Linear and Nonlinear Modeling of Long Waves Propagating around Channel Bends

By Aimin Shi[1] and Michelle H. Teng[2], M. ASCE

Abstract

In the present study, the transmission and reflection of long waves traveling through right-angled channel bends of constant depth and width are investigated by using numerical simulation based on the linear and nondispersive long wave equations. The present linear results are compared with the results obtained by Shi, Teng and Wu (1998) based on the weakly nonlinear and dispersive Boussinesq equations. The objective is to examine the difference between linear and nonlinear modeling of long waves propagating through curved channels.

Introduction

It is of practical interest to coastal engineers to understand how tides and other long ocean waves are transmitted and reflected through curved river inlets and harbors. In the past, several excellent studies were carried out on the related subject including the studies by Rostafinski (1976) on long acoustic waves in curved ducts, by Webb and Pond (1986) on Kelvin waves in channel bends, and by Kirby, Dalrymple and Kaku (1994) on short water waves through wide channel bends. Most of the previous studies were based on the linear wave theory and focused on waves propagating through smoothly curved channels, except in Miles' (1947) pioneer study where analytical solutions were obtained for linear acoustic waves traveling through sharp-cornered 90°-bends. Miles' solution is valid for long waves propagating in relatively narrow ducts whose width is less than half the wavelength.

In recent studies by Shi and Teng (1996), Shi, Teng and Wu (1998), the previous analytical studies were extended to investigate the propagation of a solitary wave through both smoothly curved and sharp-cornered 90°-bends by using numerical simulation based

[1] Researcher, Environmental Fluid Dynamics Research, Jamesburg, NJ 08831, USA.
[2] Associate Professor, Department of Civil Engineering, University of Hawaii at Manoa, Honolulu, HI 96822, USA.

on Wu's (1981) weakly nonlinear and dispersive Boussinesq equations. Solitary waves propagating through various narrow and wide channel bends were studied. Two parameters, namely, the bending curvature and the ratio of channel width b to effective wavelength λ_e, were found to be the dominant factors that affect the transmission and reflection of a solitary wave through curved channels. For solitary waves traveling through sharp-cornered 90°-bends, the transmission (reflection) coefficient, i.e., ratio of the leading transmitted (reflected) wave amplitude along the channel centerline to the initial wave amplitude, was found to depend on a single dimensionless parameter, namely, b/λ_e. Quantitatively, the transmission (reflection) coefficient was found to decrease (increase) as b/λ_e increases. Based on the numerical results, empirical power laws for predicting the transmission and reflection coefficients as functions of b/λ_e were obtained. For solitary waves traveling through smooth channel bends, the initial wave was observed to be almost completely transmitted in both narrow and wide bends with little backward reflection.

In the present study, the same cases studied in Shi, Teng and Wu (1998) will be revisited by applying the linear nondispersive long wave equations. The objective is to investigate the difference between linear and nonlinear modeling of long waves propagating through channel bends.

Governing Equations

In the recent study of Shi, Teng and Wu (1998), the numerical simulation of long waves propagating through curved shallow water channels of constant depth was based on the generalized weakly nonlinear and dispersive Boussinesq model (i.e., the gB model)

$$\zeta_t + \nabla \cdot [(h+\zeta)\nabla\phi] = 0 \tag{1}$$

$$\phi_t + \frac{1}{2}(\nabla\phi)^2 + \zeta - \frac{h^2}{3}\nabla^2\phi_t = 0 \tag{2}$$

where ζ is the wave elevation relative to the unperturbed water surface, $h\,(=1)$ the water depth, ϕ the depth-averaged velocity potential, t time, and $\nabla = (\partial_x, \partial_y)$ with x, y being the spatial coordinates in the longitudinal and lateral directions, respectively. All the variables are in dimensionless form, with length scaled by h, and time by $\sqrt{h/g}$. The boundary conditions were unperturbed water surface at $x = \pm\infty$, and zero normal velocity at the channel walls.

In the present study, the following linear and nondispersive long wave equations

$$\zeta_t + h\nabla^2\phi = 0 \tag{3}$$

$$\phi_t + \zeta = 0 \tag{4}$$

are applied along with the same boundary conditions as described above.

The long waves studied are solitary waves whose initial wave profile and speed are given by (Teng 1997, Teng and Wu 1992, Shi, Teng and Wu 1998):

$$\zeta(x,t)=\frac{\alpha \operatorname{sech}^2\beta(x-x_0-ct)}{1+\alpha\tanh^2\beta(x-x_0-ct)} \tag{5}$$

$$c=\left\{\frac{6(1+\alpha)^2}{\alpha^2(3+2\alpha)}[(1+\alpha)\ln(1+\alpha)-\alpha]\right\}^{1/2} \tag{6}$$

where α is the wave amplitude, $\beta=[3\alpha/4(1+0.68\alpha)]^{1/2}$, c the wave speed, and x_0 the initial wave position. Here the effective wavelength λ_e of a solitary wave is defined as the wavelength within which the wave elevation ζ is greater than 1% of the amplitude α. Based on (5), λ_e can be calculated by

$$\lambda_e=\frac{2}{\beta}\ln\frac{(1+0.01\alpha)^{1/2}+0.99^{1/2}}{(0.01+0.01\alpha)^{1/2}} \tag{7}$$

Numerical Results

In Shi, Teng and Wu (1998), the Boussinesq model (1), (2) was solved by using an iterative predictor-corrector finite difference scheme (Wang, Wu and Yates 1992) to simulate the propagation of solitary waves through 90°-channel bends. The numerical results for solitary waves traveling through sharp-cornered 90°-bends revealed an interesting phenomenon that in a narrow channel bend, the initial wave is almost completely transmitted with little reflection, while in a wide channel, the amplitude of the reflected wave becomes much greater than the transmitted wave. It was also found that the transmission and reflection coefficients depend on only one dimensionless parameter, namely, the ratio of channel width b to wavelength λ_e. This implies that, when studying long wave transmission and reflection through channel bends, whether a channel is "wide" or "narrow" is judged by comparing the channel width with the wavelength, rather than with the water depth. Based on the numerical results, empirical power laws for predicting the transmission and reflection coefficients through sharp-cornered 90°-bends were obtained as

$$\frac{\alpha_T}{\alpha}=\begin{cases}1, & 0<b/\lambda_e<0.2\\ 0.28(b/\lambda_e)^{-0.72}, & 0.2\le b/\lambda_e<1.0\end{cases} \tag{8}$$

$$\frac{\alpha_R}{\alpha}=\begin{cases}1.19(b/\lambda_e)^{0.9}, & 0<b/\lambda_e<0.4\\ 0.58(b/\lambda_e)^{0.14}, & 0.4\le b/\lambda_e<1.0\end{cases} \tag{9}$$

where α is the initial wave amplitude, and α_T and α_R are amplitudes of the leading transmitted and reflected waves, respectively. For solitary waves traveling through smoothly curved channel bends, it was found that the waves are almost completely transmitted with little backward reflection in both narrow and wide channels.

In the present study, the linear and nondispersive long wave equations (3), (4) are solved by using the 4^{th}-order Runge-Kutta scheme. The scheme is first tested on solitary waves traveling in a straight channel where closed-form solution (5), (6) exists. Our results show that after a solitary wave travels for about 60 water depths, the amplitude changes by only 0.1%. The accuracy of the numerical simulation is also examined by monitoring the mass and energy conservation at each computational step. Both the Runge-Kutta scheme and the scheme by Wang et al. (1992) are found to conserve mass and energy accurately. In all the simulations, including the cases involving waves traveling through sharp-cornered channel bends, the maximum errors in mass and energy conservation are 1.2% and 4.2% with Wang et al.'s scheme in solving the nonlinear Boussinesq model, and 2.2% and 1.9% with the Runge-Kutta scheme in solving the linear wave equations.

Numerical results of a solitary wave of initial amplitude $\alpha = 0.3$ propagating through a sharp-cornered 90°-bend of width $b = 5$ are shown in Fig. 1 (linear results) and Fig. 2 (nonlinear results). Figures 1 (a) and 2 present the two-dimensional wave field at different time instants based on the linear and nonlinear results, while Fig. 1 (b) and (c) show the detailed comparison between the linear (dashed line) and nonlinear (solid line) results for transmitted and reflected wave profiles along the channel centerline. From these results, we observe that, the Boussinesq model can predict more detailed (e.g., Fig.1 (a) and Fig.2, near the sharp corner) and more realistic (e.g., Fig.1 (c), the transmitted wave profile) wave features than the linear wave equations. In addition, there are some quantitative differences between the two models in predicting the transmitted and reflected wave amplitude and speed. The linear wave equations predict a slightly smaller (larger) transmitted (reflected) wave amplitude, and a slower speed which is expected. Despite these small differences, the linear and nonlinear wave models are seen to be fundamentally consistent with each other in predicting the transmission and reflection of solitary waves propagating through sharp-cornered channel bends. This consistency is further shown in Fig. 3 (a) and (b) where numerical results from fifteen simulations involving different initial wave amplitude and different channel width are plotted together. In this figure, the connected solid and dashed lines are based on the least-square fitting of the numerical data points, and the solid line also represents the empirical power laws given by (8) and (9). We can see that, similar to the nonlinear results based on the Boussinesq model, the data points for the transmission (and reflection) coefficient based on the linear and nondispersive model (3) and (4) also fall along one curve when plotted against b/λ_e, hence revealing the same similarity parameter that governs the phenomenon.

The linear results for a solitary wave of initial amplitude $\alpha = 0.3$ traveling through a smooth 90°-bend of width $b = 5$ are presented in Fig. 4 (a) and (b). Comparing with the nonlinear results (Shi, Teng and Wu 1998, Fig.9 (a), p.171), we find that the nonlinear and dispersive Boussinesq model again provides more detailed wave features, such as the lateral wave variation in the trailing region, than the linear model. In addition, based on the linear equations, the back of the transmitted wave is slightly steeper than the front of the wave, which is less realistic than the wave profile predicted by the Boussinesq model (see comparison in Fig.4 (b)). Nevertheless, both models predict that for long waves traveling through smoothly curved channels, the initial wave is almost completely transmitted with little backward reflection. In addition, the values for the leading transmitted wave amplitude predicted by the two models are quite consistent.

In our numerical simulation, the wave speed based on both models is calculated. Here we define the average wave speed as the total longitudinal distance traveled along the channel centerline divided by the corresponding travel time. Detailed numerical results on the wave speed of an initial solitary wave of $\alpha = 0.3$ traveling in different curved channels are presented in Table 1. These results show that the linear waves travel with critical wave speed, slower than the nonlinear waves, which is consistent with the wave theory.

	Channel Width b	Linear Wave Speed	Nonlinear Wave Speed
Smooth 90^o-Bends	1	0.998	1.133
	5	0.980	1.122
	10	0.940	1.093
Sharp-Cornered 90^o-Bends	1	0.996	1.154
	5	1.006	1.142
	10	1.092	1.183

Table 1. Comparison between wave speeds based on the linear and nonlinear models

Conclusion

For long waves propagating through smooth and sharp-cornered channel bends, it is found that the weakly nonlinear and dispersive Boussinesq model and the linear nondispersive long wave equations are fundamentally consistent with each other in predicting the amplitude of the leading transmitted and reflected waves. Beyond this fundamental consistency, we also find that the Boussinesq model can predict slightly more detailed wave features and more realistic wave profiles.

Acknowledgement

The numerical computations were performed on Cray c90 at the San Diego Supercomputer Center which is supported by the U.S. National Science Foundation. Helpful suggestions and discussions from Professor Howell D. Peregrine and Professor Robert Dalrymple at the 26th International Conference on Coastal Engineering in Copenhagen are greatly appreciated.

References

Kirby, J.T., Dalrymple, R.A. and Kaku, H. (1994). "Parabolic approximations for water waves in conformal coordinate systems." *Coastal Engrg.*, 23, 185-213.

Miles, J. W. (1947). "The diffraction of sound due to right-angled joints in rectangular tubes." *J. Acoust. Soc. Am.*, 19(4), 572-579.

Rostafinski, W. (1976). "Acoustic systems containing curved duct sections." *J. Acoust. Soc. Am.*, 60, 23-28.

Shi, A. and Teng, M.H. (1996). "Propagation of solitary wave in channels of complex configurations." *Proc. of 2nd Int. Conf. on Hydrodynamics,* Balkema, Rotterdam, The Netherlands, 349-354.

Shi, A., Teng, M.H. and Wu, T.Y. (1998). "Propagation of solitary wave through significantly curved shallow water channels." *J. of Fluid Mech.*, 362, 157-176.

Teng, M.H. and Wu, T.Y. (1992). "Nonlinear water waves in channels of arbitrary shape." *J. of Fluid Mech.*, 242, 211-233.

Teng, M.H. (1997). "Solitary wave solution to Boussinesq equations." technical note, *J. of Waterway, Port, Coastal and Ocean Engrg.*, 123(3), 138-141.

Wang, K.H., Wu, T.Y. and Yates, G.T. (1992). "Three-dimensional scattering of solitary waves by vertical cylinder." *J. of Waterway, Port, Coastal and Ocean Engrg.*, 118(5), 551-556.

Webb, A.J. and Pond, S. (1986). "The propagation of a Kelvin wave around a bend in a channel." *J. of Fluid Mech.*, 169, 257-274.

Wu, T.Y. (1981). "Long waves in ocean and coastal waters." *J. of Engrg. Mech.*, 107(3), 501-522.

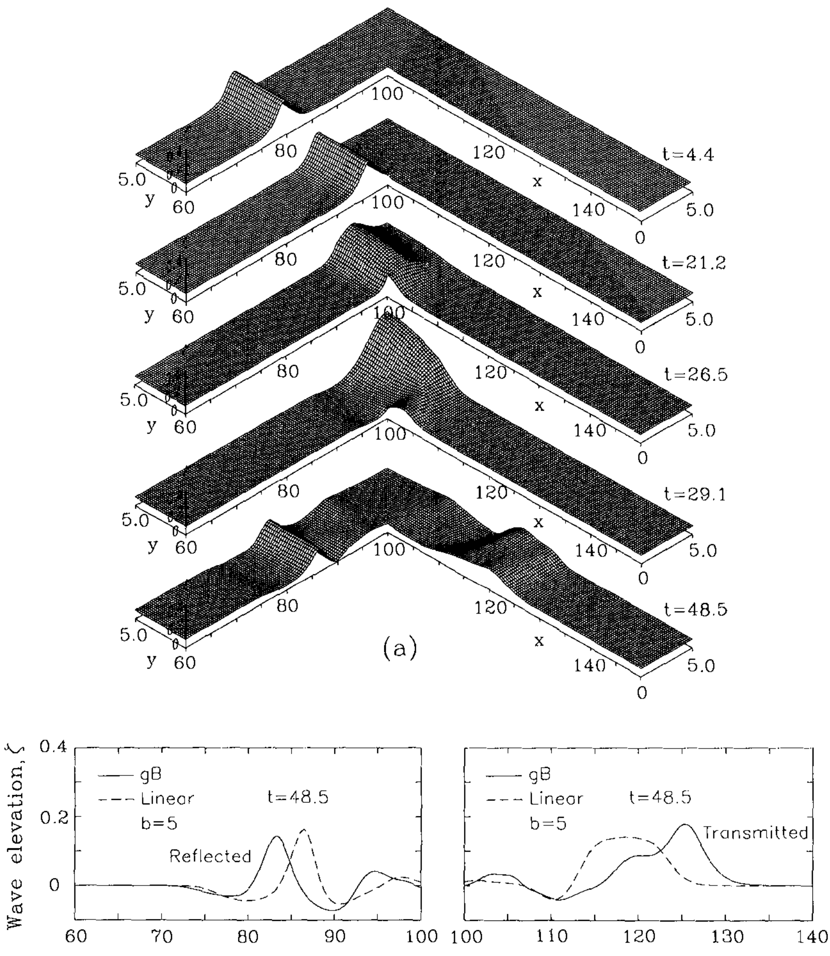

Figure 1. Propagation of a solitary wave of initial amplitude $\alpha = 0.3$ through a sharp-cornered channel bend of width $b = 5$. (a) wave field at different time instants based on the linear nondispersive wave equations; (b) and (c): comparison between the linear (dashed line) and the nonlinear (solid line) results for the reflected and transmitted wave profiles along the channel centerline.

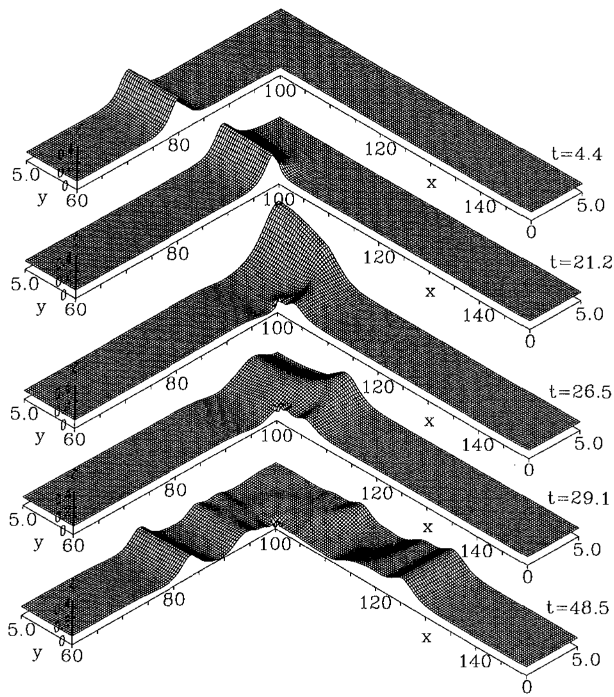

Figure 2. Propagation of a solitary wave of initial amplitude $\alpha = 0.3$ through a sharp-cornered channel bend of width $b = 5$ based on the weakly nonlinear and dispersive Boussinesq model.

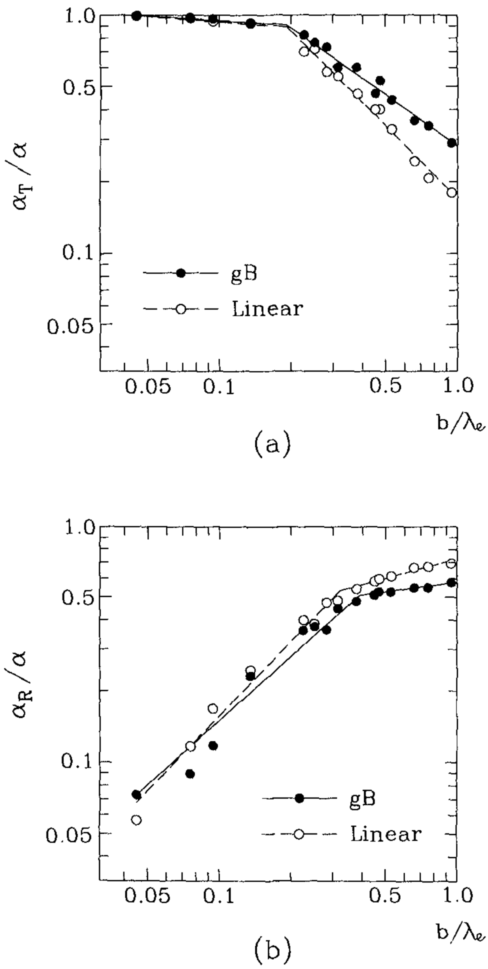

Figure 3. Plots of (a) transmission coefficient and (b) reflection coefficient v.s. the ratio of channel width b to effective wavelength λ_e for solitary waves propagating through sharp-cornered 90°-bends.

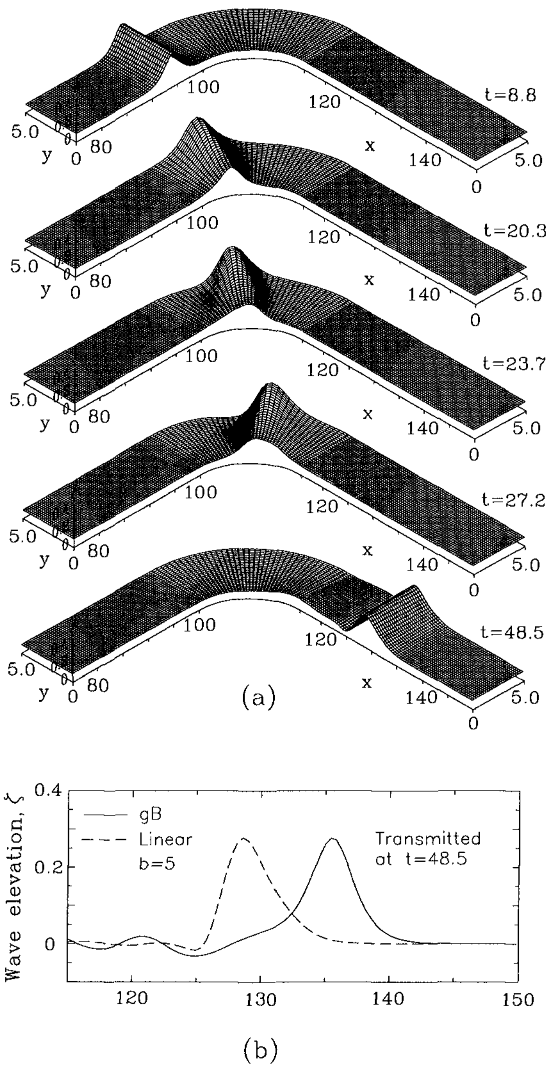

Figure 4. Propagation of a solitary wave of initial amplitude $\alpha = 0.3$ through a smooth channel bend of width $b = 5$. (a) wave field at different time instants based on the linear nondispersive wave equations; (b): comparison between the linear (dashed line) and the nonlinear (solid line) results for the transmitted wave profiles along the channel centerline.