CHAPTER 92

NONLINEAR WAVE DYNAMICS IN THE SURF ZONE

O. R. Sørensen[1], P. A. Madsen[1] and H. A. Schäffer[1]

Abstract

In this paper we shall study and model various nonlinear interaction processes in the surf zone on the basis of a two-dimensional time-domain depth-integrated Boussinesq type model, which features the possibility of wave breaking and a moving boundary at the shoreline. Phenomena to be studied include breaking and runup of bicromatic wave groups on gentle slopes with emphasis on the evolution of surface elevations, the shoreline motion and the low frequency waves generated and released in the surf zone. The model results are compared with laboratory experiments by Mase (1994), and the agreement is generally found to be very good. Special details in the surf beats are investigated, e.g. the motion of the breaker line and of the shoreline, change of wave group modulation through the surf zone, and the sensitivity of surf beats to group frequency, modulation rate and bottom slope.

1. Introduction

Shoaling, breaking and runup of irregular wave trains in shallow water is a nonlinear process involving a number of complicated details. Triad interactions between harmonics in shallow water lead to substantial cross spectral energy transfer in relatively short distances, and during shoaling still more energy will be transferred into bound sub-harmonics and super-harmonics, which will travel phase locked to the primary wave train.

In a number of situations, however, part of the energy may be released as free harmonics, e.g. during the passage over submerged bars or during the process of breaking. When wave breaking occurs, the primary waves and their super-

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[1] International Research Centre for Computational Hydrodynamics (ICCH). Located at the Danish Hydraulic Institute, Agern Allé 5, DK-2970 Hørsholm, Denmark

harmonics will start dissipating, and this will allow for a gradual release of the bound sub-harmonics and result in free long waves moving towards the shoreline from which they will be almost fully reflected.

To some extent bound long waves will persist in the surf zone depending on the degree of groupiness in this region. The groupiness of the incident waves will generally be partially transmitted into the surf zone, and it may even be reversed if the higher waves in a wave group decay sufficiently during breaking to appear as the lower waves further inshore.

The individual waves in a wave group will break at different depths, which will result in a horizontal oscillation of the breakpoint. The excursion of the breakpoint will depend on the groupiness of the incident waves, and while a linear sinusoidal motion can be expected for a weakly modulated bichromatic group, this is somewhat modified for a fully modulated group where the shoreward extreme of the breakpoint will in principle be at the shoreline. The motion of the breakpoint will result in large time-varying radiation stress gradients in the region of incipient breaking, and this will act as a local forcing of free long waves at the group frequency and its higher harmonics (Symonds et al (1982)). These will propagate in the onshore as well as in the offshore direction. Since the breakpoint forced long waves are basically due to variations of the starting point for the setup, the phenomenon is sometimes referred to as dynamic setup.

The present paper concentrates on the investigations of nonlinear interaction processes in the surf zone using a time-domain Boussinesq type model, which resolves the primary wave motion as well as the long waves. A detailed description of the model can be found in Madsen et al. (1996), while a short description of the model is given in Chapter 2. In Chapter 3 the numerical model is verified with respect to the evolution of surface elevation and shoreline motion due to incident bicromatic wave trains. In Chapter 4 special details of the surf beat mechanism are investigated.

2. Model Description

The numerical model is based on a special type of depth-integrated equations derived by Madsen et al. (1991) and Madsen and Sørensen (1992) and incorporates enhanced linear dispersion characteristics and shoaling properties, which are important for an accurate representation of the nonlinear energy transfer (Madsen and Sørensen, 1993).

The inclusion of wave breaking is based on the surface roller concept for spilling breakers following the formulation by Schäffer et al. (1993). The effect of the roller on the wave motion is taken into account by excess momentum terms originating from a non-uniform velocity profile due to the presence of the roller. The excess momentum terms are a function of the roller thickness and the roller celerity. The instantaneous roller thickness at each point is determined based on a futher development of the heuristic, geometrical approach by Deigaard (1989). Breaking is predicted to occur when the local slope of the surface elevation exceeds an initial critical value, $\tan\phi_B$, using $\phi_B = 20$ deg. During the transition from initial breaking to a bore-like stage in the inner surf zone, the critical angle is assumed to gradually change from ϕ_B to a smaller terminal angle, ϕ_0 which is taken to be 10 deg. The roller thickness is determined as the water above the tangent of slope $\tan\phi$ and the resulting thickness is multiplied by a shape factor which is taken to 1.5. The roller celerity is determined interactively from the instantaneous wave field. The basic approximation is that the waves can locally be assumed to be regular and progressive.

The moving boundary at the shoreline is treated numerically by replacing the solid beach by a permeable beach characterized by an extremely small porosity. This allows for the determination of wave motion in the swash zone and it provides a shoreline condition which like a natural beach tends to be reflective for waves of small steepness viz. the low frequency waves. Radiation of short and long period waves from the offshore boundary is allowed by use of absorbing sponge layers.

Bottom friction is described by the classical quadratic friction law with a friction coefficient f_w of 0.01.

3. Evolution of Surface Elevations and Shoreline Motion

In recent publications Mase (1994,1995) presented experimental results for shoaling, breaking and runup of various types of bichromatic wave trains on a gentle slope. Fig. 1 illustrates the experimental setup. The flume consists of a 10 m horizontal section with a water depth of 0.47 m and a 12 m section with a constant impermeable 1/20 slope. Twelve wave gauges of capacitance type are installed at still water depths of 47, 35, 30, 25, 20, 17.5, 15, 12.5, 10, 7.5, 5 and 2.5 cm, and they are denoted WG1 to WG12. Furthermore, a runup meter is placed at the shoreline. Mase considered a range of wave conditions. Here we shall concentrate on the case with the two primary frequencies $f_1 = 0.95f_m$ and $f_2 = 1.05f_m$, where the mean frequency, f_m was taken as 0.3, 0.4, 0.5, 0.6, 0.8, 1.0 and 1.2 hz, and with the amplitudes $a_1 = a_2 = 0.025$ m.

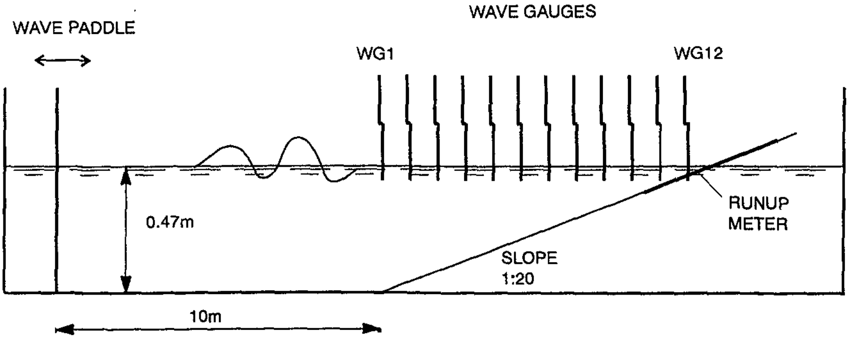

Fig. 1 Sketch of physical wave flume (Mase, 1994)

The variation in mean frequency from 1.2 hz to 0.3 hz represents a significant change in wave conditions, which are listed in Table 1 in terms of h/L_0, H_0/L_0, and ζ, where L_0 is the deep water wave length based on the mean frequency, H_0 is an estimate of the deep water wave height and h is the water depth taken at the toe of the slope. ζ is the surf similarity parameter defined as the beach slope divided by the square root of the deep water wave steepness. Based on Galvin's (1968) classification of wave breaking, Battjes (1974) found that spilling breakers would occur for $\zeta < 0.5$, while plunging would occur for $0.5 < \zeta < 3.3$. Hence, the test cases considered by Mase (1994) cover both regimes, and this will lead to rather different spatial evolutions of wave energy during the shoaling and breaking process. A further analysis of the measurements shows that for decreasing values of the mean frequency, there is a gradual increase in the motion at the group frequency and a pronounced increase in the primary wave motion at the shoreline.

Mean frequency (hz)	h/L_0	H_0/L_0	ζ	Breaker type
1.2	0.43	0.046	0.23	Spilling
1.0	0.30	0.032	0.28	Spilling
0.8	0.19	0.021	0.35	Spilling
0.6	0.11	0.012	0.47	Spilling
0.5	0.075	0.008	0.56	Plunging
0.4	0.048	0.005	0.70	Plunging
0.3	0.027	0.003	0.93	Plunging

Table 1 Wave characteristics of test cases by Mase (1994)

The wavemaker was controlled by a signal generated off-line using linear theory. Thus, advanced features as compensation for generation of spurious sub-harmonics and super-harmonics as well as active absorption of free waves reflected from the slope were not included. Furthermore, the measured frequencies and amplitudes deviated slightly from the target. Altogether, this prevents us from modelling the exact experimental setup and consequently we have chosen to place the numerical model boundary at the position of the first wave gauge (WG1) i.e. at the toe of the sloping beach. The energy of the primary frequencies and their super-harmonics propagates mainly onshore while sub-harmonic energy will propagate offshore as well as onshore. Neglecting the low frequency part of the incident waves, we use the following procedure: The measured signal at WG1 is analysed by FFT, the low frequency motion is removed and the remaining signal is converted into a flux boundary condition using linear theory. At the position of WG1 the waves are generated internally and re-reflection from this boundary is avoided by using a 1 m wide sponge layer offshore from the line of generation. In order to resolve the super-harmonics in shallow water a grid size of 0.02 m and a time step of 0.01 s is used in the simulations. Numerical simulations have been performed for mean frequencies in the interval between 0.3 and 1.0 hz. The computed evolution of surface profiles and the resulting shoreline motion agree quite closely with the measurements, indicating that phenomena like dispersion, wave-wave interaction and dissipation due to breaking are well represented by the present model. As an illustration of this Fig. 2 shows for the case of $f_m = 0.3$ hz the measured and computed time series of water surface elevations at locations WG8, WG10 and WG12 as well as the motion of the shoreline converted into vertical displacement. For clarity the measurements are shifted relative to the computational results by 0.08m in the figure. All cases were simulated with the same set of standard model parameters, and the quality of the agreement is very good for all values of f_m. This is actually a bit surprising, considering the fact that the roller concept used in the breaker model is primarily suited for spilling type of wave breaking.

The transient shoreline motion is shown in Fig. 3 for the mean frequency $f_m = 0.3$, 0.6 and 1.0 hz. Again, the agreement between the present model and the measurements is very good. The different wave conditions, ranging from plunging type to spilling type wave breaking, can be seen clearly in the shoreline motion: For the lower mean frequencies the swash of the individual primary waves is quite distinct at the shoreline, while in the case of higher mean frequencies the group induced subharmonic motion dominates the swash oscillations.

It appears that the type of shoreline motion is governed by the type of wave breaking and consequently the surf zone similarity parameter, ζ can be used as an indicator. Hence we can conclude that ζ can be used to characterize the type of shoreline motion, and that plunging/surging breakers result in individual swash, while spilling breakers result in low-frequency dominated shoreline motion.

4. Investigations of Surf Beat

Release of incident bound long waves as well as long-wave generation by a time-varying breakpoint result in a long wave emitted from the surf zone. The amplitude of this wave is a measure of the surf beat activity and it was studied in a series of laboratory experiments by Kostense (1994). The numerical model has been verified against the experimental data by Kostense and the comparison was most satisfactory. For more details see Madsen et al. (1996). Here we shall concentrate on special details of the surf beat mechanism. The following topics are adressed: Trajectories of surface rollers; the motion of the breaker line and of the shoreline; the change of wave group modulation through the surf zone; the sensitivity of surf beats to group frequency, modulation rate and bottom slope.

For this investigation we shall concentrate on incident bichromatic wave groups which are effectively linear at the seaward boundary. This has the advantage that the determination of the amplitude of the outgoing free long waves due to the surf beat can be determined very accurately by simple means. The

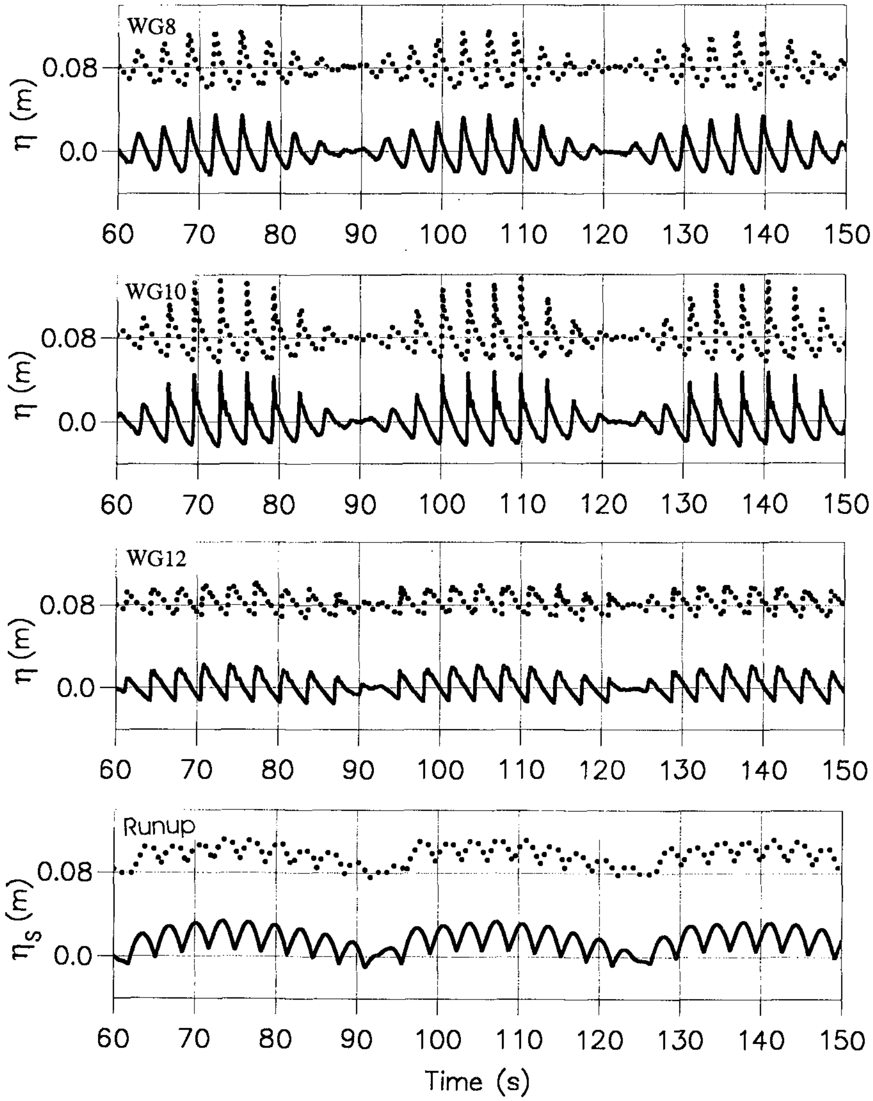

Fig. 2 Surface elevation (η) and swash oscillation (η_s, vertical displacement).

Case: WP2 , f_m=0.3 hz i.e. f_1=0.285 hz and f_2=0.315 hz.

____ Present model

...... Experimental data by Mase (1994), shifted relative to computed results by 0.08m.

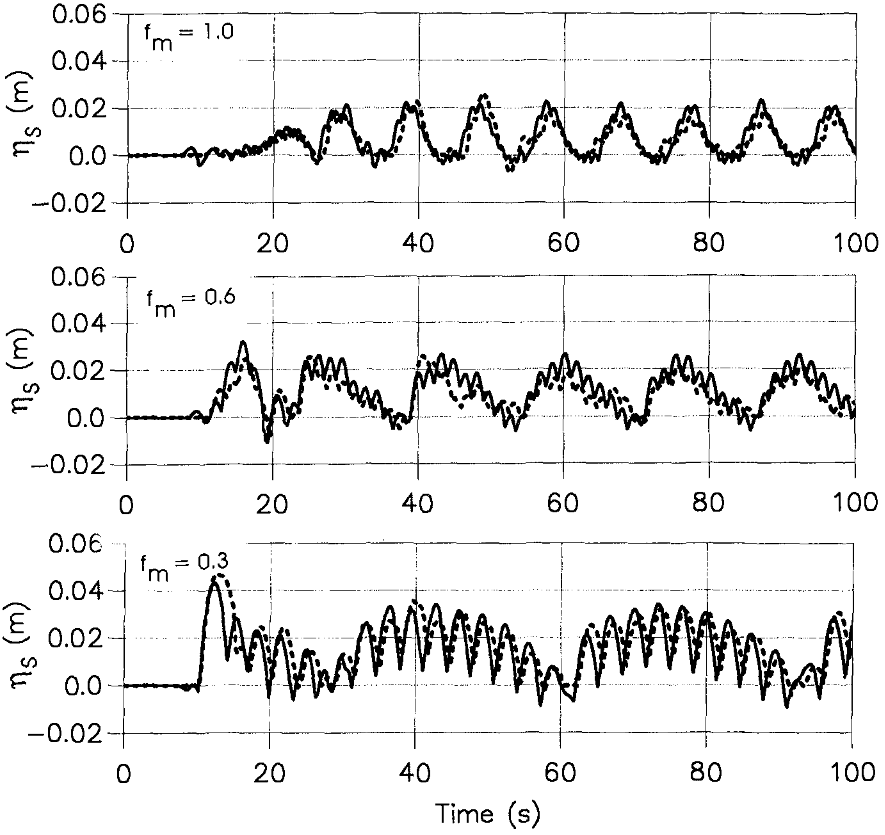

Fig 3 Transient Shoreline motion (η_s, vertical displacement).
Case WP2: f_m=0.3 hz, 0.6 hz and 1.0 hz.
____ Present model
...... Experimental data by Mase (1994)

bathymetry used for this study consists of a horizontal part of 10 m with a depth of 1.2 m and a part with a constant sloping beach (various slopes are considered). The grid size, the time step and other model parameters are identical to those used in Chapter 3. Again, bichromatic wave trains are considered. The two primary frequencies are given by $f_1 = f_m + \Delta f/2$ and $f_2 = f_m - \Delta f/2$, where the mean frequency $f_m = 0.60$ hz and the group frequency, Δf is varied in the range 0.02-0.15 hz. The sum of the amplitudes is fixed ($a_1 + a_2 = 0.08$m) and two different initial modulation rates, σ ($= a_2/a_1$) are considered. In all tests waves are generated inside the model domain and re-reflection from the offshore boundary is avoided by using a sponge layer seawards of the point of generation.

In the first test case the beach slope is $h_x=1/40$, the still water shoreline is at $x=58$m, while two wave groups with $\sigma=0.2$ and $\sigma=1.0$ are considered. As mentioned above, the sum of a_1 and a_2 at the point of wave generation is constant which implies that the highest waves in each of the two wave groups are of equal size. Hence, we may expect that the outermost breakpoint position, $x_{B,outer}$ will be almost the same for the two wave groups, while the innermost breakpoint, $x_{B,inner}$ will be quite different as it is determined from the smallest waves in the wave group. This is confirmed by Figs 4a-b, which show the computed trajectories of

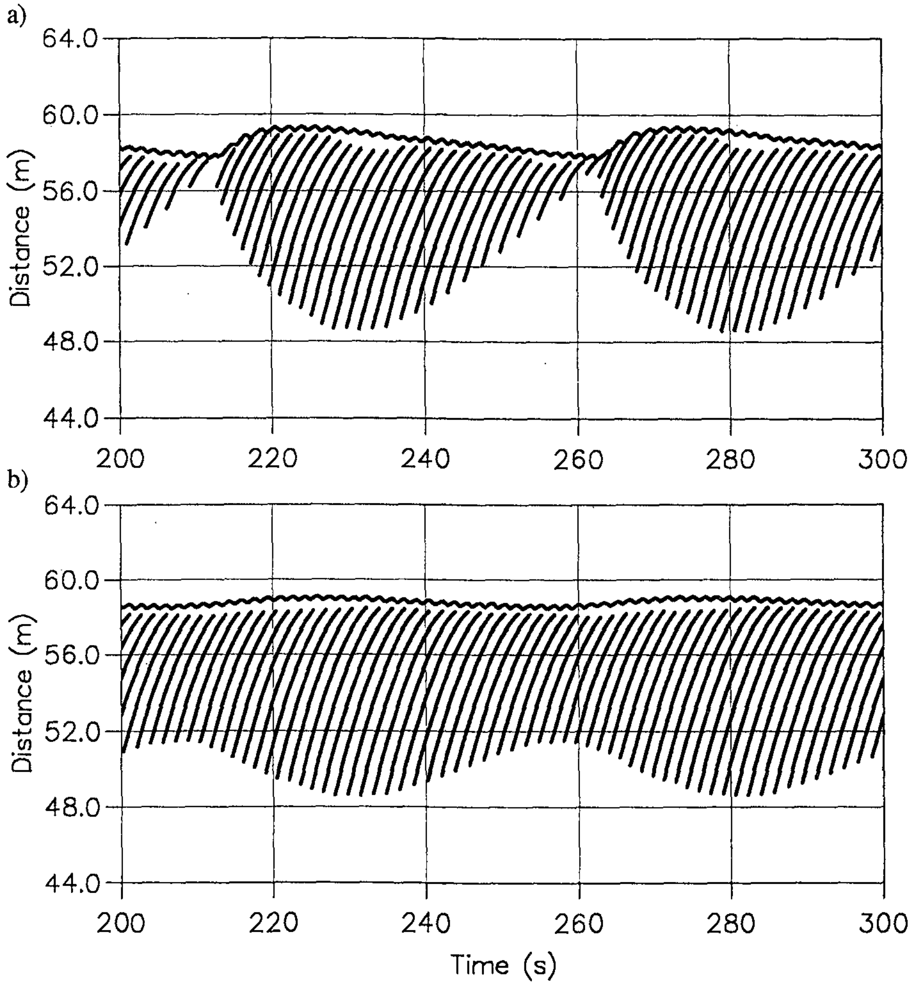

Fig. 4 Trajectories of surface rollers and shoreline motion.
Group frequency, $\Delta f=0.02$ hz. Bottom slope=1/40, $f_m=0.60$ hz, $f_1=f_m+\Delta f/2$, $f_2=f_m-\Delta f/2$; $a_1+a_2=0.080$m; a) Modulation rate, $a_2/a_1=1.0$ b) Modulation rate, $a_2/a_1=0.2$

the surface rollers and the shoreline motion for a group frequency $\Delta f=0.02$hz in which case the group motion is resolved by 30 mean periods. We notice that $x_{B,outer}$ is practically identical for the two modulation rates, while $x_{B,inner}$ strongly depends on σ. The breakpoint excursion between these two limits is seen to be well resolved by the number of rollers being detected and traced. The almost sinusoidal time-variation of x_B for $\sigma=0.2$ is significantly distorted as the modulation is increased to $\sigma=1.0$, the reason being that $x_B(t_2)=x_{B,inner}$ is delayed relative to $x_B(t_1)=x_{B,outer}$ with half a group period *plus* the travelling time

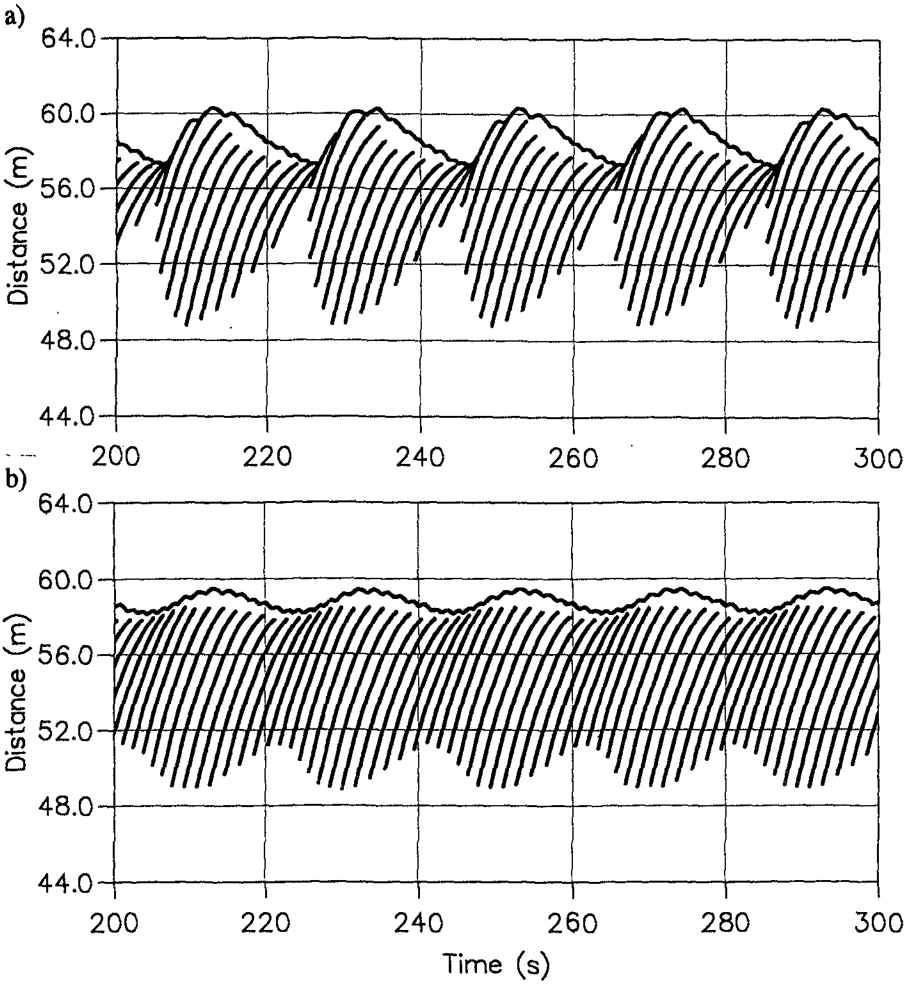

Fig 5 Trajectories of surface rollers and shoreline motion.
Group frequency, $\Delta f=0.05$ hz. Otherwise as Fig 4.
a) Modulation rate, $a_2/a_1=1.0$ b) Modulation rate, $a_2/a_1=0.2$

neccessary for the lowest waves to cover the distance from $x_{B,outer}$ to $x_{B,inner}$. This delay was also discussed by Symonds et al. (1982). In Figs. 5a-b the group frequency is increased to $\Delta f=0.05$hz. Apparently, this leads to a small shoreward shift of $x_{B,outer}$ and a seaward shift of $x_{B,inner}$ leading to a minor reduction in the breakpoint excursion for both modulation rates. It turns out that this trend is amplified for increasing ratios of the group frequency to the mean frequency ($\Delta f/f_m$) and when this number is larger than say 1/3 the effective excursion produced by the model almost vanishes.

The shoreline excursion in Figs 4 and 5 is seen to be dominated by low frequency motion while the individual bores almost vanish. The fully modulated case obviously results in the strongest motion, as would be expected from both second order theory for bound waves and from the theory of the generating mechanism due to breakpoint oscillations. The non-sinusoidal variation of the breakpoint position is also found in the breakpoint forcing and thereby in the surf beat motion. Thus, (in Figs 4a and 5a) the shape of the resulting shoreline motion indicates substantial energy on harmonics of the group frequency i.e. $2\Delta f$ etc.

The modulation of the wave groups changes slightly during the processes of shoaling and nonlinear interactions, but much more rapid variations can be expected in the surf zone. In numerical modelling this obviously depends on the type of breaker model applied and as an example the classical assumption of a saturated surf zone (with wave amplitudes proportional to the local water depth) leads to a vanishing modulation. Schäffer (1993), on the other hand, discussed the possibility of a reversion of modulation and such a reversion is indeed possible in the present model, because of the difference between the initial limiting breaker angle for non-breaking waves (20 deg) and the final breaker angle (10 deg) for waves already breaking.

Fig. 6 shows high-pass filtered timeseries of computed surface elevations at three locations for the case of $\sigma=0.2$ and $\Delta f=0.05$ hz. The high-pass filtering ($f>2\Delta f$) makes it easier to spot the modulation of the primary waves without the influence of the low frequency motion. At $x=49$ m the highest waves have just started breaking and the modulation is similar to the input wave train. At $x=50.8$ m the former highest waves (indicated by I) have been reduced by breaking and are now approximately of the same size as the former lowest waves (indicated by II). Hence the modulation has almost vanished at this location. At $x=52$ m a relatively strong reversion of modulation has occurred and the former highest waves are now the lowest waves in the wave group. A similar trend can actually be observed in Mase's (1994) fully modulated experimental data. We can therefore conclude that the reversion is not just an artifact of the numerical model.

We shall now turn to an investigation of the surf beat sensitivity to group frequency, modulation rate and bottom slope. Emphasis is put on the amplitude of the resulting outgoing free long waves determined on the horizontal section seawards from the surf zone. Figs. 7 and 8 show the computed amplitudes versus the group frequency. In Fig. 7 the amplitude is shown for the bottom slope $h_x = 1/40$ and for the two modulation rates $\sigma = 0.2$ and 1.0. Fig. 8 showns the amplitude for the case of full modulation ($\sigma=1.0$) and for the three bottom slopes

h_x = 1/20, 1/30 and 1/40. The curves are seen to oscillate with the group frequency, a trend which is in qualitative agreement with the theory of Symonds et al. (1982), who showed the importance of the relative phase between the long waves reflected from the shoreline and the long waves directly generated in the seaward direction by the moving breakpoint mechanism. The computed values of Δf, for which minimum response occurs, are much larger than the values predicted by Symonds et al. This trend is, on the other hand, supported by Kostense's (1984) measurements as well as by Schäffer's (1993) analytical work.

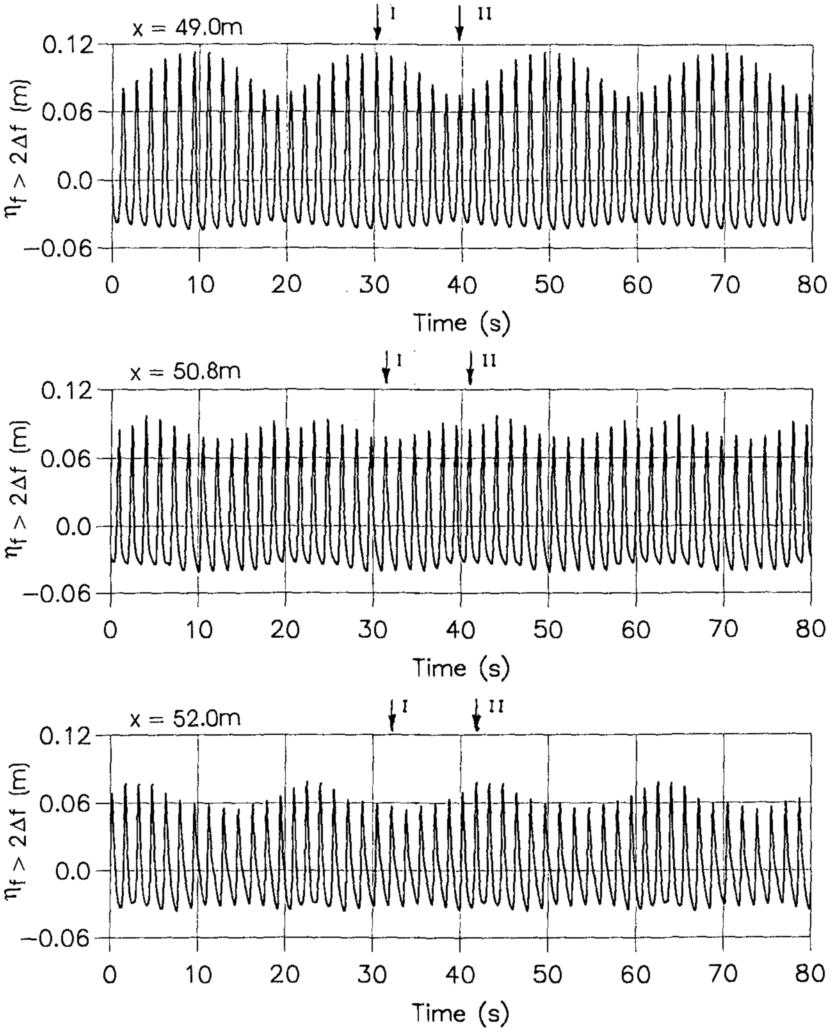

Fig 6 Time series of computed high-pass filtered (f > $2\Delta f$) surface elevations at three locations. Δf=0.05 hz, a_2/a_1=0.2. Otherwise as Fig 4.

It is therefore likely that the deviation from the prediction of Symonds et al. is due to their omission of the incident bound long waves. Fig. 8 shows that the values of Δf, for which the amplitude of the outgoing long wave is minimum, increases for increasing values of the slope. The effect of the modulation rate is small. Fig. 7 shows that when σ is increased from 0.2 to 1.0 the local minima occur for slightly higher values of Δf.

From Figs 7 and 8 it is seen that the maximum amplitude of the outgoing free long wave increases significantly for increasing modulation rate while there only is a small increase of the amplitude for increasing values of the slope. The main reason for this trend can be related to the incoming bound long waves. Second order theory (valid on a horizontal bottom) predicts that the long waves bound to the incoming wave groups will be proportional to the product of a_1 and a_2, which indicates a scaling of $\sigma/(1+\sigma)^2$ i.e. they will be almost twice as large with $\sigma=1.0$ as compared to $\sigma=0.2$. Another obvious trend is that the local maxima decrease for increasing group frequencies. This is connected with the overall decrease in thereflection from the shoreline. When $f_m/\Delta f$ is decreased one will expect that the individual rollers connected with the short waves will have a dissipative effect on the long waves. However, the numerical treatment at the shoreline can also been shown to be increasingly dissipative for higher group frequencies. Further investigation is needed in order to quantify the reliability of the numerical model in this regime. A more detailed discussion of these results is given by Madsen et al (1996).

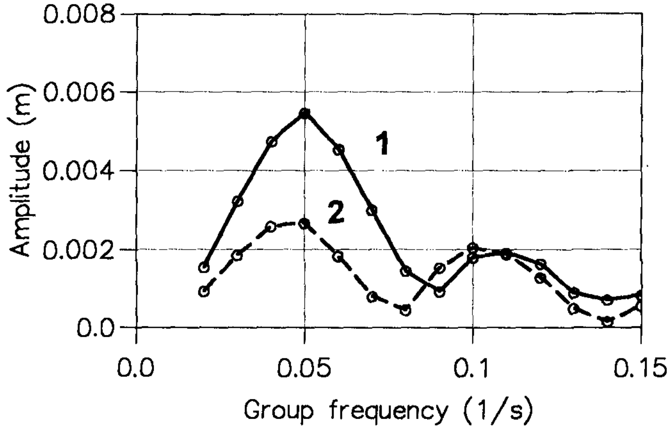

Fig 7 Amplitude of outgoing free long wave as a function of modulation and group frequency. Bottom slope, $h_x=1/40$.
1: $a_2/a_1=1.0$; 2: $a_2/a_1=0.2$

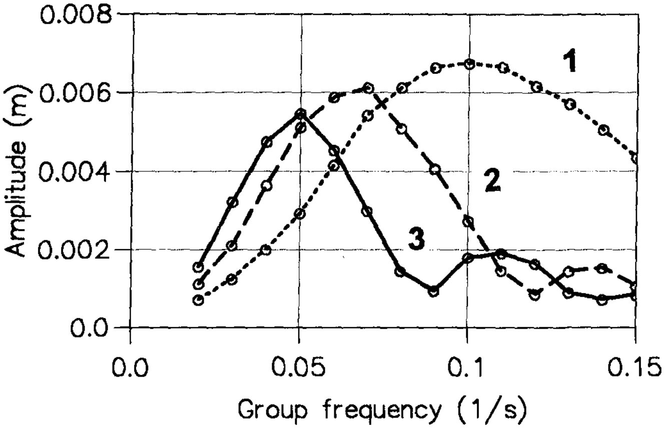

Fig 8 Amplitude of outgoing free long wave as a function of group frequency. Full modulation ($a_2/a_1=1.0$). 1: $h_x=1/20$; 2: $h_x=1/30$; 3: $h_x=1/40$

Related and future research

This work is part of an extensive research effort at ICCH within the field of Boussinesq-type modelling. Concurrently with the present applications to strictly cross-shore motion, the investigation of situations in two horizontal dimensions as presented at the 24th ICCE (Sørensen et al. 1994) is continued. Improvements, e.g. with regard to the celerity of the surface roller as well as comparison with laboratory experiments on nearshore circulations are in progress. For applications in one as well as in two horizontal dimensions the main discrepancy between measurements and computation of e.g. the wave height variation appears to be the nonlinear shoaling close to the breakpoint rather than the following decay in the surf zone. This is one of the reasons for pursuing a higher order Boussinesq model as also presented at this conference (Madsen et al. 1996). In the near future the breaking model will be coupled with this higher order model. This will provide a tool for studying waves and current in a region extending from quite deep water and all the way to the shoreline. Related developments accounting explicitly for waves on ambient currents is also documented at this conference (Chen et al. 1996).

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References

Battjes, J.A. (1974), "Surf Similarity", In Proc. of the 14th Coastal Eng. Conf., ASCE, pp 466-480.

Deigaard, R. (1989), "Mathematical modelling of waves in the surf zone", Prog. Report 69, ISVA, Technical University, Lyngby, Denmark, pp 47-59.

Galvin, C.J. Jr. (1968), "Breaker Type Classification on three Laboratory Beaches", Journal of Geophysical Research, Vol. 73, No. 12, pp 3651-3659.

Kostense, J.K. (1984), "Measurements of surf beat and set-down beneath wave groups", In Proc. of the 19th Coastal Eng. Conf., pp 724-740.

Madsen, P.A., R. Murray and O.R. Sørensen (1991), "A new form of the Boussinesq equations with improved linear dispersion characteristics. Part 1", Coastal Engineering, Vol 15, pp 371-388.

Madsen, P.A. and O.R. Sørensen (1992), "A new form of the Boussinesq equations with improved linear dispersion characteristics. Part 2: A slowly varying bathymetry", Coastal Engineering, Vol 18, pp 183-204.

Madsen, P.A. and O.R. Sørensen (1993), "Bound waves and triad interactions in shallow water", Ocean Engineering, Vol 20, pp 359-388.

Madsen, P.A., H.A. Schäffer and O.R. Sørensen (1996), "Surf zone dynamics simulated by a Boussinesq type model. Part 1: Model description and cross-shore motion of regular waves", Submitted to Journal of Geophysical Research.

Madsen, P.A., O.R. Sørensen and H.A. Schäffer (1996), "Surf zone dynamics simulated by a Boussinesq type model. Part 2: Irregular waves and surf beat", Submitted to Journal of Geophysical Research.

Mase, H. (1994), "Uprush-backrush interaction dominated and long wave dominated swash oscillations", In Proc. of Int. Symposium: Waves - Physical and Numerical Modelling, Vancouver, Canada, pp 316-325.

Mase, H. (1995), "Frequency down-shift of swash oscillations compared to incident waves", Journal of Hydraulic Research, Vol 33, No 3, pp 397-411.

Schäffer, H.A. (1993), "Infragravity waves induced by short-wave groups", J. Fluid Mech. (1993), Vol. 247, pp. 551-588.

Schäffer, H.A., P.A. Madsen and R. Deigaard (1993), "A Boussinesq model for waves breaking in shallow water", Coastal Engineering, Vol 20, pp 185-202.

Symonds, G., G.A. Huntley and A. J. Bowen (1982), "Two dimensional surf beat: Long wave generation by a time-varying break point", J. Geophys. Res, 87, C1, pp 492-498.

Sørensen, O.R., H.A. Schäffer, P.A. Madsen and R. Deigaard (1994), "Wave breaking and induced nearshore circulations", In Proc. 24th Coastal Eng. Conf., Kobe, Japan, pp. 2583-2594.