORATORY AND FIELD DATA

In summary, the previously derived EP shapes under breaking and non-breaking waves may be written, respectively,

$$h = Ax^m, \qquad 0 \le x \le x_b \tag{9}$$

$$h = \left(h_b^{1/n} + B(x - x_b)\right)^n, \qquad x \ge x_b \tag{10}$$

where A and B are shape parameters, and subscript b denotes the break point. The values of the powers were determined to be $m=2/3$ (compare Bruun 1954 and Dean 1977), whereas n was in the range 0.15-0.30, depending on the mechanisms assumed to control profile equilibrium. The shape parameter A has been related to grain size (or fall speed), whereas B is a function of the sediment characteristics as well as the offshore wave conditions (in the general case). Descriptions of beach profiles that involve regions governed by different predictive relationships have previously been proposed by Everts (1978) and Inman et al. (1993).

Eqs. 9 and 10 were least-square fitted towards measured profiles in the laboratory and the field to evaluate how well they are able to characterize the profile shape in the surf zone and offshore zone. The distinction between these two zones was typically made based on the presence of a nearshore bar (compare Inman et al. 1993). Only the shape parameters A and B were optimized in the fitting procedure, whereas x_b and h_b were visually determined based on the observed profiles introducing an element of subjectivity in the calculations. The value of the power n was also varied, but $n=0.3$ provided the best overall fit. This value is somewhat larger than what was typically found from the theoretical analysis, although it still within the range of realistic values. The theoretical EP models were essentially based on monochromatic (or representative) wave conditions and the effects of wave randomness were not explicitly addressed. In the following, the least-square fit was carried out some distance away from the bar region, where the effects of randomness are expected to be most pronounced. Thus, seaward of the bar mostly non-breaking waves prevail and shoreward of the bar fully broken waves dominate, implying that a monochromatic wave description should be appropriate as a first approximation.

Laboratory Data

Data obtained in the German Large Wave Tank were employed to evaluate how well the derived EPs could describe measured profile shapes at near-equilibrium conditions (Dette et al. 1998). The tests used here for comparison involved random waves according to a TMA spectrum with an $H_{mo}=1.2$ m and $T_m=5$ sec. In total data from three tests were used and the experimental conditions were similar in all tests except for the foreshore slope. In all cases, both in the surf zone and offshore zone, it was possible to obtain a close fit between the equations and the data (one example given in Fig. 3). The estimated A-value in the surf zone (A=0.12 $m^{1/3}$) was in good agreement with what is to be expected for the grain size employed (median grain size 0.33 mm). The B-values varied somewhat more (0.44-0.78 $m^{7/3}$), which partly depended on the amount of material deposited in the bar area. A larger amount implied a steeper seaward bar face and the need for a larger B-value to properly fit both the steep bar face and the more gently sloping profile seaward of the bar.

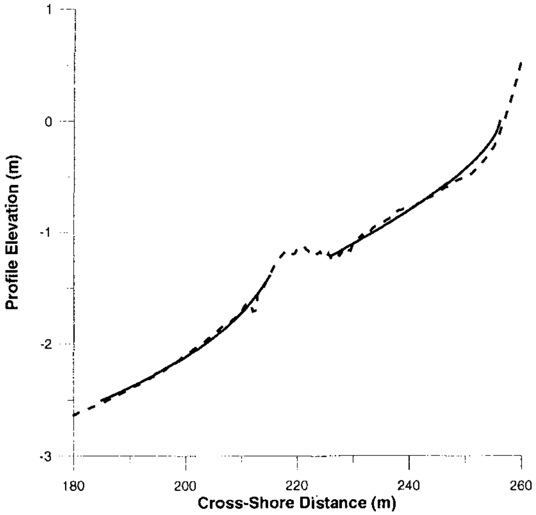

Figure 3. Comparison between theoretical and measured EPs from Test B2 in the German Large Wave Tank.

Field Data

Profile measurements from several locations around the United States were employed to test the derived EP shapes for field conditions. Data sets from Ocean City, Long Island (Fire Island, Westhampton Beach, and Ponds), Cocoa Beach, and Silver Strand were employed in this comparison representing a wide variety of wave and beach conditions. In most cases a more or less clear breakpoint bar was present along the profiles that provided a natural separation point between the surf zone and offshore zone. As an example, Fig. 4 displays a fit towards a profile measured at Silver Strand (Larson and Kraus 1992), where the estimated parameter values were A=0.21 $m^{1/3}$ and B=12.3 $m^{7/3}$. The obtained A-parameter value is somewhat high considering the grain size at the site; however, the profile shape near the shoreline reveals a feature indicative of accretion that complicates the picture.

In Fig. 4 the bar feature was quite suppressed and the two EPs could be joined at a matching point without too much deviation from the measured profile around this point. This was not always the case, but frequently a pronounced bar feature appeared that produced a region where it was not possible to obtain an acceptable fit. Fig. 5 illustrates one such example from Ocean City (Stauble et al. 1993), where a large bar was present creating a region where the composite EPs fail to describe the shape. On the contrary, extrapolating the EPs so that they intersect at a match point could be a useful way of defining the bar feature. The shape parameters for the fit described in Fig. 5 were A=0.14 $m^{1/3}$ and B=3.7 $m^{7/3}$.

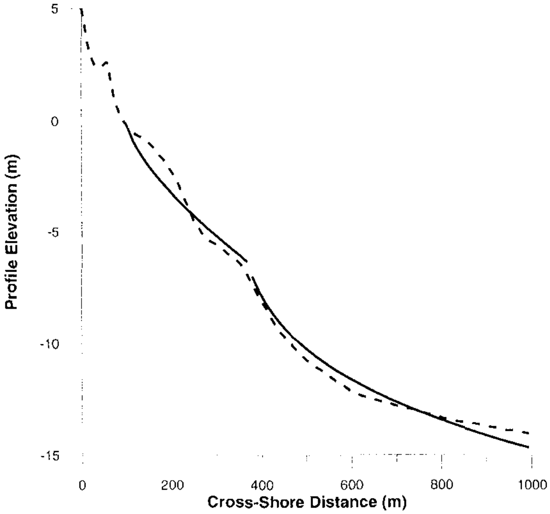

Figure 4. Comparison between theoretical and measured EPs from Silver Strand, USA.

As indicated by the profile fits presented in Figs. 3-5 the optimal value of the shape parameter B varied substantially between the studied surveys. In order to usefully employ the composite EP for predictive purposes, a formula is needed that will yield B as a function of the governing factors (A is predicted from grain size according to Moore (1982) or Dean (1987)). A closer examination revealed a distinct correlation between h_b and B (see Fig. 6), implying that knowledge of the depth at the location where wave breaking typically occurs can be used to compute B. Several different empirical relationships were developed, encompassing both linear and power-type functions (see Fig. 6), each explaining about 70% of the variation in the data (in total 41 profiles from the above-mentioned sites were used in the fit). The following equation is dimensionally consistent and results if the EPs (Eqs. 9 and 10) are scaled with h_b:

$$B = 0.142 h_b^{7/3} \quad (11)$$

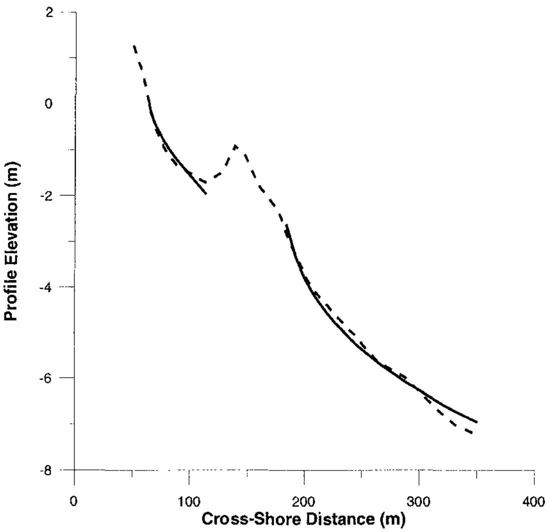

Figure 5. Comparison between theoretical and measured EPs from Ocean City, USA.

The rational between Eq. 11 is that the typical breaking wave height (and, thus, water depth at breaking) is the main quantity that scales profile behavior allowing for intercomparison between profile shapes at different sites. Eq. 11 may be regarded as a first-order approximation to compute the shape parameter controlling the EP in the offshore for profiles with one more or less well-developed bar. It should be pointed out that the analysis presented here of the field profiles contains some amount of uncertainty, not only concerning objectively determining the fitting parameters, but also regarding how close the measured profiles were to equilibrium conditions at the time of survey.

CONCLUDING REMARKS

One theoretical model to calculate the EP under breaking waves and three models to calculate the EP under non-breaking waves were briefly discussed in this paper. The theoretical models were derived based on descriptions of wave and sediment transport processes, providing physical justification for the equilibrium conditions. The approach for breaking waves resulted in a power function with a value on the power of 2/3, which is in accordance with existing field data and Dean's formulation of equilibrium based on constant energy dissipation per unit water volume across the profile. The first formulation of equilibrium for non-breaking waves was based on an assumption about the overall behavior of the offshore profile shape regarding the dissipation of wave energy, whereas the other two formulations relied on assumptions about the predominant sediment transport mechanisms controlling the EP shape. For the two latter models, one approach started from a micro-scale sediment transport formula, whereas the other approach encompassed a conceptual description of the processes governing sediment transport in the offshore. A power function was obtained for all three approaches with the value of the power being around 0.25. Thus, these models confirm the common observations that the EP shape in the offshore has a lower curvature than in the surf zone.

Comparisons with field data showed that the theoretically derived composite EP could well describe the measured profiles over a wide range of water depths. However, in some cases, when a marked bar was present along the profile, the composite EP failed to accurately describe the shape in the bar region. The simple theoretical EP models derived here were not designed to resolve the complex flow and sediment transport conditions in the bar region, but mainly to describe the profile behavior under a steady bore propagating shoreward or a purely oscillatory wave in the offshore. It might be possible to heuristically include the bar region by some additional assumptions regarding the equilibrium conditions in this region.

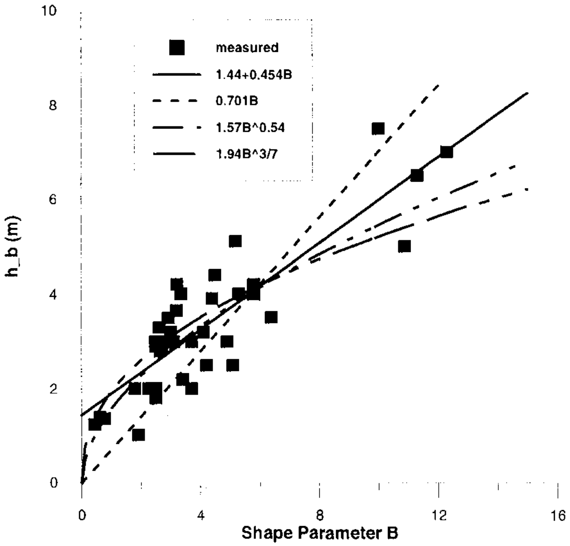

Figure 6. Relationship between the shape parameter for the EP in the offshore and the depth at breaking.

Two of the approaches to calculate the EP shape under non-breaking waves were based on balances between the onshore and offshore transport. If this balance is not fulfilled there will be a net sediment transport and the magnitude of this transport is determined as the difference between the onshore or offshore rate. Although the calculation of the net transport is straight-forward, it might be advantageous to formulate the local transport rate in terms of a deviation from the equilibrium shape. In the case such a formulation is employed, EP shapes obtained from laboratory and field data can be used to determine some of the unknown coefficients in the transport formulas (or eliminate the coefficients completely from the formulas).

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