

## Abstract

*In the design of coastal structures, the choice of the safety level for which the structure has to be designed is a major problem. This is also the case for vertical breakwaters. This paper applies the concept of economic optimisation to derive the appropriate safety level and at the same time the optimal geometry. Application to a design case shows that it can be economically optimal not to distribute the acceptable failure probability equally over all failure modes, but rather let one or two failure modes determine the total probability of failure.*

## Introduction

In Europe the interest in and the importance of vertical breakwaters is growing. A central point is the optimal geometry, e.g. a ratio of the width and height of a vertical breakwater in the sense that the total lifetime costs are minimised. For a given safety level it is possible to choose the width and height of the breakwater in such a way that the construction costs are minimised. In practice however one has to determine the optimal level of safety.

In general there are two boundary conditions for the acceptable safety level:

- The individual acceptable risk. The probability accepted by an individual to die in case of collapse of a structure;
- The societal acceptable risk. The probability of occurrence of a certain number of casualties in case of collapse of a structure.

In addition to these limits, it is possible in some cases to derive the optimal probability of failure based on an economic analysis. In the case of a breakwater without amenities the probability of loss of life due to failure is very small, but the economic losses can be severe. Therefore an economical point of view for optimising the structures design is suitable and sufficient.

In this paper the concept of economic optimisation is applied to a fictitious design case of a vertical (caisson) breakwater. The relation between a full probabilistic optimisation procedure and the simpler approach of minimising the construction costs for a given safety level is shown. It is also shown that the system probability of failure of an optimal designed breakwater is largely determined by only

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one or two failure modes. Wave transmission imposes a constraint on the caisson height.

Van Dantzig (1956) was the first to apply economic optimisation for the determination of the optimal safety level. He applied the method to derive the optimal height of the dike protecting the central part of the Netherlands. A wide variety of applications is found nowadays, primarily in engineering and economics. However, the only application to vertical breakwaters known to the authors is by Burcharth et al (1995). In the paper by Burcharth a vertical breakwater is optimised using an objective function which assumes the caisson costs proportional to the caisson weight. The only design variable considered in Burcharth's paper is the caisson width.

In this paper, a more realistic caisson cross-section consisting of a concrete floor and cap and a mixed sand/concrete body is used. The objective function consists of a part that describes the construction costs and a part that describes the expected costs of failure. The construction costs are described as a function of the volumes of sand, concrete and rubble stone in the breakwater cross section. The risk part contains a part that describes damage due to serviceability limit states (SLS) and a part that describes damage due to ultimate limit states (ULS). As design variables the caisson height, the caisson width and the height of the rubble foundation are chosen in order to find the optimal breakwater geometry.

Practical experience shows that very often the system probability of failure of a structure is determined by a single failure mode (weak link). This paper will show that it is economically optimal to have such weak links in the design of a vertical breakwater.

### Economic Optimisation

An economic optimal design is defined as the design for which the total lifetime costs are minimal. The total lifetime costs consist of the construction costs and the expected value of the damage costs. In case of a vertical breakwater the total lifetime costs are a function of:

- The vector of design variables ( $\bar{x}$ )
- Initial costs, not depending on the design variables ( $I_0$ );
- Construction costs as a function of the design variables ( $I(\bar{x})$ );
- Costs per day in case of serviceability failure ( $C_{SLS}$ );
- The probability of serviceability failure per day ( $P_{f,SLS}(\bar{x})$ )
- Costs per event in case of ultimate limit state failure ( $C_{ULS}$ );
- The probability of ultimate limit state failure per year ( $P_{f,ULS}(\bar{x})$ )
- Maintenance costs for the breakwater per year ( $C_{main}$ );
- The net interest rate per year ( $r$ );
- The yearly rate of economical growth, expressing growth and development of the harbour ( $g$ );
- The lifetime of the structure in years ( $N$ ).

In this paper the maintenance costs have been neglected.

The optimal set of design variables is found by minimising the cost function:

$$I(\vec{x}) = I_0 + I(\vec{x}) + \sum_{n=1}^N \left( \frac{365 C_{SLS} P_{f,SLS}(\vec{x}) + C_{ULS} P_{f,ULS}(\vec{x})}{(1+r'-g)^n} + \frac{C_{maint}}{(1+r')^n} \right) \tag{1}$$

Several minimisation procedures have been developed which can be used to minimise function (1) (Press et al, 1990). For the calculation of the failure probabilities also several methods exist (Ditlevsen and Madsen,1996). Some points to consider in the choice of the algorithms are mentioned in the remainder of this paper.

The Design Case

In this paper, a fictitious design case has been considered. A caisson breakwater has to be designed for a water depth of 25 m with respect to mean sea level (MSL). The subsoil consists of sand. An overview of the conceptual design cross section is given in Figure 1. The exact sizing of all elements in this design cross section will be derived by economic optimisation.

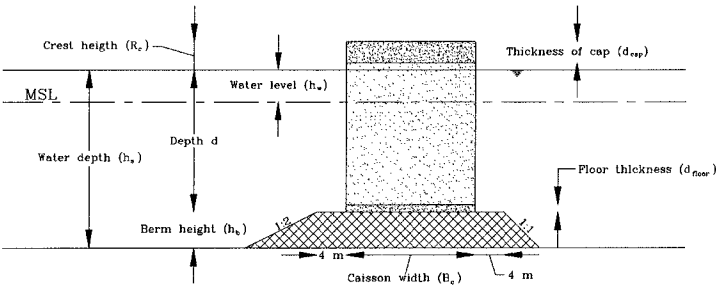


Figure 1: Overview of breakwater cross section

The total length of the breakwater is assumed to be 6000 meters. The height and width of the caisson as well as the berm height are chosen as design variables. The height of the concrete cap and floor are kept constant. The cap and floor consist of concrete only. The zone between cap and floor consists of filling sand and concrete. In the optimisation procedure the number of walls inside the caisson is unknown. A fixed percentage of concrete of 10 % has been used in the weight and cost calculations. The costs of concrete, filling sand and rubble are estimated by practising engineers. An overview is given in Table 1.

| Material     | Costs [\$/m <sup>3</sup> ] |
|--------------|----------------------------|
| Concrete     | 250                        |
| Filling sand | 5                          |
| Rubble       | 70                         |

Table 1: Overview of material costs

Due to these cost figures and the properties of the breakwater cross section, it is more expensive to increase the width of the caisson then the height of the caisson. Increasing the caisson width results in a larger volume of the concrete cap and floor

which results in a faster increase of the concrete volume in comparison to an increase of the caisson height.

In case of failure of the breakwater, the damage will consist of structural damage to the breakwater itself and economical damage due to the interruption of harbour processes. In this study for the damage in monetary terms, figures have been used which originate from a similar study for a rubble mound breakwater (Delft University of Technology, 1995).

| Failure type | Damage description          | Damage amount (US \$)                                |
|--------------|-----------------------------|------------------------------------------------------|
| SLS          | Economic damage per day     | 750,000                                              |
| ULS          | Structural damage per event | 9,000,000 + 20 % of construction costs of breakwater |
|              | Economic damage per event   | 555,000,000                                          |

Table 2: Overview of damage costs

All damage is considered from the viewpoint of the harbour authorities. Economic damage denotes the economic damage due to interruption of harbour operations, loss of port dues, claims and costs of alternative transport. Structural damage consists of a (fixed) part that denotes damage to harbour inventory and a (variable) part denoting the damage to the breakwater itself.

#### Boundary Conditions and Wave Force Model

The boundary conditions for the breakwater consist of hydraulic boundary conditions (wave height, water levels) as well as of geotechnical boundary conditions (friction angles and densities). An overview of the relevant boundary conditions and the assumed distributions is given below.

| Variable                                   | Description                                                   | Distribution type | Shift                | Scale                 | Shape | Power |
|--------------------------------------------|---------------------------------------------------------------|-------------------|----------------------|-----------------------|-------|-------|
| <b>Yearly hydraulic conditions</b>         |                                                               |                   |                      |                       |       |       |
| $H_s$                                      | Significant wave height                                       | Gumbel            | 3 m                  | 0.25 m                | -     | 1     |
| $s$                                        | Wave steepness                                                | Normal            | 5 %                  | 1 %                   | -     | 1     |
| $R$                                        | Ratio between significant wave height and maximum wave height | Rayleigh          | -                    | 1                     | 2     | 3000  |
| $h_w$                                      | Water level with respect to MSL                               | Weibull           | 2.2 m                | 0.8 m                 | 2     | 1     |
| <b>Daily hydraulic conditions</b>          |                                                               |                   |                      |                       |       |       |
| $H_s$                                      | Significant wave height                                       | Gumbel            | 1.5 m                | 0.25 m                | -     | 1     |
| $h_w$                                      |                                                               | Normal            | 0 m                  | 1 m                   | -     | 1     |
| <b>Subsoil properties</b>                  |                                                               |                   |                      |                       |       |       |
| $\phi_{\text{subsoil}}$                    | Friction angle of subsoil                                     | Normal            | 35°                  | 1.8°                  | -     | 1     |
| $\delta$                                   | Friction angle between rubble and caisson bottom              | Normal            | 30°                  | 1.5°                  | -     | 1     |
| $\phi_{\text{rubble}}$                     | Friction angle of rubble                                      | Normal            | 40°                  | 2°                    | -     | 1     |
| $\gamma_{\text{rubble}}$                   | Density of rubble                                             | Deterministic     | 21 kN/m <sup>3</sup> | -                     | -     | -     |
| $\gamma_{\text{subsoil}}$                  | Density of subsoil                                            | Deterministic     | 21 kN/m <sup>3</sup> | -                     | -     | -     |
| <b>Properties of caisson cross section</b> |                                                               |                   |                      |                       |       |       |
| $\gamma_{\text{fill}}$                     | Density of caisson fill                                       | Normal            | 18 kN/m <sup>3</sup> | 1.8 kN/m <sup>3</sup> | -     | 1     |
| $\gamma_{\text{concr}}$                    | Density of concrete                                           | Deterministic     | 24 kN/m <sup>3</sup> | -                     | -     | -     |

Table 3: Overview of boundary conditions

The wave loading on the caisson is calculated by means of the method of Goda (1985) which is extended by Takahashi (1996) to include impact conditions.

Failure Modes

In this study a total of six failure modes have been implemented in the optimisation procedure. These six failure modes are:

- Wave transmission (SLS);
- Sliding of the caisson over the rubble foundation (ULS);
- Exceedance of the maximum allowable eccentricity of the resultant vertical force (ULS). The eccentricity has to be limited to ensure sufficient rotation capacity for the other ULS failure modes to be valid;
- Straight sliding plane through the rubble foundation (ULS);
- Sliding of rubble foundation over the subsoil (ULS);
- Failure of the subsoil (ULS).

Wave transmission effectively describes the functionality of the breakwater. It is included by applying the transmission model of Goda (1969) and providing an acceptable significant wave height in the harbour basin.

The last five failure modes describe different forms of foundation failure. These models are taken from a report written under the European Marine Science and Technology program (de Groot et al, 1996). In the report two sets of "feasibility level models" are given. One set is applicable to caissons placed on low rubble mounds and the other is applicable to caissons placed on high rubble mounds. De Groot provides no clear definition of low and high rubble mounds. Furthermore, in an optimisation process all options are open and therefore the behaviour of all alternatives should be accurately described. Therefore, the set of failure modes in this study consists of the union of the two sets given in de Groot.

Observation of the list of failure modes and the cost function shows that there are two main sources of damage, i.e. damage due to excessive wave transmission and damage due to instability of the caisson. Instability of the caisson is the result of a series system containing all the ULS failure modes mentioned above. In a fault tree, instability of the caisson is described as in Figure 2.

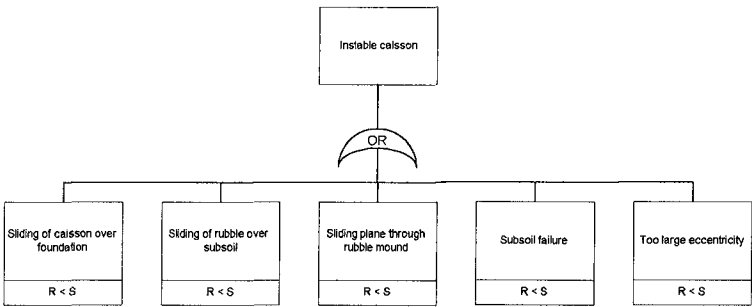


Figure 2: Fault tree for ultimate limit states

Several methods are available to provide reliability estimates for this kind of systems of failure modes. The choice of the algorithm is not an arbitrary one, as will be seen later.

### Deterministic Optimisation of the Caisson Design

If all input is treated deterministically, it is possible to derive the required width of the caisson as a function of the crest height of the caisson. Since the construction costs are a function of the height and width of the caisson, the construction costs can be minimised for a given design wave height and water depth, resulting in optimal caisson dimensions. Analysis of this case is useful since it is an integral part of the full probabilistic optimisation, as will be shown later. The disadvantage of this procedure is that the optimal design is still dependent on the choice of the (deterministic) design conditions and is therefore in fact still open.

The deterministic optimisation procedure has been applied to a breakwater design according to Figure 1, using design wave heights of  $H_s = 3.74$  m for SLS and  $H_d = 9.64$  m for ULS. The design variable berm height has been fixed to 6 m. The required caisson width is expressed as a function of the caisson height for each limit state. The construction costs have been calculated for every design alternative, resulting in Figure 3.

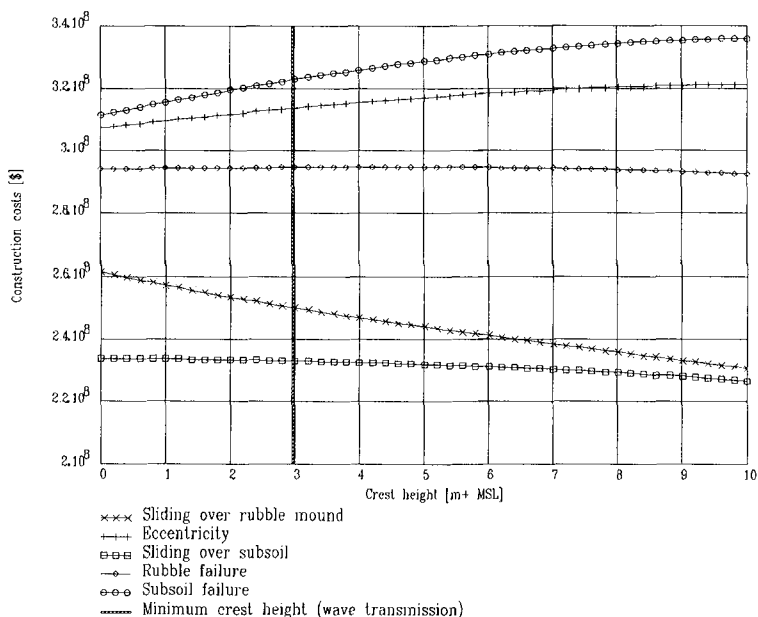


Figure 3: Construction costs as a function of the crest height of the breakwater (berm height 6 m)

Subsoil failure requires the largest caisson width in all cases and thus decides the construction costs. Assuming an acceptable significant wave height behind the breakwater of 0.50 m, wave transmission imposes a constraint on the crest height. This constraint is shown in Figure 3 by the vertical line. This shows that the function of the breakwater (in this case described by wave transmission) is an essential part of the optimisation of the breakwater design.

Probabilistic Optimisation and its Relation to Deterministic Optimisation

In the previous section the optimal breakwater design was determined for chosen design wave heights. However, also the design wave heights (significant wave height for SLS and design wave height for ULS) can be made subject to economic optimisation by specifying them as distribution functions (all other input is still treated deterministically). This approach can be considered an intermediate step between the deterministic optimisation of the previous section and the full probabilistic approach of the next section.

Using the fault tree (figure 2) it is possible to calculate the probability of caisson instability due to any of the given failure modes. Wave transmission is treated separately, resulting in an ultimate significant wave height which indicates when excessive wave transmission occurs. Since in this case the daily and extreme wave heights are the only random variables, substitution of the ultimate wave heights in their respective distributions results in estimates of the failure probabilities. The failure probabilities are used in equation (1) to derive the expected value of the damage costs. Following this procedure for several caisson height and width combinations provides the total lifetime costs as a function of the caisson dimensions. Figure 4 shows a contour plot of the total lifetime costs for a breakwater using a berm height of 6 m.

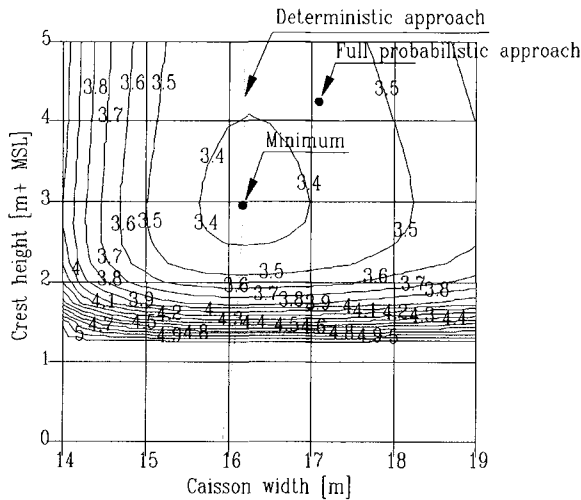


Figure 4: Contour plot of total lifetime costs (random wave height only, costs in 10<sup>8</sup> US \$)

In the previous section it was shown that for fixed values of the berm height, water depth and design wave heights, it is possible to derive the required caisson width as a function of the caisson height. The result of this deterministic approach is also shown in Figure 4 as the dotted line. The optimal design found in the probabilistic procedure lies exactly on this curve. This shows that minimisation of the construction costs for a fixed probability of failure is included in the probabilistic optimisation procedure. In figure 4 the point "full probabilistic approach" denotes the result of the full probabilistic optimisation, described in the next section.

In the probabilistic procedure, wave transmission causes a sharp increase of the lifetime costs with decreasing crest height, rather than a fixed constraint. This effect is also visible in Figure 4.

### A Procedure for Full Probabilistic Optimisation

In the previous section the probability of failure of the breakwater was considered equal to the probability of exceedance of a certain wave height. In practice however the failure probability depends on several parameters which might show random behaviour. Optimising a breakwater design taking into account several uncertainties requires a numerical optimisation procedure, which consists of the following components:

- An algorithm for the minimisation of functions in several dimensions;
- A numerical description of the cost function;
- A procedure for the calculation of the system probability of ULS failure;
- A procedure for the calculation of the probability of SLS failure.

Figure 5 shows a scheme of the co-operation between the main components.

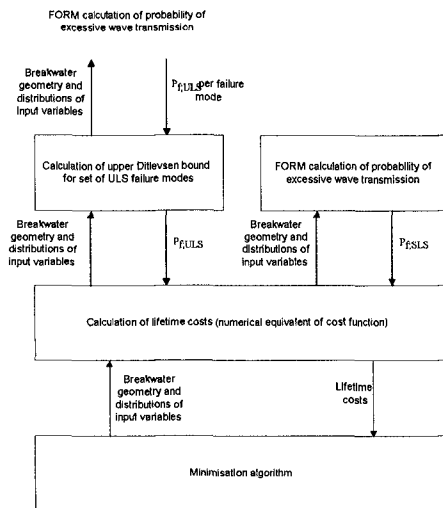


Figure 5: Structure of optimisation process

Minimisation algorithms can with advantage be obtained from several sources (See for instance: Press et al, 1990). The choice of the algorithm should be made with care. In general, algorithms that use derivatives of the function are the most efficient. Application of such an algorithm requires the function to be continuously differentiable. In this case the deterministic optimisation has shown that in most cases the optimal design is found exactly in the intersection point of two failure modes. Therefore, the derivatives can be expected to be discontinuous and the use of them in the minimisation procedure has therefore been avoided. The direction set method developed by Powell and implemented by Press et al (1990) has been used. The minimisation procedure is the central part of the optimisation procedure since it controls all the calls made to the cost function and therefore also all the calls to the probabilistic method.

The numerical description of the objective function is the equivalent of formula (1) expressed in programming language. The cost function needs the failure probabilities as input. To obtain the failure probabilities the procedure makes a call to a probabilistic algorithm and passes all relevant input to this procedure.

Several methods for the calculation of the system probability of failure exist (See for instance: Ditlevsen and Madsen, 1996). For application in an optimisation program the algorithm has to fulfil two requirements:

- The algorithm should provide reliability estimates in a relatively short time;
- The resulting estimates of the failure probabilities have to be stable, meaning that recalculation of the failure probability for the same geometry should yield a result that only differs in the range of numerical inaccuracies.

The first requirement is important because of the large number of reliability calculations that are to be made in the course of the optimisation procedure (typically 100 to 500). A probabilistic procedure that takes long calculation times will slow down the optimisation too much.

The second requirement is necessary because of the co-operation between a minimisation procedure and a probabilistic procedure. Especially Monte Carlo methods provide reliability estimates that slightly vary from calculation to calculation, even for the same dimensions of the breakwater. This variation is inherent to the Monte Carlo method and presents no problem if the reliability estimates are not used in an optimisation procedure. When using Monte Carlo estimates in an optimisation, the slightly varying failure probabilities cause variations of the cost function, which cause convergence problems for the minimisation procedure.

In this paper first order reliability methods have been used to calculate the Ditlevsen bounds of the system probability of failure (Hasofer and Lind, 1974; Ditlevsen, 1979; Hohenbichler, 1983). The upper bound of the failure probability according to Ditlevsen is used in the cost function as the system probability of ULS failure. The probability of serviceability failure is calculated by means of a first order reliability method.

### Results of Full Probabilistic Optimisation

The procedure described in the previous section has been used to derive optimal caisson dimensions for several berm heights using all input given in Table 3. The results have been used to determine an overall optimal breakwater cross section, i.e. a breakwater cross section with an optimal berm height, crest height and caisson width. Figure 6 shows the optimal total caisson height and the caisson width for several berm heights. For a berm height of 6 m the optimal width and height are 17.20 m and 23.25 m respectively. This is larger then in the case with only random wave heights due to the added uncertainty in the subsoil properties and the water level (see also figure 4).

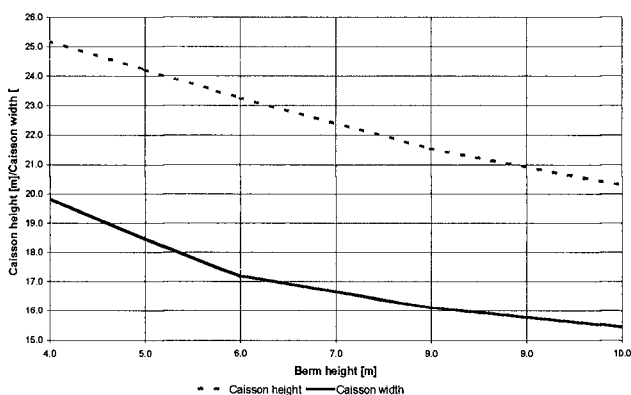


Figure 6: Optimal caisson dimensions as a function of the berm height

Figure 7 shows three alternative breakwater designs with different berm heights and corresponding optimal caisson dimensions.

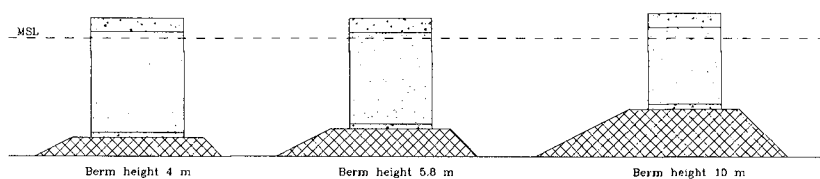


Figure 7: Optimal designs for three berm heights

Both Figure 6 and Figure 7 show that up to a berm height of approximately 6 m there is a strong decrease of the caisson width with increasing berm height. For higher berms this reduction is considerably less.

Figure 8 shows the failure probability per failure mode as well as the overall probability of ULS failure for every calculated alternative.

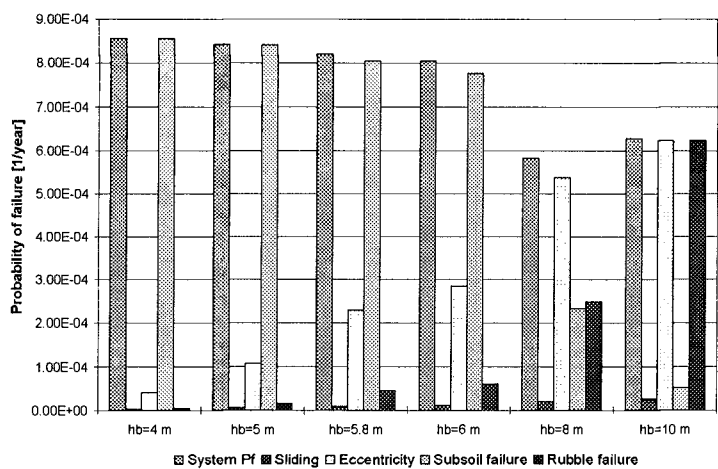


Figure 8: Overview of failure probabilities for alternative designs

Figure 8 shows that in most cases the system probability of failure is determined by one failure mode only (subsoil failure). In these cases the height is governed by wave transmission, in the way shown in Figure 4. Since there is a clearly dominant failure mode, the system probability of failure can also be accurately calculated by taking the fundamental upper bound of the failure probability instead of the Ditlevsen bound.

Figure 9 gives an overview of total lifetime costs as a function of the berm height. It should be noted that every point denotes the costs of an optimal caisson corresponding to that berm height.

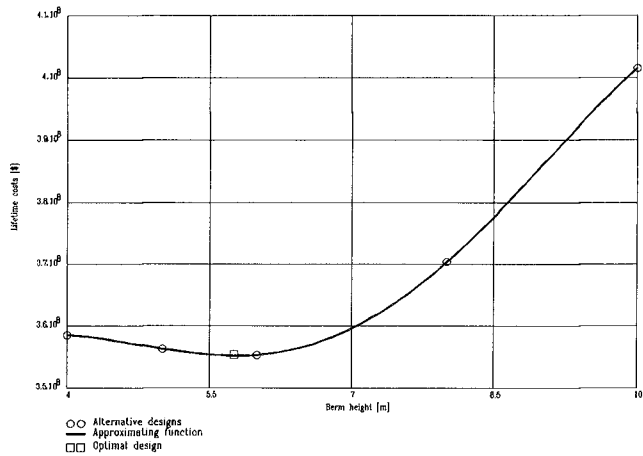


Figure 9: Lifetime costs as a function of the berm height

Apparently, the optimal design is found in the vicinity of the point in Figure 6 where the large width reduction due to higher berms is virtually over. This leads to the conclusion that in this case the only (economical) justification of the rubble foundation is a reduction of the loading on the subsoil.

The calculation results lead to the following conclusions regarding the optimal breakwater geometry for the design case:

1. The optimal design consists of a berm with a height of 5.8 m with respect to the sea bottom and a caisson with a total height of 23.5 m (crest height MSL +4.3 m) and a width of 17.5 m;
2. The optimal geometry is decided by wave transmission (SLS) and subsoil failure (ULS);
3. The system probability of ULS failure of the optimal design virtually equals the probability of subsoil failure. This indicates that subsoil failure is a very clear "weak link" in the design;
4. Due to the presence of a weak link, the system probability of failure can be accurately calculated by taking the fundamental upper bound instead of the Ditlevsen bound.

## Conclusions

This paper considers the application of economic optimisation to the design of vertical breakwaters. Regarding the procedure itself the following conclusions can be drawn:

- The concept of economic optimisation provides a rational way of supporting the choice of the optimal safety level;
- Minimisation of the construction costs of a breakwater for a given safety level is an integral part of the full probabilistic optimisation procedure.

Regarding the optimal geometry for the design case, the following conclusions are justified:

- The crest height of the optimally designed caisson breakwater is determined by wave transmission;
- The system probability of ULS failure virtually equals the probability of subsoil failure. Subsoil failure is therefore a very clear "weak link" in the optimal design. The optimisation procedure shows that it is not economical to distribute the acceptable probability of failure equally over all failure modes;
- The only (economical) justification of a rubble berm in this case is reduction of the loading of the subsoil, thus decreasing the probability of failure of the structure.

Regarding the implementation of the procedure in programming language, the following points are important:

- Ready-at-hand minimisation algorithms can with advantage be used;
- Minimisation algorithms which use derivatives of the cost function should be avoided;

- The probabilistic algorithms to be implemented in the optimisation procedures should provide stable estimates of the failure probabilities, in order to avoid convergence problems of the minimisation algorithm. Stability in this case meaning that a repeated calculation for the same geometry should provide a result for the probability of failure that only differs within the range of numerical accuracy.

### Acknowledgement

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