CHAPTER 85

A Statistical Approach for Modeling Triad Interactions in Dispersive Waves

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Abstract

The feasibility of a statistical approach to model the effect of triad interactions on the evolution of wave spectrum is investigated. The approach is based on the Zakharov kinetic integral for resonant triad interactions in capillary gravity waves. For application to dispersive gravity waves, the kinetic integral is modified by inclusion of a spectral filter (smeared delta function), to allow for the cross-spectral energy transfers in dispersive wavefields, with bandwidth to be determined empirically. Numerical investigation of the resulting expression indicates that the energy flux from the spectral peak region toward higher harmonics increases with decreasing water depth.

The interaction integral has been cast into an energy source/sink term and implemented in an energy balance equation that describes the evolution of a unidirectional energy spectrum in shoaling regions. The evolution model is investigated using observations of harmonic generation. Qualitatively the comparisons have shown the ability of the model to generate higher harmonics and a consequent upward shift in the mean frequency. However, quantitatively the model performance needs improvement.

1. Introduction

The evolution of wave spectra in shallow water is significantly affected by the cross-spectral energy transfers between various wave components due to triad

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interactions. Boussinesq equations have been used to establish evolution equations for the complex amplitudes of waves propagating over slowly varying topography, simulating harmonic generation (Madsen and Sørensen, 1993).

For computational efficiency in practical applications, phase-averaged energy-based models are preferred. Abreu et al. (1992) have presented a statistical model for the nonlinear evolution of the frequency-directional spectrum, suitable as a source term in a spectral energy balance equation. The model is based on the nondispersive, nonlinear shallow-water equations and an asymptotic closure (Newell and Aucoin, 1971) for directionally spread nondispersive waves. The restriction to nondispersive waves is easily violated in practical application. The consequence of this is an unwanted behavior of the high-frequency part of the spectrum (dispersive waves).

The purpose of this paper is to investigate the feasibility of a statistical approach to model the average effects of triad interactions in dispersive surface gravity waves. The arrangement of this paper is as follows. In section 2, the kinetic integral for triad wave interactions in gravity waves is presented. The kinetic integral is numerically investigated and the results are analyzed in section 3. In section 4, the kinetic integral is used as an energy source/sink term in a spectral evolution model for investigation against observations of harmonic generation. Finally a discussion and conclusions are given in section 5.

2. Kinetic integral for triad interactions in surface gravity waves

The nonlinear triad interactions in surface gravity waves are treated mathematically using the Zakharov kinetic integral (Zakharov, 1968). The evolution equation of the spectral "energy" density n_k due to the triad interaction between $(\boldsymbol{k},\omega)$, $(\boldsymbol{k}_1,\omega_1)$ and $(\boldsymbol{k}_2,\omega_2)$, where $\boldsymbol{k}$ and ω are the wavenumber vector and the angular frequency respectively, is

$$\frac{dn_k}{dt} = 4 \iint d\boldsymbol{k}_1 d\boldsymbol{k}_2 [V_{k12}^2 N_{k12}\, \mu_{k-1-2}\, \delta_{k-1-2} - 2 V_{1k2}^2 N_{1k2}\, \mu_{k-1+2}\, \delta_{k-1+2}] \tag{1}$$

Here V is the interaction coefficient and

$$N_{k12} = n_1 n_2 - n_k (n_1 + n_2) \tag{2}$$

in which the density n_k is related to the surface elevation wavenumber energy spectrum $E(\boldsymbol{k})$ by

$$n(\boldsymbol{k}) = \frac{(2\pi)^2\, g}{\omega(\boldsymbol{k})} E(\boldsymbol{k}) \tag{3}$$

The factor μ_{k-1-2} in (1) is defined as

$$\mu_{k\text{-}1\text{-}2} = \frac{\Omega}{(\omega_k - \omega_1 - \omega_2)^2 + \Omega^2} \tag{4}$$

Ω is an auxiliary frequency parameter which is small compared to the spectral peak frequency ($\Omega \ll \omega_p$). The factor μ functions as a frequency filter. It is useful to gain insight in its behavior. Using the shorthand notation $\Delta\omega = \omega_k - \omega_1 - \omega_2$ and μ for $\mu_{k\text{-}1\text{-}2}$, Fig. 1 shows μ (in Hz^{-1}) plotted versus $\Delta\omega$ (in Hz) for various values of Ω. The plot indicates that the filter becomes narrower and more spiky by decreasing the value of Ω, but the integral always equals π independent of Ω.

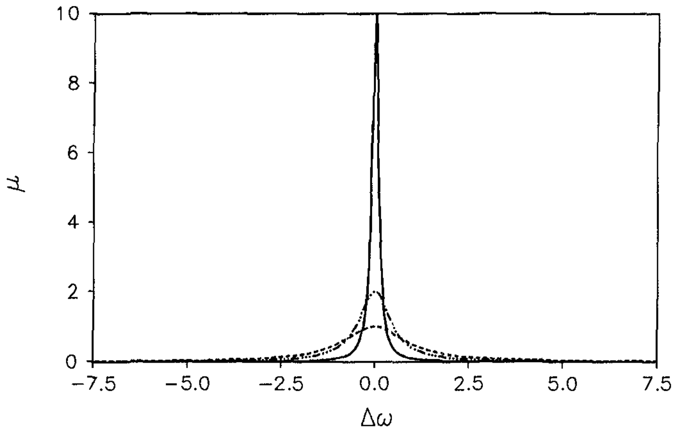

Fig. 1 μ plotted versus $\Delta\omega$ for various values of Ω. Dashed line: $\Omega=1.0$; Dot-dashed line: $\Omega=0.5$; Solid line: $\Omega=0.1$.

In the limit of $\Omega \to 0$, the filter becomes a dirac delta function and $\mu_{k\text{-}1\text{-}2} = \pi\delta(\omega_k - \omega_1 - \omega_2)$. Zakharov et al. (1992) use this limit, which upon substitution in (1) results in the kinetic integral for resonant three-wave interactions. The kinetic integral for resonant interactions gives nonzero contributions only for waves satisfying the resonance conditions

$$\begin{aligned} \omega(\boldsymbol{k}) - \omega(\boldsymbol{k}_1) - \omega(\boldsymbol{k}_2) &= 0 \\ \boldsymbol{k} - \boldsymbol{k}_1 - \boldsymbol{k}_2 &= 0 \end{aligned} \tag{5}$$

Exact resonance cannot be satisfied for surface gravity waves in water of arbitrary depth (Phillips, 1960; Hasselmann, 1962) with the following frequency dispersion

$$\omega^2 = gk\tanh(kh) \tag{6}$$

Thus for practical applications in intermediate depth, a formulation which allows for a degree of phase mismatch between the interacting waves is required to model the cross-spectral energy transfers.

For the problem of off-resonant energetic triad interaction, Holloway (1980) suggested to use (4) for μ_{k12} with a small but finite value of Ω instead of its limit for $\Omega \rightarrow 0$. Thus instead of replacing equation (4) by a dirac delta function we use a smeared delta function (finite value of Ω) to allow for spectral energy transfers in dispersive wavefields. Here we treat Ω as a constant to be determined empirically. The feasibility of this approach will be investigated in the following.

Substituting equation (4) into equation (1) and integrating in $\boldsymbol{k}$ space yield the time evolution of the spectral "energy" density n_k due to triad interaction between components $\boldsymbol{k}$, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$

$$\frac{dn_k}{dt} = 4 \int dk_1 \{ V^2_{k12} N_{k12} \frac{\Omega}{(\omega_k - \omega_1 - \omega_2)^2 + \Omega^2} - 2\, V^2_{1k2} N_{1k2} \frac{\Omega}{(\omega_k - \omega_1 + \omega_2)^2 + \Omega^2} \} \tag{7}$$

This is the final equation describing the slow time evolution of the wave spectrum due to triad interactions. Assuming that the spectral energy is of second-order in nonlinearity: $n(\boldsymbol{k}) \sim \epsilon^2$, one can define the time scale $t_1 = t_p \epsilon^{-2}$ for the slow variation of the wave spectrum. An appropriate value for the filter band-width Ω should be of the order $\Omega \cong t_1^{1} = \epsilon^2 \omega_p$.

In application to spectral wave models based on the energy (or action) balance equation, source/sink terms are normally expressed in terms of energy (or action) density function $E(\omega,\theta)$ of the sea surface elevation. The relation between the wavenumber spectrum $E(\boldsymbol{k})$ and the frequency-directional spectrum $E(\omega,\theta)$ is

$$E(\boldsymbol{k}) = \frac{c c_g}{\omega} E(\omega,\theta) \tag{8}$$

The evolution equation of the frequency-directional energy spectrum $E(\omega,\theta)$ can be found by substitution of (8) into (7) and rearranging

$$\frac{dE(\omega_k,\theta_k)}{dt} = 16\pi^2 g \int_0^{2\pi} \int_0^{\infty} d\omega_1 d\theta_1 \frac{c_2 c_{g2}}{\omega_1 \omega_2^2} (T^+_{k12} - 2T^-_{1k2}) \tag{9}$$

Here

$$T^+_{k12} = V^2_{k12} [\frac{\omega_k^2}{c_k c_{g,k}} E_1 E_2 - \frac{\omega_2^2}{c_2 c_{g2}} E_k E_1 - \frac{\omega_1^2}{c_1 c_{g1}} E_k E_2] \frac{\Omega}{(\omega_k - \omega_1 - \omega_2)^2 + \Omega^2} \tag{10}$$

$$T^-_{1k2} = V^2_{1k2} [\frac{\omega_1^2}{c_1 c_{g1}} E_k E_2 - \frac{\omega_2^2}{c_2 c_{g2}} E_k E_1 - \frac{\omega_k^2}{c_k c_{g,k}} E_1 E_2] \frac{\Omega}{(\omega_k - \omega_1 + \omega_2)^2 + \Omega^2} \tag{11}$$

Note that the units of $E(\omega,\theta)$ is m^2/Hz/rad. The interaction coefficient V is given by

$$
\begin{aligned}
V_{k12} = \frac{g^{1/2}}{8\pi\sqrt{2}} \quad & \{ \, [\boldsymbol{k} \cdot \boldsymbol{k}_1 - (\omega_k \omega_1 / g)^2] \, (\omega_2/\omega_k\omega_1)^{1/2} \\
& + [\boldsymbol{k} \cdot \boldsymbol{k}_2 - (\omega_k \omega_2 / g)^2] \, (\omega_1/\omega_k\omega_2)^{1/2} \\
& + [\boldsymbol{k}_1 \cdot \boldsymbol{k}_2 + (\omega_1 \omega_2 / g)^2] \, (\omega_k/\omega_1\omega_2)^{1/2} \, \}
\end{aligned}
\tag{12}
$$

Equations (9-11) describe the time evolution of wave energy spectrum $E(\omega,\theta)$ due to triad interactions. The first term of the integrand T^{+}_{k12} represents the sum interaction ($\boldsymbol{k}=\boldsymbol{k}_1+\boldsymbol{k}_2$), the second T^{-}_{1k2} the difference interaction ($\boldsymbol{k}=\boldsymbol{k}_1-\boldsymbol{k}_2$). The present formulation is directionally coupled and thus allows for both colinear and noncolinear interactions. Note that the resonance condition (5) does not necessarily have to be fulfilled in (7).

3. Numerical investigation of energy transfer rate

Aim and method of discretization

The general purpose of the investigations is to study the characteristics of the kinetic integral and the dependence of the nonlinear rate (NLR) of energy transfer on the filter bandwidth Ω and relative depth kh. The Jonswap spectrum is used to describe the frequency distribution of the wave energy and a $\cos^2$-distribution is used for the directional spreading. The kinetic integral is calculated by means of simplest trapezium method of integration.

For a sufficiently fine grid (ω,θ) resolution, the values obtained for the kinetic integral do not significantly depend on the choice of grid. This choice is important from the point of view of good resolution of spectral form $E(\omega,\theta)$ and covering a proper interval of frequencies and directions for the output rate dE/dt. After some test calculations, we used a logarithmic frequency distribution with 48 discrete frequencies in the range $0.265 \leq \omega/\omega_p \leq 4$, and a uniform directional grid with 24 discrete components.

Results of calculations and analysis

The calculations of the kinetic integral have been carried out for a range of relative depths $k_p h=3$, 1.2, 0.6, and 0.3. Two values for the nondimensional parameter Ω/ω_p are considered, these are 0.01 and 0.1.

Fig. 2 shows NLR for Jonswap spectrum for $k_p h=0.3$ and $\Omega/\omega_p=0.01$. The results are given in one-dimensional form, i.e., integrated over directions. The results show an energy flux from the region near the primary peak toward the higher harmonics. The behavior of the NLR indicates the following two features. First the maximum of the positive lobe occurs at $f/f_p=1.9$ (less than the location of the first harmonic $2f_p$). This is due to the fact that a triad of waves is considered such that two wave components $(\omega_1,\boldsymbol{k}_1)$ and $(\omega_2,\boldsymbol{k}_2)$ can force a motion at the vector sum or difference wavenumber $k=k_1 \pm k_2$ and frequency $\omega(k)$. This consideration implies that the first harmonic appears in the wavenumber spectrum at $2k_p$, which corresponds to a peak in the frequency spectrum at $\omega < 2\omega_p$ (in intermediate water). The second feature is that the area under the positive lobe is smaller than the area under the negative lobe, implying attenuation in the total energy.

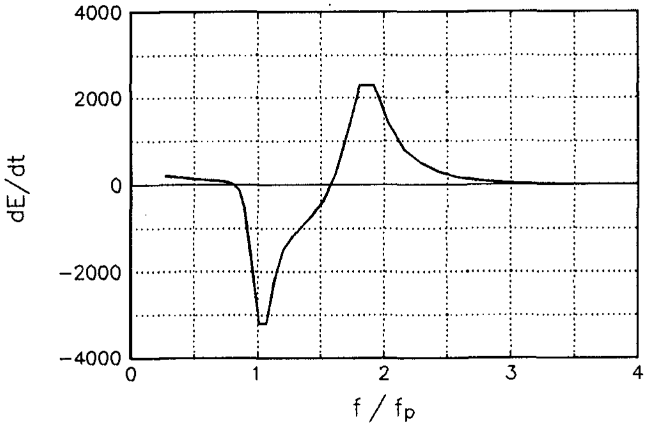

Fig. 2 Nondimensional rate of energy transfer in Jonswap spectrum, $\Omega/\omega_p=0.01$, $k_ph=0.3$, directional spreading; $\cos^2$-distribution.

For quantitative analysis of the calculations we used the following characteristics:
- maximum value of positive lobe of two-dimensional NLR: MT^+;
- maximum absolute value of negative lobe of two-dimensional NLR: MT^-;
- ratio of total NLR (two-dimensional NLR integrated over frequency and direction) to the absolute value of the total negative part of NLR: D

Table 1 summarizes the results for various relative depths and Ω-values. Note that the parameter D is a measure of energy conservation within the system. Positive values of D indicate energy gain and negative values indicate energy attenuation. Table 1 indicates that NLR is roughly proportional to the value of Ω.

Table 1 *Statistics of nonlinear rate (NLR) of energy transfer for Jonswap spectrum with direction spreading of* $\cos^2$*-distribution*

k_ph	3		1.2		0.6		0.3	
Ω/ω_p	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
MT^+	0.1	1.1	1.1	10.7	48	401	2,031	11,071
MT^-	-0.6	-6.1	-1.9	-18.4	-64	-548	-2,566	-17,646
D	-0.48	-0.48	-0.26	-0.32	+0.16	-0.13	+0.12	-0.07

The parameter D measures the percentage of energy gain/loss from the total energy flux across the spectrum. In deep water D reaches large values but there the energy transfers are weak, so that the nonconservation of energy is weak also. As the water depth decreases in shallow water, the NLR increases strongly. In shallow water, although the values of D decrease, the energy attenuation becomes significant because of the strong increase in the NLR values. In fact for $\Omega/\omega_p = 0.01$, D indicates an energy gain in shallow water ($k_p h = 0.6$, 0.3).

The preceding analysis permits to state the following characteristics of NLR of energy transfers due to off-resonant triad interactions:

1. NLR strongly depends on the relative depth $k_p h$. With decreasing $k_p h$, the intensity of NLR increases.
2. NLR has a non-conservativity feature. Generally it results in an energy attenuation in intermediate and shallow water depths.
3. The intensity of NLR varies in proportion to Ω.

4. Spectral evolution

4.1 Model formulation and implementation

Assessment of the characteristics of the kinetic integral for triad interactions requires verification with observations. For simulation of the spectral evolution, we need to develop a spatial evolution model for the energy spectrum with a source/sink term representing the effect of triad wave interactions. Since the observations used here are measured in flume experiments that are characterized by long-crested waves, the following one-dimensional energy balance equation is used

$$\frac{d}{dx}[c_{g,k}\, E(\omega_k)] = S_k \qquad (13)$$

Here $E(\omega_k)$ is the frequency energy density, $c_{g,k}$ is the one-dimensional group velocity and S_k is the net source/sink term. To implement the effect of triad wave interactions in equation (13), the kinetic integral (9-11) is cast in an energy source/sink term for unidirectional waves as follows:

$$S_{tr}(\omega_k) = 16\pi^2 g \int_0^\infty d\omega_1 \frac{c_2 c_{g2}}{\omega_1 \omega_2^2} \left(T^+_{k12} - 2T^-_{1k2}\right)$$

$$T^+_{k12} = V^2(\omega_k,\omega_1)\left[\frac{\omega_k^2}{c_k c_{g,k}} E_1 E_2 - \frac{\omega_2^2}{c_2 c_{g2}} E_k E_1 - \frac{\omega_1^2}{c_1 c_{g1}} E_k E_2\right] \frac{\Omega}{(\omega_k-\omega_1-\omega_2)^2+\Omega^2} \qquad (14)$$

$$T^-_{1k2} = V^2(\omega_1,\omega_k)\left[\frac{\omega_1^2}{c_1 c_{gl}} E_k E_2 - \frac{\omega_2^2}{c_2 c_{g2}} E_k E_1 - \frac{\omega_k^2}{c_k c_{g,k}} E_1 E_2\right] \frac{\Omega}{(\omega_k-\omega_1+\omega_2)^2+\Omega^2}$$

The energy source/sink term given in (14), for the effect of triad wave interactions, represents a (positive/negative) contribution to the temporal rate of change of spectral density. The energy balance equation (13) comprises a set of first-order ordinary differential equations that describe the evolution of the energy spectrum $E(\omega)$. Giving the initial energy spectrum at the upwave boundary, equation (13) has been numerically integrated using a fourth-order Runge-Kutta method.

4.2 Simulation of spectral evolution

In this section, the evolution model (13) together with the triad source term (14) is investigated using observations for harmonic generation in random waves propagating over a shallow bar (Beji and Battjes, 1993) as well as over a beach profile (Arcilla et al., 1994).

The investigation of the nonlinear rate (NLR) of energy transfers presented in section (3) has shown that the present formulation is not conservative. To ensure energy conservation in the simulation of spectral evolution the following *ad hoc* method is used. First the NLR of energy transfer is estimated as a first guess using the present formulation. Next the integral (of NLR) over the spectrum which represents the total energy gain/loss I is computed. The NLR of energy transfer is then rescaled by adding (or subtracting) the quantity I. If I is negative, implying energy loss, then the area of negative lobe is reduced with I in proportion to the values of the NLR. On the other hand if I is positive, implying energy gain, then the area of positive lobe is reduced with I in proportion to the values of the NLR.

To simulate energy dissipation due to wave breaking over a beach profile, the energy balance equation (13) is supplemented with a source term for depth induced wave breaking after Eldeberky and Battjes (1996), in which the total energy dissipation due to breaking in random waves is calculated according to Battjes and Janssen (1978) and spectrally distributed in proportion to the spectral levels.

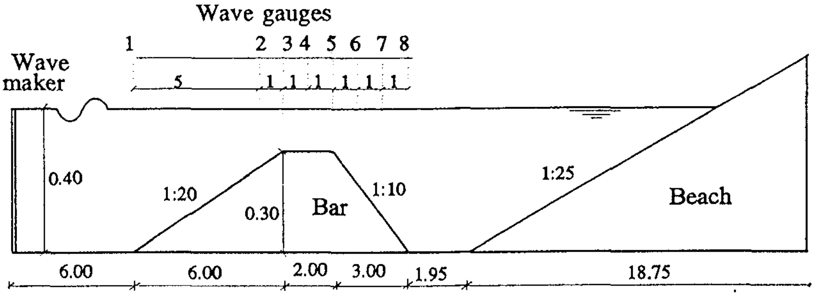

Fig. 3 Layout for the experimental setup of Beji and Battjes (1993). All lengths are expressed in meters.

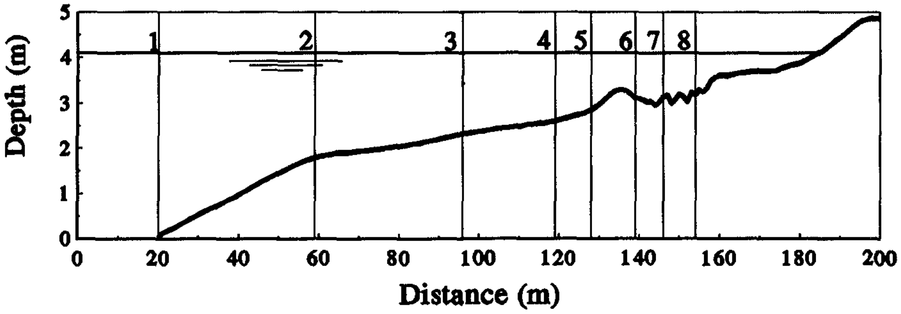

Fig. 4 Bed profile and locations of wave gauges (Arcilla et al., 1994)

To examine the sensitivity of the spectral evolution to the choice of Ω, two values are used in the computations: $\Omega/\omega_p=0.01$ and 0.1. The computed spectra are compared to the measured ones in nonbreaking waves propagating over a bar (Fig. 3) and breaking waves over a beach profile (Fig. 4). The results are given in Figs. 5 and 6 respectively. The comparisons indicate the following characteristics of the triad source term:

1. The intensity of energy transfers from the primary spectral peak to the higher frequencies is mainly controlled by the choice of Ω-value. Increasing Ω-value results in stronger energy transfers, extended to higher frequencies.
2. The energy transfers to higher harmonics are underestimated when $\Omega/\omega_p=0.01$, and overestimated when $\Omega/\omega_p=0.1$. The latter results in an unwanted behavior for the energy spectrum at the high frequency range (spectral tail).
3. The second spectral peak (in frequency-domain) is shifted to a lower frequency compared with observation. It appears at a frequency less than two times the primary peak. This is ascribed to the fact that triads are considered such that $k=k_1+k_2$, which results in $\omega_k<\omega_1+\omega_2$ in intermediate water depths.

4.3 Sensitivity to the filter bandwidth

The previous results for the computed spectral evolution have shown dependence of the NLR of energy transfer on the choice of the filter bandwidth Ω. Additional numerical simulations for wave propagation over a shallow bar and beach profile have been carried out with different values of Ω. To evaluate the variation in the spectral evolution for various values of Ω, the variations in the mean frequency of the spectrum are computed. The mean frequency of the energy spectrum is defined as

$$f_m = \frac{\int \omega E(\omega)\, d\omega}{2\pi \int E(\omega)\, d\omega} \qquad (15)$$

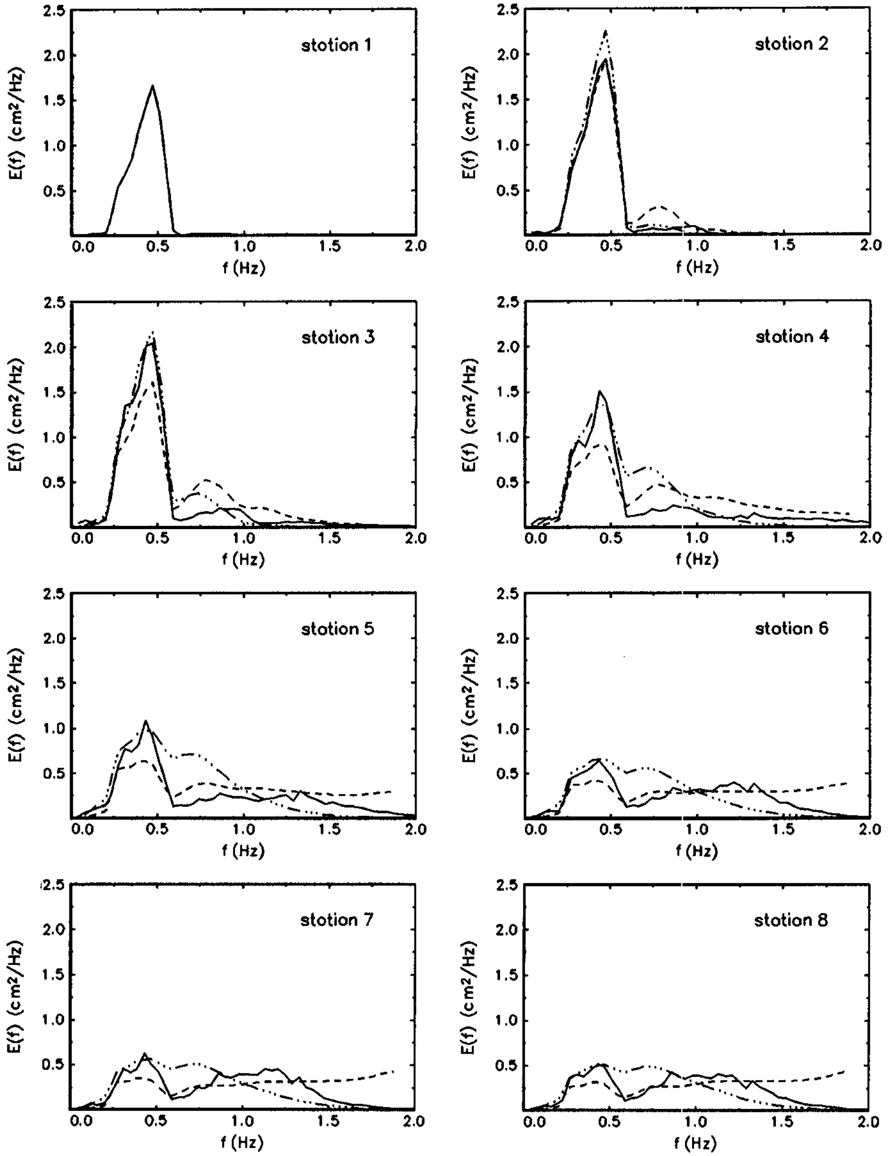

Fig. 5 Energy spectra from experiments (solid lines) and from the evolution model: with $\Omega/\omega_p=0.01$ (dashed lines) and $\Omega/\omega_p=0.1$ (dot-dashed lines) for waves propagating over a shallow bar.

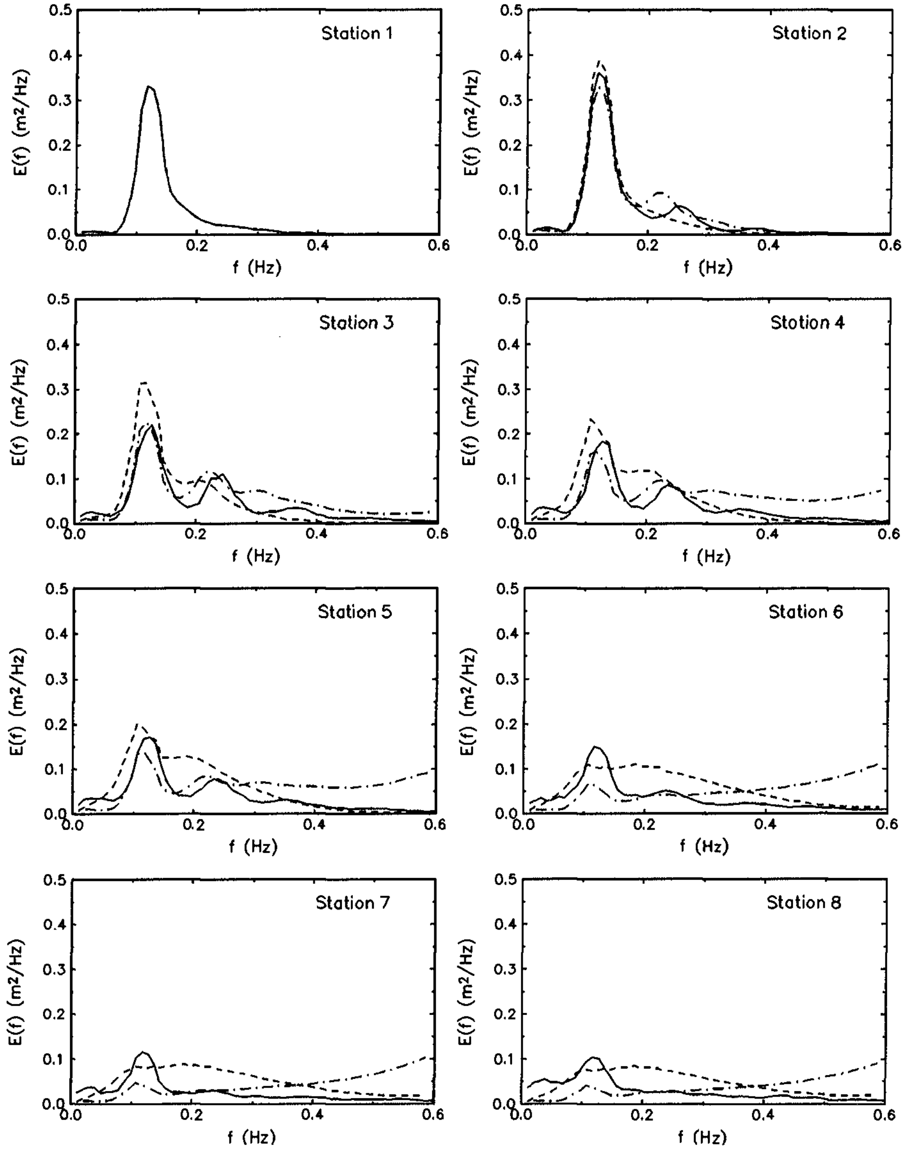

Fig. 6 Energy spectra from experiments (solid lines) and from the evolution model: with $\Omega/\omega_p=0.01$ (dashed lines) and $\Omega/\omega_p=0.1$ (dot-dashed lines) for waves propagating over a beach profile.

Fig. 7 shows the observed variations in the mean frequency in waves passing over a shallow bar and those computed by the evolution model with different values of Ω/ω_p. The observed variation in the mean frequency shows a rapid increase (from 0.43 Hz to 0.85 Hz) over the upslope side and the horizontal part of the bar, which is ascribed to generation of higher harmonics. Beyond the bar crest, the mean frequency remains at a high level without significant change. The computed variations in the mean frequency using the evolution model show a strong dependence on the value of the parameter Ω. The larger the value of Ω/ω_p, the stronger the energy transfers and hence the shift in the mean frequency to higher harmonics. From the results one can see that the best choice for the parameter Ω/ω_p, for best simulation of the observed shift in the mean frequency, in this case is between 0.01 and 0.03.

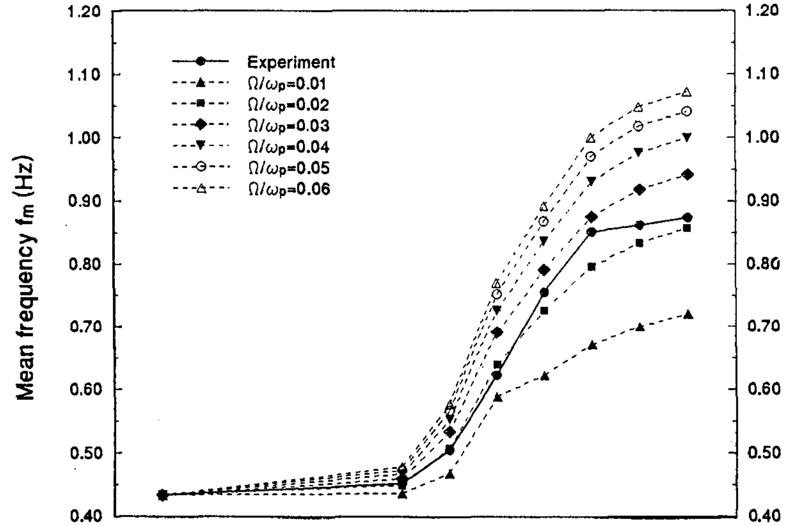

Fig. 7 Spatial variation of mean frequency f_m in waves propagating over a shallow bar. Solid line: experiment; Dashed lines: evolution model using different values for Ω/ω_p

Fig. 8 shows the observed variations in the mean frequency in waves propagating over a beach profile and those computed by the evolution model with different values of Ω. The observed variation in the mean frequency shows a rapid increase in intermediate water from 0.14 Hz to 0.21 Hz due to harmonic generation. In very shallow water, the mean frequency nearly attains a constant level. The computed variations in the mean frequency using the evolution model show a strong dependence on the value of Ω. In intermediate water depths (between stations 1 and 3) computations with $\Omega/\omega_p=0.04$ and 0.05 seem to best match the observed shift in the mean frequency. In shallow water, all computations with different values of Ω result in a trend which differs strongly from the observed one, significantly overestimating the mean frequency.

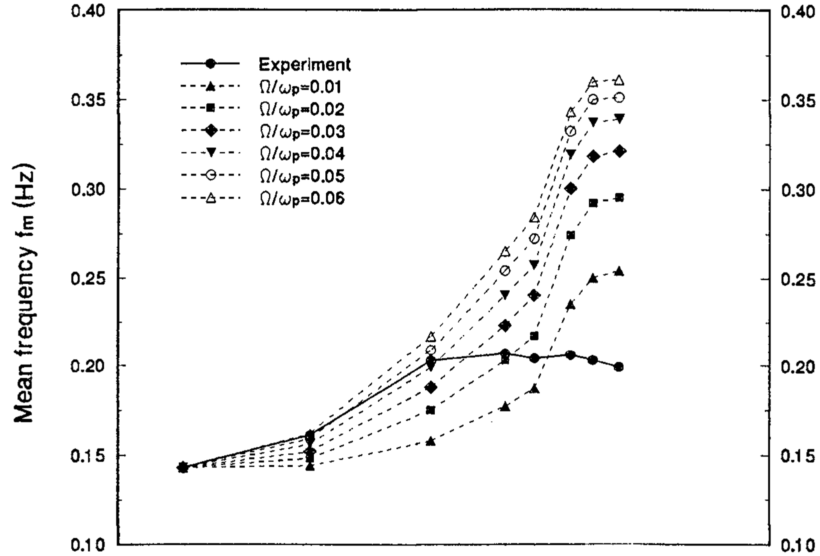

Fig. 8 Spatial variation of mean frequency f_m in waves propagating over a beach profile. Solid line: experiment; Dashed lines: evolution model using different values for Ω/ω_p

5. Discussion and conclusions

The feasibility of a statistical approach to model the average effects of triad interactions on the evolution of wave spectrum is investigated. The approach is based on the Zakharov kinetic integral for resonant triad interactions in capillary gravity waves. For application to surface gravity dispersive waves, the kinetic integral is supplemented with a frequency filter to allow for the off-resonant energetic interactions. The filter bandwidth Ω is of small but finite value resulting in a smeared delta function. Ω is treated as a constant to be determined empirically. The interactions integral is used as a source term in an evolution model to simulate observations of harmonic generation in waves propagating over a shallow bar as well as over a beach profile. In general the comparisons have shown the ability of the model to generate higher harmonics and a consequent upward shift in the mean frequency.

The consequences of treating the filter bandwidth Ω as a constant have resulted in an energy attenuation and unguaranteed spectral evolution in some cases. Holloway (1980) proposed to treat Ω as a prognostic variable with magnitude related to the rate of interaction of the three components involved in the interaction and increases with increasing nonlinearity. In Holloway's approach, the magnitude of Ω for the interaction between l, m, and n needs to be determined first by solving three equations representing the interaction rates of the three components. These three equations for each possible triad together with the spectral evolution equations represent a closed set of equations guiding the evolution of the energy density spectrum.

In principle the approach of Holloway (1980) sketched above may be applied to provide an estimate for the parameter Ω. This may achieve a better prediction of the evolution of the energy spectrum. On the other hand the extensive computational efforts required to resolve the closed set of equations are a concern. For computational efficiency in practical applications, we recommend a parametrized source term for triad wave interactions (Eldeberky, 1996, Chapter 7 and Eldeberky and Battjes, 1997)

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References

Abreu, M., A. Larraza, and E.B. Thornton, 1992, Nonlinear transformation of directional wave spectra in shallow water, *J. Geophys. Res.*, 97, pp. 15579-15589.

Arcilla, A.S., J.A. Roelvink, B.A. O'Connor, A.J.H.M. Reniers, and J.A. Jimenez, 1994, The Delta flume '93 experiment, *Proc. Coastal Dynamics Conf. '94*, Barcelona, Spain, pp. 488-502.

Battjes, J.A., and J.P.F.M. Janssen, 1978, Energy loss and set-up due to breaking of random waves, *Proc. Coastal Eng. Conf.*, 16th, pp. 569-587.

Beji, S., and J.A. Battjes, 1993, Experimental investigation of wave propagation over a bar, *Coastal Eng.*, 19, pp. 151-162.

Eldeberky, Y., and J.A. Battjes, 1996, Spectral modeling of wave breaking: Application to Boussinesq equations, *J. Geophys. Res.*, 101(C1), pp. 1253-1264.

Eldeberky, Y., 1996, Nonlinear transformation of wave spectra in the nearshore zone, *Ph.D. thesis*, Delft University of Technology, The Netherlands.

Eldeberky, Y., and J.A. Battjes, 1997, Parameterized energy formulation for triad wave interactions, In preparation.

Hasselmann, K., 1962, On the non-linear energy transfer in a gravity wave spectrum, Part 1. General theory, *J. Fluid Mech.*, 12, pp. 481-500.

Holloway, G., 1980, Oceanic internal waves are not weak waves, *J. Physical Ocean.*, 10(6), pp. 906-914.

Madsen, P.A., and O.R. Sørensen, 1993, Bound waves and triad interactions in shallow water, *J. Ocean Engng.*, 20(4), pp. 359-388.

Newell, A.C., and P.J. Aucoin, 1971, Semidispersive wave systems, *J. Fluid Mech.*, 49(3), pp. 593-609.

Phillips, O.M., 1960, On the dynamics of unsteady gravity waves of finite amplitude. Part 1, *J. Fluid Mech.*, 9, pp. 193-217.

Zakharov, V.E., 1968, Stability of periodic waves of finite amplitude on the surface of a deep water, Engl. trans. *Soviet Physics, J. Appl. Mech. Tech. Phys.*, 4, pp. 86-94.

Zakharov, V.E., V.S. L'vov, and G. Falkovich, 1992, *Kolmogorov spectra of turbulence, I. Wave Turbulence*, Springer verlag.