CHAPTER 46

A comparison of analysis methods for wave pressure data

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ABSTRACT

A comparison of a range of wave pressure analysis methods has been conducted using both numerical data generated by a Fourier steady wave method and experimental data gathered in a wave flume. The results show that the local methods (local sinusoid approximations and local polynomial approximations) are more accurate than the traditional global linear spectral method. A new local polynomial approximation method shows improvement compared to similar methods developed previously.

INTRODUCTION

Pressure transducers have been used for many years by coastal engineers for measuring the wave climate at a site of interest. These instruments are well suited to this purpose, being unobtrusive and robust. Problems do occur, however, when the surface elevation is inferred from the pressure data. The problem is fundamentally a poorly posed one, the subsurface pressure data being an attenuated representation of the flow conditions. In addition, the nonlinearity of waves in the coastal zone means that the traditional linear analysis methods may be inaccurate for such an application. The aim of the present study is to evaluate existing analysis techniques and if possible to improve the accuracy and robustness of those that are more suitable. Both numerical and experimental data was used in the analysis.

The methods examined include one global method and several local methods. The global method is the linear spectral method, described at length by Bishop and

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Donelan (1987). The local methods are the first order Local Sinusoidal Approximation (LSA) method, the empirical LSA method (Nielsen, 1989), the fully nonlinear Local Polynomial Approximation (full LPA) method (Fenton, 1986) and the simple LPA method (Fenton and Christian, 1989). A new method, using LPA, is presented and tested - the nonlinearly optimised LPA (NOLPA). There is a new local Fourier approximation method (Barker and Sobey, 1996) based upon the surface method of Sobey (1991), which is being published in these proceedings.

ANALYSIS METHODS

Linear spectral method

The most common analysis technique used in this area is the linear spectral method, which has been presented and comprehensively discussed by Bishop and Donelan (1987). Briefly, the spectrum of the water surface elevation, $S_s(\omega)$ is related to the spectrum of the dynamic pressure at the pressure transducer, $S_p(\omega)$ by:

$$S_s(\omega) = \left[\frac{N(\omega)}{K_p(\omega)}\right]^2 S_p(\omega) \ , \tag{1}$$

where ω is the angular frequency of each Fourier component, $N(\omega)$ is an empirical correction factor and $K_p(\omega)$ is the pressure response factor defined as:

$$K_p(\omega) = \frac{\cosh k(\omega) y_p}{\cosh k(\omega) D}, \tag{2}$$

where D is the mean water level (MWL) and y_p is the height of the pressure transducer from the sea bed.

The authors have, for the purpose of this paper set $N(\omega)$ to 1 for all ω. Bishop and Donelan consider the presence of $N(\omega)$ as an attempt to compensate for poor measurements, instruments and/or analysis methods. As the authors are using 'exact' nonlinear waves and pressure traces to conduct these tests none of the above need be considered and only potential inadequacies in the linear spectral method will be highlighted.

It was necessary to determine a maximum ω above which the pressure response factor was not applied, as doing so would cause the method to 'blow-up' when $K_p(\omega)$ became small. To determine this limit, the ratio of the spectral amplitude to $K_p(\omega)$ was determined at each ω. When this ratio began to increase at frequencies above the spectral peak the method had started to 'blow-up'.

Local sinusoidal approximation methods

The first of the computer-based local methods to emerge were the two developed simultaneously by Nielsen (1989). Both are known as LSA methods in which a sine curve is passed through three points from the pressure data which are adjacent or with a small number of intermediate points between them. This sine curve is used to determine the 'local' frequency. The water surface elevation at the instant in time of the central point can then be calculated by one of two methods. One method applies a transfer function derived from stretched linear theory (the first order LSA), the other applies a semi-empirical transfer function.

First order local sinusoidal approximation

This transfer function is expressed:

$$\hat{\eta}_n = \frac{p_n}{\rho g} \frac{\cosh k_n \left(D + \frac{p_n}{\rho g} \right)}{\cosh k_n z}, \tag{3}$$

where $\hat{\eta}_n$ is the water surface elevation corresponding to the nth central pressure reading, p_n is the nth central pressure reading, k_n is the nth wave number derived from the local frequency calculated from the three pressure readings and ρ is the water density.

The local frequency is determined by:

$$\hat{\omega}_n^2 = \frac{-p_{n-M} + 2p_n - p_{n+m}}{p_n (M\delta)^2}, \tag{4}$$

which is an estimate corrected by:

$$\omega_n^2 = \hat{\omega}_n^2 \left[1 + \frac{1}{12} (\hat{\omega}\delta)^2 \right]. \tag{5}$$

Empirical local sinusoidal approximation

Nielsen derived this transfer function from Dean (1974):

$$\hat{\eta}_n = \frac{p_n}{\rho g} \left[A\left(\frac{y_p}{D} \right) \frac{-p_{n-M} + 2p_n - p_{n+M}}{p_n g (M\delta)^2} \left(D + \frac{p_n}{\rho g} - y_p \right) \right] \tag{6}$$

where M is a positive integer the value of which can be estimated by $M \approx \sqrt{D/g}/\delta$ and is a multiplier which smoothes noisy data by selecting more widely spaced points instead of adjacent ones in the frequency calculation, δ is the sampling period of the data, and $A(y_p/D) = 0.67 + 0.34 y_p/D$ and accounts for the height of the pressure transducer above the sea bed.

Both the above are extremely simple to apply with little computational effort required. In this study M was set to 1 when used with numerically generated data as no noise was present in the input and the authors felt that this gave a better indication of the method's robustness. With real data M was calculated by the above equation.

Local polynomial approximations

The other two local methods are LPA techniques and were developed by Fenton (1986) and Fenton and Christian (1989). Both utilise the principle of low-degree polynomial approximation, partly based on least-squares approximation methods and partly on solving locally the full nonlinear equations of motion.

Fully nonlinear local polynomial approximation

The approach followed for the full LPA (Fenton, 1986) was to approximate the complex velocity potential as follows:

$$\begin{aligned} w(x,y,t) &= \phi(x-ct,y) + i\psi(x-ct,y) \\ &= c_e(z-ct) + \sum_{j=0}^{J} \frac{a_j}{j+1}(z-ct)^{j+1}, \end{aligned} \tag{7}$$

where $z=x+iy$, c_e is the Eulerian current and the surface elevation is given by

$$\eta(x,t) = \sum_{j=0}^{J} b_j (x-ct)^j \ . \tag{8}$$

From equation (7) ϕ satisfies Laplace's equation identically throughout the flow and the bottom boundary condition ($v(x,0,t)=0$) is satisfied if the coefficients a_j and b_j are real.

To satisfy the necessary boundary conditions on the free surface the steady kinematic equation is invoked such that:

$$\psi(x-ct, \eta(x-ct)) = -Q, \tag{9}$$

where Q is a constant, and the steady Bernoulli equation:

$$R = \frac{1}{2}\left|\frac{dw}{d(z-ct)}\right|_s^2 + \eta, \tag{10}$$

where R is the Bernoulli constant and s denotes the surface y=η.

Bernoulli's equation is also written about the position $(0,y_p)$, the position of the pressure transducer, expressed as a Taylor series in x-ct:

$$p(0,y_p,t) = R - \frac{1}{2}\left|\frac{dw}{d(z-ct)}\right|_{y_p}^2 - y_p = \sum_{j=0}^{J} p_j(-ct)^j \, . \tag{11}$$

The p_j are calculated using a least squares fit across K data points where K is an odd integer greater than or equal to $J + 3$ with $(K - 1)/2$ data points each side of the point of interest. For the above method $K = 21$ was found to give good results for both smooth and noisy data and is the value used in these tests. $J = 4$ was found to be the optimum degree of approximation.

By manipulation of equations (7), (8), (9), (10) and (11) and isolating powers of $(x$-$ct)$, a system of nonlinear equations in terms of the unknown a_j and b_j is obtained. The solution of these equations is performed for each point in the pressure series using direct iteration to achieve convergence. The surface elevation data obtained was then passed though a simple 3-point smoothing routine. Space does not permit explanation of the details regarding the solution of these equations for the a_j and b_j coefficients.

Simple local polynomial approximation

The simple LPA (Fenton and Christian, 1989) is somewhat simpler in that a point value is used to describe each value of η at $t = 0$ as opposed to the polynomial expansion across the window in the former method. The resulting solution is much simplified with the extraction of a system of quite manageable nonlinear equations. Unfortunately, there are only 6 equations in that set with a total of seven unknowns. It was necessary to introduce more equations to be able to find a solution. The first assumption required is that the wave speed c is given by long wave theory:

$$c = \sqrt{g\eta}\,, \tag{12}$$

and the second is that the main fluid velocity component a_0 is given by:

$$a_0 = -c. \tag{13}$$

For the results that follow the value of K was set to 17.

Nonlinearly optimised local polynomial approximation

This method (NOLPA) was developed using some of the ideas mentioned in the two sections above. The same basic idea of approximating the velocity potential by a low degree polynomial over a small window of data remains, but the method of solving this nonlinear problem is quite different.

The velocity potential ϕ is represented by equation (7) and substituted into Bernoulli's equation at the pressure transducer

$$\frac{p}{\rho}(x,y_p,t) = R - \frac{1}{2}\left(u_p^2 + v_p^2\right) - gy_p, \tag{14}$$

where u_p and v_p are the horizontal and vertical velocities ($\partial\phi/\partial x$ and $\partial\phi/\partial y$ respectively) at the pressure transducer (y=y_p). Neglecting the kinematic boundary condition, this equation can be solved across K pressure points (K is an odd integer greater than or equal to J+3) using a nonlinear least squares solver such as the Levenberg-Marquardt method (Grace, 1994). The actual function to be optimised by this method is written

$$F = \sum_{k=1}^{K}\left[R - \frac{1}{2}u_{pk}^2 - \frac{1}{2}v_{pk}^2 - gy_p - \frac{p_k}{\rho}\right]^2. \tag{15}$$

The initial estimates of the unknowns R/gD, $c/\sqrt{gd}$, a_0 - a_J were set to the still water values {1.5, 1, -1, 0, 0, 0, 0} and the time vector t_k (embedded in the velocity equations) was scaled such that the temporal length of the window was between -1 and +1. This, along with the appropriate rescaling if the system on equations approximates orthogonality in the velocity potential (Fenton, 1994), a property not held by the polynomial chosen.

Once this solution has been obtained, the location of the surface can be calculated. Bernoulli's equation at the unknown surface η

$$F_{\eta t} = R - \frac{1}{2}u_{\eta t}^2 - \frac{1}{2}v_{\eta t}^2 - g\eta_t \tag{16}$$

is a nonlinear equation in one unknown and can be solved at any time within -1 and +1. In the results that follow the surface estimates shown were calculated in the middle of the window (t = 0).

When dealing with experimental data each window was smoothed using an equally weighted moving average filter which was made larger or smaller depending upon the quality of the pressure data used. $K = 11$ was used for numerically generated smooth data while $K = 17$ was used for real data. As with the other LPA method it was found to be unnecessary to use a degree of approximation greater then $J = 4$.

It was sometimes required, when analysing extreme waves, such as in Figure 4, that the length of the window be made a smaller proportion of the wavelength than was normally used to allow convergence of the least squares routine. To give an adequate number of data points in the smaller window, the number of points actually used in the solution was increased by cubic spline interpolation, adding data points in between the originally sampled data, in a similar manner to Sobey (1991) with his surface based local Fourier method.

Methodology

The methods described above were tested with data from a Fourier method for generating steady waves (Fenton, 1988) and with experimental data obtained in a wave flume. The input from the Fourier method was generated for three different wavelengths, λ/D=6.31, 10 and 19.95 all with 64 data points per wavelength plus 5 extra points at each end of the data. The waves shown are extreme with a height/depth ratio (H/D) of 0.6. The gathering of the experimental data is described below.

Experimental Procedure

Experiments were performed in the large wave flume in the Department of Mechanical Engineering at Monash University. The facility is 52m long, 2.5 m wide with two working sections of 4m and 2.5m connected by a ramp. The shallower working section was modified for a working depth of 1.5m with a false floor. It is capable of generating both regular and irregular waves using a feedback system to obtain the desired sea state. There is a wave absorbing beach at the far end which allows less than 10% reflection across the range of working frequencies.

Four pressure transducers were set into a false wall in the flume at y_p/D ratios from 0 to 0.5. Two more transducers were mounted further 'downstream' at y_p/D of 0 and 0.5 to obtain phase data if required. The surface elevation above the first column of transducers was measured using a capacitance wave probe.

Results

Numerically generated data

Figure 1 shows results for a short wave with $\lambda/D = 6.31$, $H/D = 0.6$ with the pressure measured on the bed ($y_p/D = 0$). It seems that LPA methods should not be used for waves shorter than this, the full LPA and the NOLPA underestimating the crest. The traditional linear spectral method and the simple LPA do not perform at all well. Considering the considerably attenuated signal that occurs with such a short wave it is significant that the two LSA methods are almost identical and predict the wave height well although the wave shape is poor. Note the failure of the LSA methods on the wave "shoulder".

Figure 2 shows a longer wave with $\lambda/D = 10$ and all other properties the same as the previous figure. At this wavelength/depth ratio, the level attenuation of the pressure signal is much less and the quality of the output from all methods is higher. Only the simple LPA underestimates the crest while the full LPA and the NOLPA perform extremely well. The LSA methods perform in a similar manner to the previous figure and the linear spectral method does not describe the trough of the wave very well.

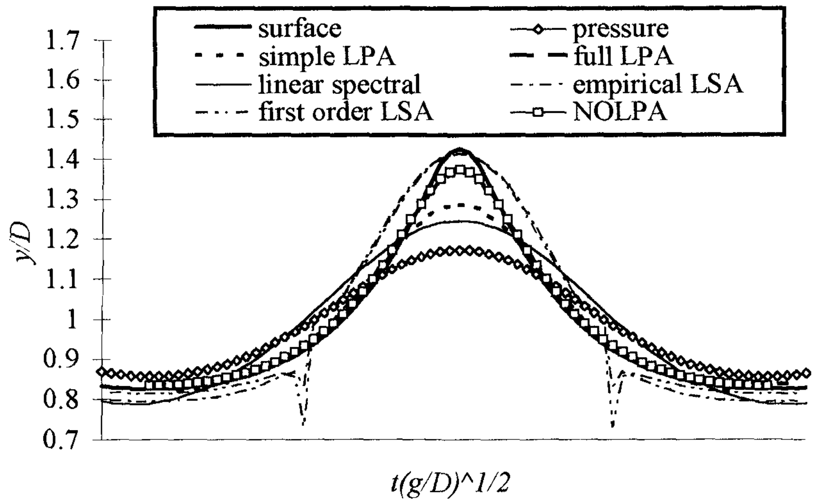

Figure 1: $\lambda/D = 6.31$, $H/D = 0.6$, $y_p/D = 0$

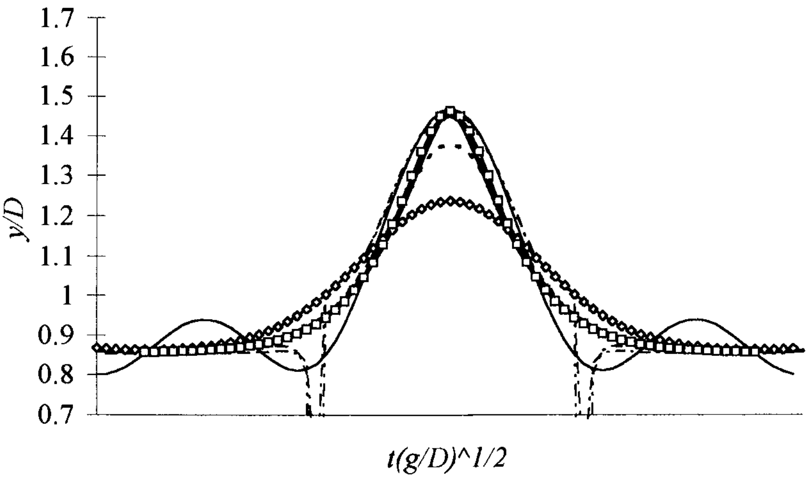

Figure 2: $\lambda/D = 10.0$, $H/D = 0.6$, $y_p/D = 0$

The wave in Figure 3 is identical to that in the previous figure, the difference being that the pressure was sampled higher in the water column, at $y_p/D = 0.5$. The most noticeable improvement is the result from the linear spectral method, the higher measurement position detecting the higher frequency components required to describe the trough of the wave. The performance of the simple LPA is much improved, while all other methods are similar in their performance to Figure 2.

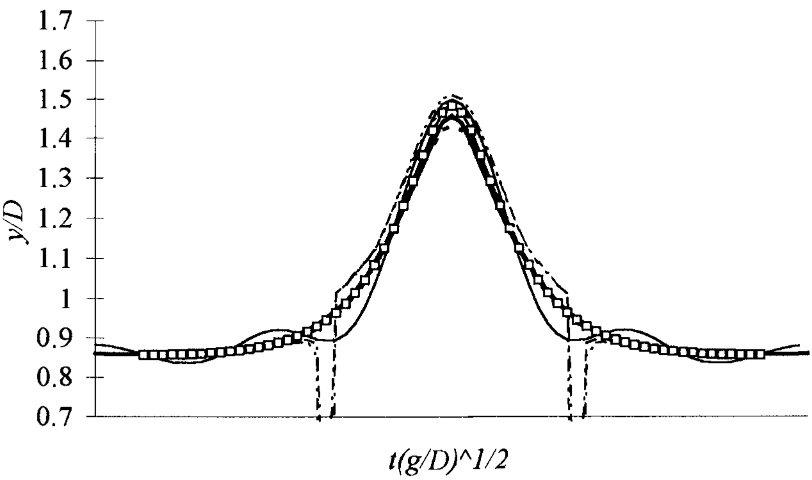

Figure 3: $\lambda/D = 10.0$, $H/D = 0.6$, $y_p/D = 0.5$

Figure 4 is a longer wave again, with λ/D = 19.95 but with the pressure, once again measured at the bottom. Once again the linear spectral method cannot describe both the sharp crest of the wave and the long flat trough, as the high frequency components attenuate completely before reaching the sea bed. The LSA methods predict both the profile and the height well, with a slight overprediction of the crest, as does the NOLPA. The simple LPA underpredicts the crest, as could be expected, while the full LPA performs very well.

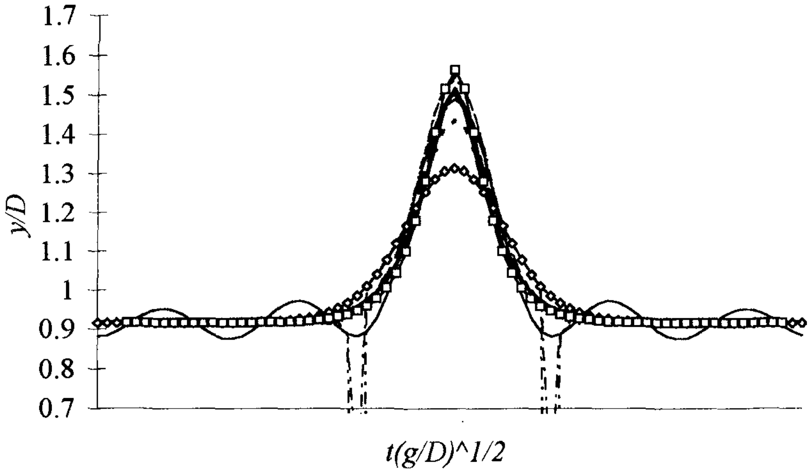

Figure 4: λ/D = 19.95, $H/D = 0.6$, $y_p/D = 0$

Experimental data

The irregular wave results shown in this section were generated from a JONSWAP spectrum with the properties: peak frequency, f_0 = 0.4 Hz, D = 1.55 m, Stokes current, $c_s = 0$, and a sampling rate of 20 Hz. The exception is Figure 7 where there is an Eulerian current, c_e, = 0.134 m/s in the direction of wave propagation. This peak frequency corresponds to a λ/D ration of approximately 6 in this water depth, a condition where the LPA methods are not expected to perform at their best. Unfortunately the flume is not capable of generating irregular waves with a lower peak frequency.

The full LPA method is not shown in these results. This method was shown to be extremely sensitive to noisy data in an earlier, numerical comparison (Townsend and Fenton, 1995), where convergence appeared to be case dependent with regard to the number of data points per window.

The almost identical performance of the two LSA methods was shown in the previous section and due to the more cluttered appearance of the experimental results, the first order LSA has not been included. The empirical LSA has also proved to be more robust when dealing with noisy data.

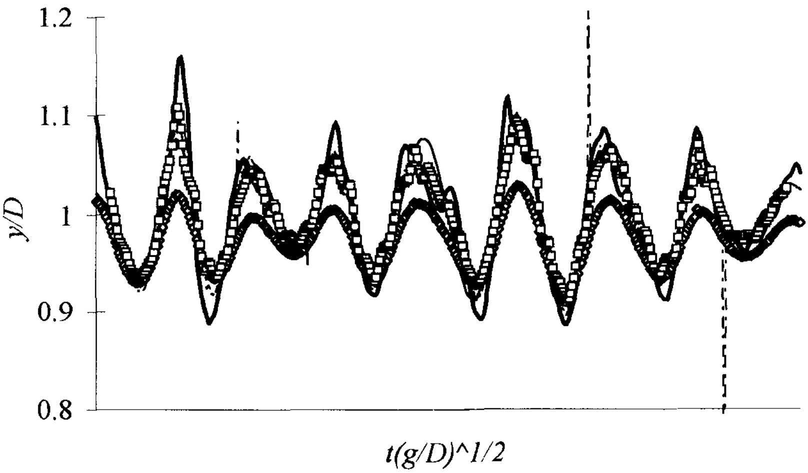

Figure 5: $y_p/D = 0$, $c_s = 0$

Figure 5 shows a zero Stokes current condition with the pressure measured at the floor of the flume. The effect of the noise in the pressure signal is evident, with the linear spectral method providing the smoothest output due to its inherent low pass filtering. The empirical LSA performs well, with the exception of the expected occasional failure, while the simple LPA and the NOLPA perform almost identically, which is well considering the shortness of the waves.

The wave trace shown in Figure 6 is the same as in Figure 5 except that the pressure data was measured almost halfway up the water column ($y_p/D = 0.484$). All methods exhibited an improvement with this data, with the signal to noise ratio being more favourable.

Figure 7 shows a wave train with a superimposed Eulerian current of 0.134 m/s. In this case the first order LSA differs from the empirical LSA in that, like the linear spectral method, it can be modified to account for the current. The linear spectral method and the first order LSA were modified by including the current in the wave number calculation. The LPA methods already had the current included in the expression for ϕ (equation 7). Surprisingly the differences between the first order LSA and the empirical LSA are quite small. The current level is the highest that can be generated in the wave flume but it seems that it is not high enough to have a significant effect on the wave properties. All other methods performed in a manner similar to the other irregular wave traces

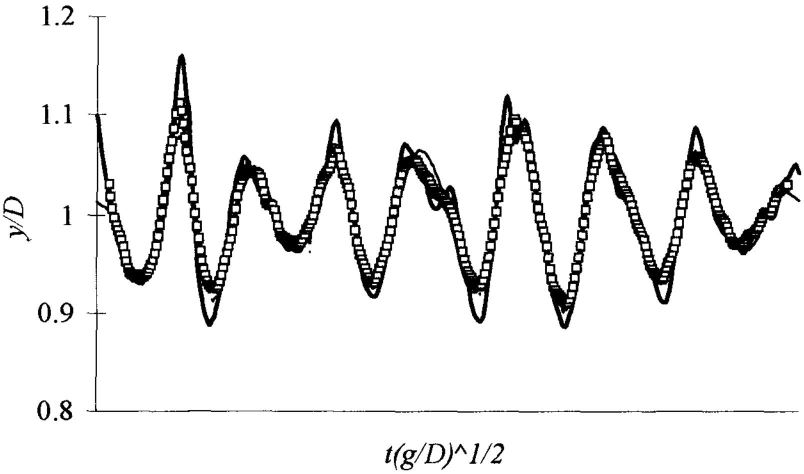

Figure 6: $y_p/D = 0.484$, $c_s = 0$

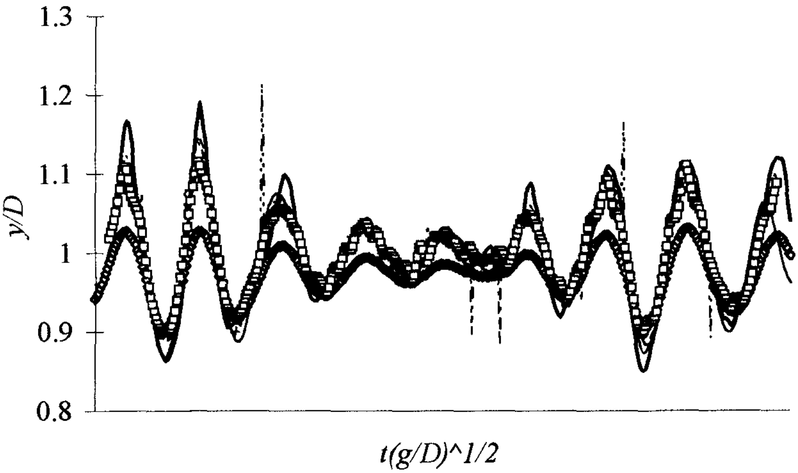

Figure 7: $y_p/D = 0$, $c_e = 0.134$ m/s

Figure 8 shows a moderately long regular wave with wave period, $T = 4.1$s and zero stokes current. This gives a λ/D ratio of approximately 12 in the wave flume. All methods perform well with this moderately long and high wave, although it can be seen that the empirical LSA is affected by the noise in the pressure signal.

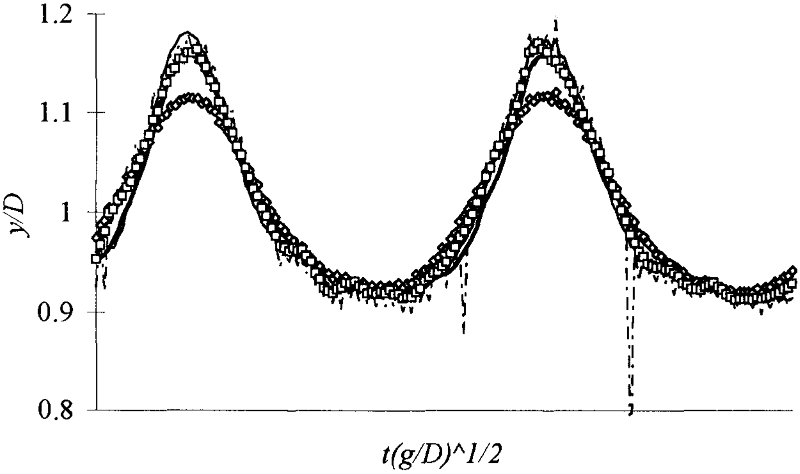

Figure 8: regular wave, $T = 4.1$ s, $y_p/D = 0$, $c_s = 0$, $\lambda/D \approx 12$

Conclusions

A comprehensive comparison of a variety of global and local wave analysis methods has been conducted. The aim was to compare the accuracy of these methods for determining wave surface elevation data from sub-surface pressure readings. Overall the local methods are more effective in dealing with highly nonlinear steady waves than is the global linear spectral method. Unsurprisingly, all methods provided better results with pressure readings taken higher up in the water column.

The linear spectral method performed better for moderate waves but behaved poorly when the waves were steep and y_p/D was small. Both the LSA methods behaved similarly, generally predicting wave heights well. These methods were affected by noise although a post analysis smoothing routine would result in fairly smooth curves. Of the two, the empirical LSA is the simpler to apply, without sacrificing any accuracy compared to the first order method. However a disadvantage of the LSA methods is the failure just above the trough or the 'shoulder' of the wave.

The simple LPA method does not perform as well as the LSA methods when predicting wave heights from pressure readings at low y_p/D values but generally describe the wave profile with greater accuracy and is reasonably accurate when y_p/D is high. It is able to deal with noise in a satisfactory manner, better in fact than the full LPA method. In addition to this, the simple LPA is much easier to derive and program than the full LPA. The fragility of the full LPA method is unfortunate as its accuracy is higher than all other methods if no noise is present.

Many of these shortcomings have been addressed by the nonlinearly optimised local polynomial approximation. Accuracy is comparable to the full LPA, it is simple to develop the theory and it is possible to deal with noise in a relatively simple manner. As with all the LPA methods, it performs better when dealing with longer waves, due to the polynomial form of the velocity potential. As a rational nonlinear method these results are promising and there is potential for further development to determine the full wave kinematics.

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