Transformation of wave groups and accompanying long waves in shallow water

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Abstract

Interaction of short waves modulation and long waves evolution is studied. Existence of free long waves is taken into account. Laboratory experiment and numerical simulation with coupled equations are performed. Mechanism of the modulation is discussed. The distance, below which the modulation may be neglected, is evaluated.

Introduction

Figure 1 shows an example of wave record obtained in Hasaki pier, PHRI, MOT in Japan. Strong grouping of short waves is present. Long period component is filtered out numerically with cut-off frequency 0.04 Hz. Magnitude of the long waves is 1/10 of wind waves. Bounded long waves to grouping waves may be evaluated by applying the following Longuet-Higgins and Stewart (1962) solution.

$$\eta_b=-Sxx(x-c_gt)/\rho(gd-c_g^2) \qquad (1)$$

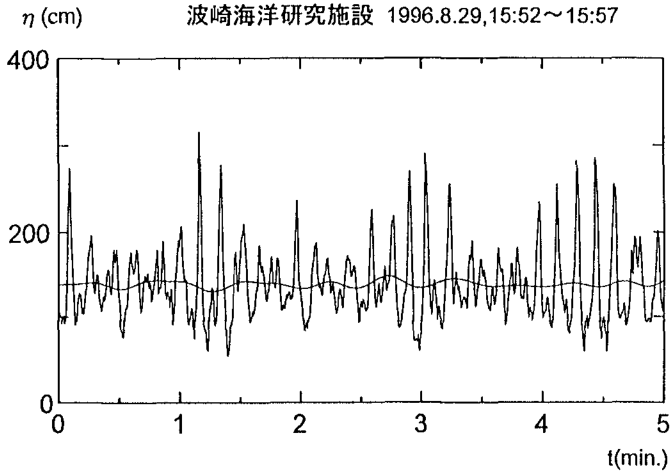

Fig. 1 Long waves in the field (where water is shallow and about 5m deep)

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where η_b is the surface profile of the bound long waves, Sxx is the radiation stress of short waves, c_g is the group velocity of short waves, d is the water depth. Rough calculation of Eq.(1) gives 1.0 m in terms of wave height for η_b, with d=5 m, wave period T=10 s, maximum amplitude of short waves a_{max}=1.1 m, minimum amplitude a_{min}=0.3 m. Observed long waves are much smaller than that of bound long waves as is widely pointed out.

Why is the observed long waves much smaller than that of bound long waves. A very plausible explanation may be found in Nagase and Mizuguchi(1996) that free long waves, being generated simultaneously while the bound waves developes in the process of shoaling, nearly cancels out the bound one. Then long waves observed is much smaller than the bound one unless those two are separated well as shown in Fig. 2.

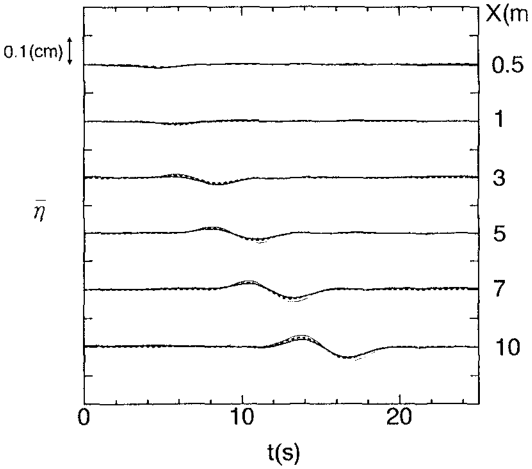


Fig. 2 Long waves accompanying a group of waves (Mizuguchi and Toita, 1996) X is the distance from the wave maker. Thick solid line: experiment, broken line; first-order theory. For information on short waves, see the reference.

In Fig. 2, no significant long waves is observed near the wave maker. Free long waves, which is needed to satisfy the boundary condition at the wave maker, is c_g/c times the bound waves, where c is the phase speed of the long waces. As they travel, long waves appear to be growing both in experiment and in theory. Free long waves, generated at the wave maker, propagate faster than the bound waves (or set-down waves) and start to separate each other. The separation can be observed only for this kind of single group of waves.

In this figure, one can also notice small difference between the experimental results and the theoretical one growing with the distance from the wave maker. The purpose of present study is to examine this difference. This difference is of second-order and very small in Fig. 2, but may be significant in some situation and worth to be investigated. The first-order theory assumes that short waves propagates with no change of group form. Previous nonlinear theory of wave modulation does not take into account of the free long waves. No systematic experimental investigation has not yet published, either.

Laboratory experiment

First we conducted a series of laboratory experiments to see how significant the change of wave group (or wave modulation) is. Figure 3 shows our experimental setup. Wave flume is 40m long and 30 cm wide and equipped with a piston-type wave maker.

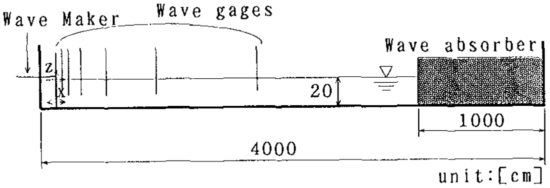


Fig. 3 Wave flume and experimental setup

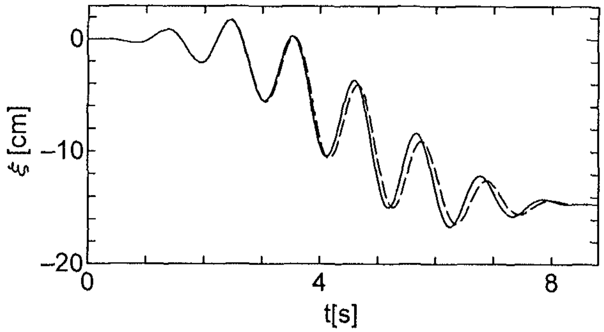


Fig. 4 An example of signal for wave maker (Case 2)
Solid line is calculated by Eq.(2) and broken line after Mizuguchi and Toita(1996).

We use single group of short waves for its simplicity as in Mizuguchi and Toita(1996). An example of wave maker signal is shown in Fig. 4. Displacement of wave maker ξ is obtained by numerically integrating the following equation.

$$d\xi/dt=u_w(0,t)+u_l(0,t)+\xi_l\partial u_w/\partial x \qquad (2)$$

where u_w and u_l denote horizontal velocities of short waves and long waves respectively. u_w is given by linear wave theory and u_l is by

$$u_l=r(c_g/d)\eta_b(x,t) \qquad (3)$$

where r is the quantity to control the generation of long waves. ξ_l is the displacement of the wave maker for long wave componenct and is given by

$$\xi_l=\int_0^t u_l(0,t)dt \qquad (4)$$

The last term in Eq.(2) is newly added to compensate rather large value of ξ_l.

Ten cases are chosen as shown in Table 1. Standard case is Case 1, where generation of free long waves is suppressed by putting r=1. Finite amplitude effects may be seen in Cases 2 and 3. Effects of dispersion may be studied both in Cases 4 and 5 with different number of waves in a group and in Cases 6 and 7 with different wave period of short waves. Effects of free long waves may be observed in Cases 8, 9 and 10. Water depth is 20 cm throught the experiment.

Table 1 Experimental cases

case	a_{max}[cm]	T[s]	N_g	r	$(\eta^*_0)_{min}$[cm]	kd	ka_{max}	Us_{max}	Uss_{max}
1	2	1.1	8	1	-0.223	0.92	0.092	9.36	11.86
2	4	1.1	8	1	-0.891	0.92	0.184	18.73	23.72
3	1	1.1	8	1	-0.056	0.92	0.046	4.68	5.93
4	2	1.1	6	1	-0.223	0.92	0.092	9.36	11.86
5	2	1.1	12	1	-0.223	0.92	0.092	9.36	11.86
6	2	0.8	8	1	-0.116	1.42	0.142	3.94	6.27
7	2	1.6	8	1	-0.475	0.59	0.059	22.52	25.09
8	2	1.1	8	0	-0.223	0.92	0.092	9.36	11.86
9	2	1.1	8	-1	-0.223	0.92	0.092	9.36	11.86
10	2	1.1	8	2	-0.223	0.92	0.092	9.36	11.86

$Us_{max} = 2a_{max}L^2/d^3$, $Uss_{max} = 2ga_{max}T^2/d^2$

Analytical description

In order to discuss the expermental results, knowledge on the theoretical background is very helpful. It is known that for long waves under grouping short waves, mass continuity equation is given as

$$\partial\eta_l/\partial t+d\partial u_l/\partial x=0 \qquad (5)$$

and momentum equation is as

$$\partial u_l/\partial t+g\partial\eta_l/\partial x=-(n'-1/4)g\partial|A|^2/\partial x \qquad (6)$$

where A is the complex amplitude of short waves. For short waves, the complex amplitude A follows

$$\partial A/\partial t+c_g\partial A/\partial x+i[\alpha\partial^2 A/\partial x^2+\beta'|A|^2 A$$

$$+\{(k/d)(c_g-c/2)\eta_l+ku_l\}A]+\text{dissipation}=0 \qquad (7)$$

where

$$\alpha=-(\partial^2\omega/\partial k^2)/2$$

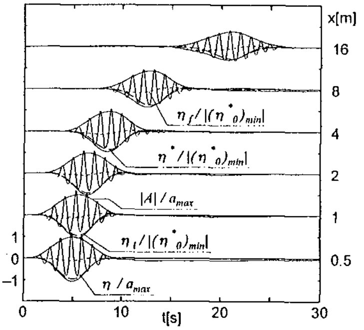
$$\beta'=(\omega k^2/16\sinh^4 kd)\{2\sinh^2 2kd(1-\tanh kd/kd)+9\}$$

Equation (7) is written explicitly for η_l and u_l. Conventional form may be found in Mei(1989). Dissipation term is modeled after Mase(1987), where dissipation due to viscosity at side walls is included in addition to that at bottom. The traditional Schrodinger-type equation for wave modulation is obtained by substituting only the forced solution of Eqs.(5) and (6), while assuming the functional form of $A(x-c_g t)$, into Eq.(7) and neglecting the dissipation term.

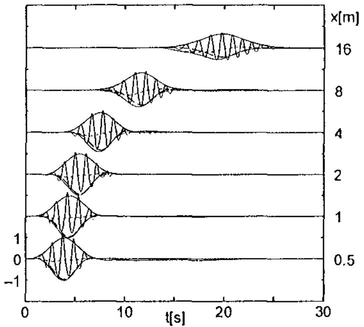
Equations (5) to (7) may describe the second-order phenomena of long wave evolution and modulation of short waves for any initial and/or boundary condition. Standard finite difference technique is employed to solve Eqs.(5) to (7) numerically. Care is needed to give the boundary condition at the wave maker to be smooth enough.

Experimental results and comparison with theories

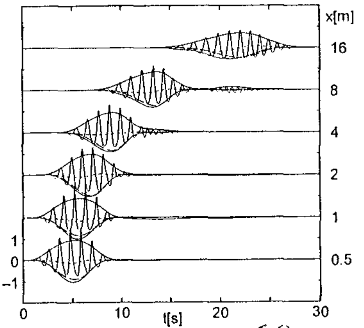
In the analysis of the laboratory data, long waves is simply filtered out numerically. Envelope of short waves is calculated by applying numerical filter to the absolute value of short wave surface fluctuation(List, 1992). Bound long



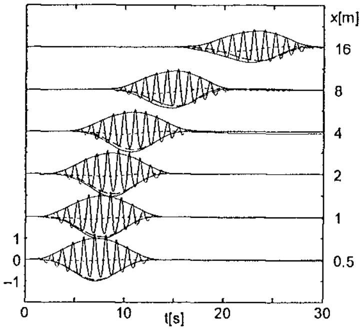
a_{max}=2cm, T=1.1s, N_g=8, r=1 (Case1)



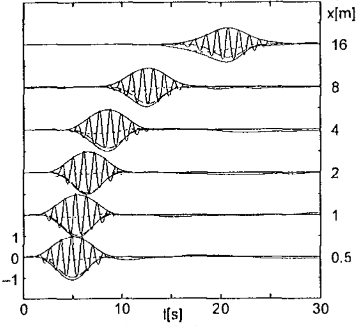
a_{max}=2cm, T=1.1s, N_g=6, r=1 (Case4)



a_{max}=4cm, T=1.1s, N_g=8, r=1 (Case2)

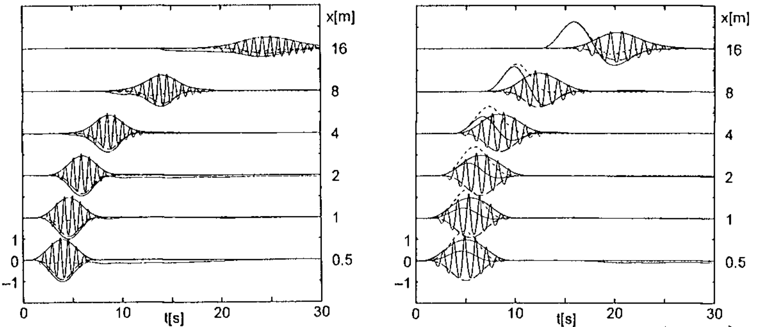


a_{max}=2cm, T=1.1s, N_g=12, r=1 (Case5)



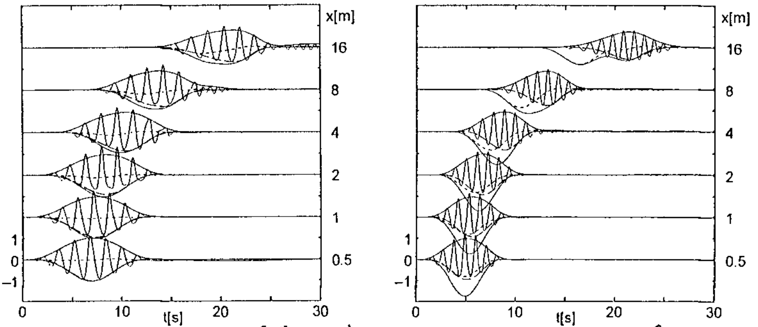
a_{max}=1cm, T=1.1s, N_g=8, r=1 (Case3)

Fig. 5a Experimental results (Case 1 to 5)



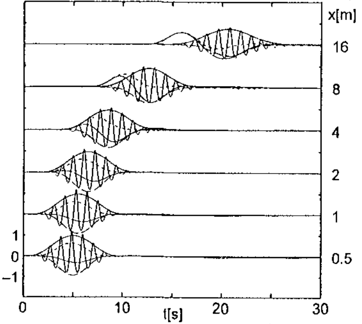
a_{max}=2cm, T=0.8s, N_s=8, r=1 ((Case 6)

a_{max}=2cm, T=1.1s, N_s=8, r=-1 ((Case 9)



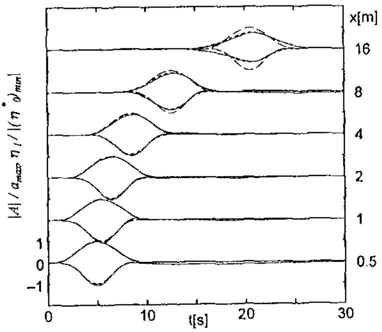
a_{max}=2cm, T=1.6s, N_s=8, r=1 (Case 7)

a_{max}=2cm, T=1.1s, N_s=8, r=2 ((Case 10)

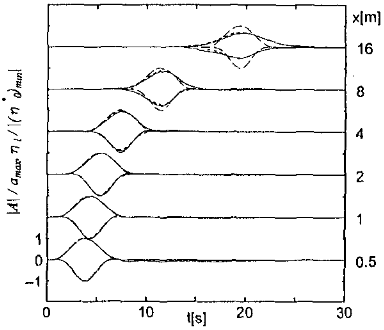


a_{max}=2cm, T=1.1s, N_s=8, r=0 ((Case 8)

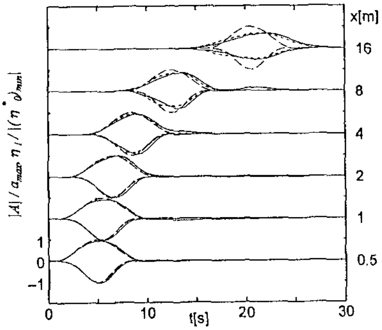
Fig. 5b Experimental results (Case 6 to 10)



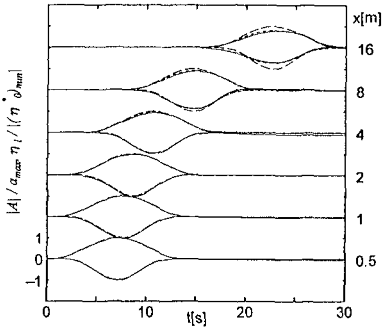
a_{max}=2cm, T=1.1s, N_s=8, r=1



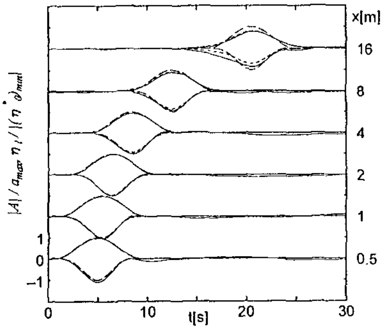
a_{max}=2cm, T=1.1s, N_s=6, r=1



a_{max}=4cm, T=1.1s, N_s=8, r=1

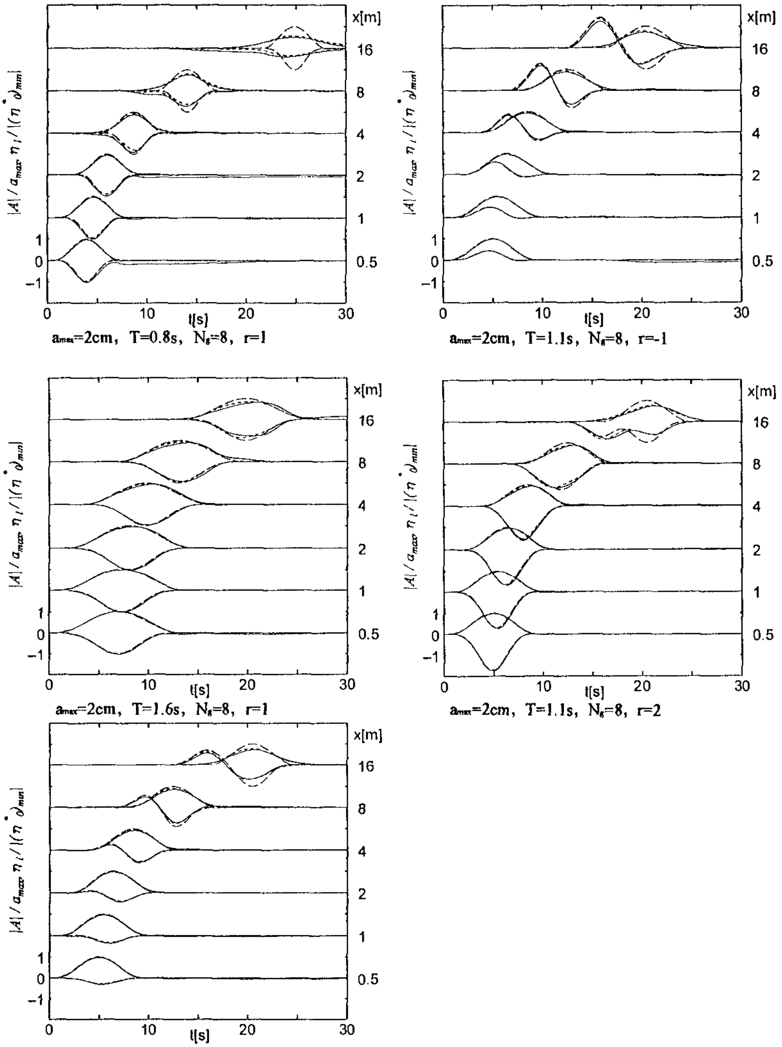


a_{max}=2cm, T=1.1s, N_s=12, r=1



a_{max}=1cm, T=1.1s, N_s=8, r=1

Fig. 6a Comparison among experiment, first- and second-order theories (Case 1 to 5). Lines in upper half of each figure show envelope profiles and theose in lower half long wave profiles.



a_{max}=2cm, T=0.8s, N_s=8, r=1

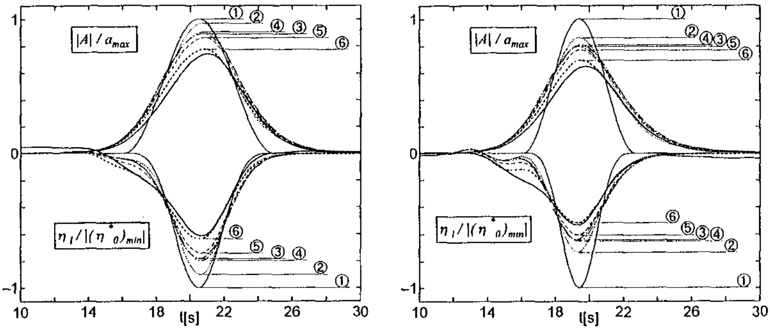
a_{max}=2cm, T=1.1s, N_s=8, r=-1

a_{max}=2cm, T=1.6s, N_s=8, r=1

a_{max}=2cm, T=1.1s, N_s=8, r=2

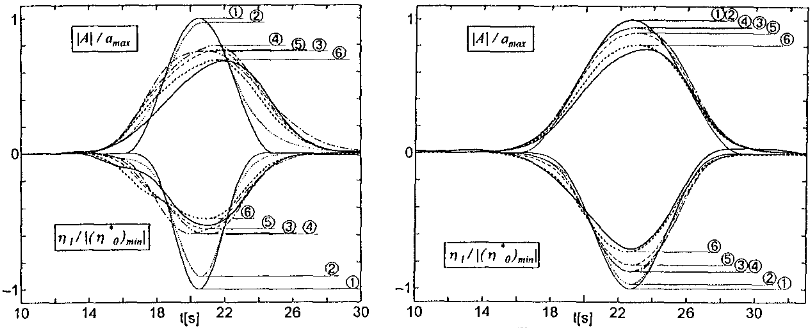
a_{max}=2cm, T=1.1s, N_s=8, r=0

Fig. 6b Comparison among experiment, first- and second-order theories (Case 6 to 10)



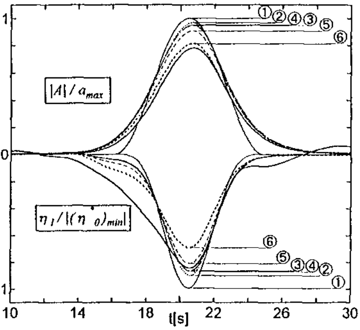
a_{max}=2cm, T=1.1s, N_s=8, r=1

a_{max}=2cm, T=1.1s, N_s=6, r=1



a_{max}=4cm, T=1.1s, N_s=8, r=1

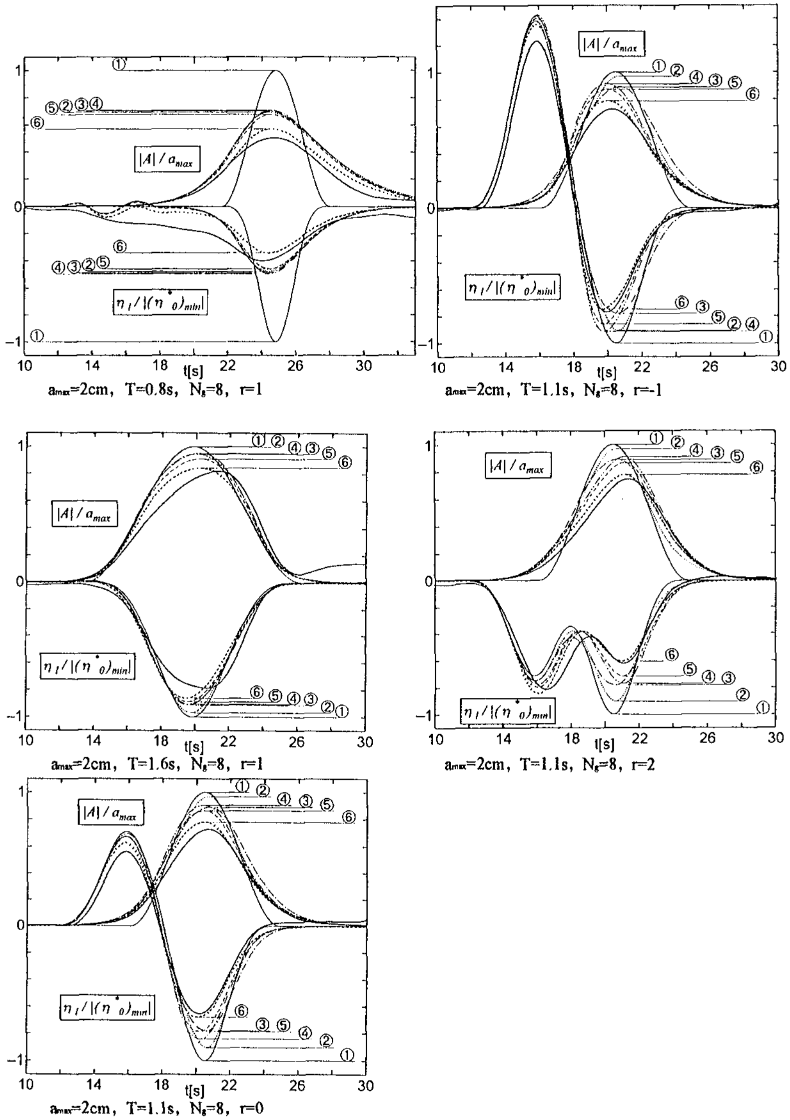
a_{max}=2cm, T=1.1s, N_s=12, r=1



a_{max}=1cm, T=1.1s, N_s=8, r=1

Fig. 7a Contribution of each term in Eq.(5) (case 1 to 5)

Lines in upper half of each figure show envelope profiles and theose in lower half long wave profiles.



a_{max}=2cm, T=0.8s, N_s=8, r=1

a_{max}=2cm, T=1.1s, N_s=8, r=-1

a_{max}=2cm, T=1.6s, N_s=8, r=1

a_{max}=2cm, T=1.1s, N_s=8, r=2

a_{max}=2cm, T=1.1s, N_s=8, r=0

Fig. 7b Contribution of each term in Eq.(5) (case 6 to 10)
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waves is calculated as LHS solution, that is by Eq.(1), with the evaluated short wave envelope. Free long waves is defined as the difference between the observed long waves and the calculated bound one.

In all cases both envelope of short waves and long wave (or set-down wave) flatten more or less as they propagate away. In Case 2, phase modulation (change of wave period of short waves) as well as amplitude modulation is clearly seen. Large velocity of bound long waves is responsible. Free second (higher) harmonics are also present, being left behind. In more linear case Case 3, change of wave group is naturally slower. In this case experimental error in long waves could be significant as its magnitude is very small. In Cases 4 and 5, effect of dispersion due to the curavature of $|A|$ is showing. In Cases 6 and 7, effect of the coefficient $\alpha(=\omega''(k))$ for linear dispersion is evident. Cases 8, 9 and 10 show that effect of free long waves may appear in another group of waves travelling ahead.

Comparisons with first- and second-order theories are made in Figs. 6(a) and (b). They may be summarized as in the following. Near the wave maker three of them, experiment, the analytical solution of the first-order theory (Mizuguchi and Toita, 1996) and numerical simulation of Eqs.(5) to (7), agree well for all cases. First-order theory starts to fail to agree with other two at some point. Even after the first-order theory fails, the coupled Eqs. (3) to (5) well describe experimental results.

Discussions on wave modulation

We evaluate degree of contribution of each term by truncating the modulation equation at different position shown below.

$$\partial A/\partial t+c_g\partial A/\partial x+i[\alpha\partial^2A/\partial x^2+\beta'|A|^2A$$
$$\text{--(1)} \qquad \text{--(2)}$$

$$+\{(k/d)(c_g-c/2)\eta_l+ku_l\}A]+\text{dissipation}=0 \qquad (7')$$
$$\text{--(3)/} \qquad \text{--(4)} \qquad \text{--(5),(6)}$$

where, in Stage (3) the bound long wave solution is used in this interaction term, in practice Schrodinger-type equation is used. In Stage (5), dissipation only at bottom is included. Stage (1) corresponds to the analytical solution of first-order. Numerical results in Stage (1) to (6) are plotted together with experimental results in Figs 7(a) and (b). Comparison are made for data at x=16m, farthest measuring point from the wave maker. Overall comparison tells that the first-order theory is rather good. For second-order effects, dissipation (in particular at side wall) is always non-negligible in this scale of experimnts. Other second-order effects plays their each role as is expected. In particular following points may be marked. In Case 2, the flattening effect of bound long waves on wave modulation is most significant, as the difference between Stage (2) to (3) is largest. The cubic nonlinear term is not important as it contributes to sharpening the grouping profile. In Cases 4 and 6, especially in the latter, effect of linear dispersion term (from Stage (1) to (2)) is dominant. Small difference between the experiment and the full thory is noticable in these magnified figures. They may come from either errors in the experiment or higher order effects in theory.

Then the distance X^*, below which rms difference in envelope profile between first-order theory and full second-order theory is less than 5% is calculated by using numerically simulated results with the coupled cuations and plotted in Fig. 8. Free waves at the wave maker are suppressed and dissipation neglected. When ka_{max} is large, flattening due to interaction with bound waves is significant and the distance is at most one-quarter of group length. When ka_{max} is small, linear dispersion term $\propto${curavature} determines the distance, which is of order one wave group length. Significant shoaling of field waves occurs in rather short distance (a few km?), which is of order of one wave group length. The first-order treatment may be sufficiently accurate.

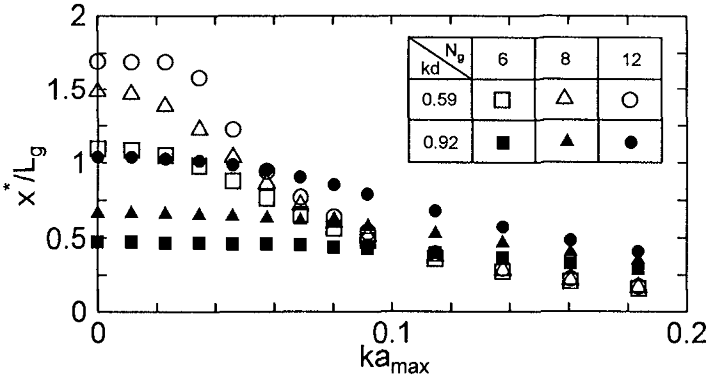

Fig. 9 Traveling distance of negligible change in envelope
L_g is the length of the wave group

Conclusions

1)Coupled equations (5) and (7) can describe well the modulation of short wave group and evolution of long waves in a wide range of experimental results.

2) The second-order effects may be insignificant for shoaling field waves,

although the effect of free long waves could not be fully assesed as the long waves travels ahead of grouping waves.

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