CHAPTER 39

GENERATION OF SECOND-ORDER LONG WAVES BY A WAVE GROUP IN A LABORATORY FLUME AND ITS CONTROL

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ABSTRACT

Generation of second-order long waves at a wave maker by non-periodic wave group is analyzed in time series by Mizuguchi(1995). The analysis, which include a way to control the long wave generation, is briefly described. Then confirmation by using a single wave packet is successfully undertaken both in numerical simulation by Boussinesq equation and in laboratory experiment.

1. INTRODUCTION

Long period waves (infra-gravity waves) are of typical time scale a few minutes. Recent studies on field waves in the nearshore zone reveals that these long waves are quite significant and cannot be neglected even for engineering purposes.

Reproduction of a field phenomenon in laboratory experiments is a way to understand it if tried or a proof of understanding if successfully done. Here we deal with a method of correct reproduction of the long waves coexisting with grouping short-period waves. Ottesen Hansen et al.(1980) presented a theoretical analysis of the bound second-order long waves in the frequency space. The long waves are calculated as the second-order difference waves of the two primary waves with slightly different frequencies in Stokes-type nonlinear analysis. They also give discussions both on the production of spurious free long waves at a wave maker and on a method to suppress them. Kostense(1984) successfully applied the method to bichromatic waves. However this approach in frequency space cannot be applied to non-periodic wave

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groups such as the initial growth stage of the wave generation and a single wave packet.

On the other hand, Longuet-Higgins and Stewart(1962) show a way to describe these long waves by applying linear long wave equation with a forcing term given by second-order quantity (radiation stress) of the short period waves. Mizuguchi(1995) follows their approach to study the behavior of the second-order long waves produced at the wave maker when grouping waves are generated in a laboratory flume. He also shows a way to control them in time domain. Here we report their experimental confirmation both numerically and physically.

2. TIME SERIES ANALYSIS OF SECOND-ORDER LONG WAVE GENERATION

Radiation stress approach employed by Mizuguchi(1995) is briefly described below. He assumes one dimensional case with a constant depth h. Then basic conservation equations are

mass: $\eta_t+(hu)_x=0$ (1)

momentum: $u_t+g\eta_x=-(Sxx/\rho)_x/h$ (2)

where η and u are surface elevation and onshore velocity for long waves. Sxx is a radiation stress component. Here the long waves are assumed to be of small amplitude. Eliminating u in Eqs.(1) and (2), we have the linear long wave equation with a forcing term.

$$\eta_{tt}-g(h\eta_x)_x=(Sxx/\rho)_{xx}. \tag{3}$$

For a wave group, which propagates in a steady form on a constant depth, Longuet-Higgins and Stewart(1962) shows Eq.(3) has the following particular solution (the bound long waves) η_*,

$$\eta_*=-Sxx(x-c_gt)/\rho(c^2-c_g^2) \quad \text{where } c^2=gh. \tag{4}$$

For the long waves under short wave groups in a laboratory flume, the general solution for x>0 (wave maker at x=0) is written as

$$\eta(x,t)=f(x-ct)+\eta_*(x-c_gt) \tag{5}$$

where f(x-ct) is a general solution of Eq.(3), which propagates in the positive direction. Uniqueness exclude other general solution, which propagates in the negative direction. Functional form of f should be determined by either initial conditions or boundary conditions. In other words the general solution $f(x-ct)$ is needed to satisfy conditions which reflect a real situation.

When generating either grouping waves or irregular waves, we normally neglect the existence of the second-order long waves. Then this natural boundary condition at the wave maker is written as

$$u=0 \quad \text{at } x=0. \tag{6}$$

Surface elevation η of Eq.(5) gives the following horizontal velocity u,

$$u(x,t)=(c/h)f(x-ct)+(c_g/h)\eta_*(x-c_gt) \tag{7}$$

For a boundary condition where $u(0,t)$ is specified, Eq.(7) yields

$$f=-n\eta_*+(h/c)u|_{x=0} \tag{8}$$

where $n=c_g/c$. For the natural boundary condition described by Eq.(6), we have the following solution

$$\eta(x,t)=\eta_*(x-c_gt)-n\eta_*[n(x-ct)]. \tag{9}$$

In addition to the bounded long waves $\eta_*(x-c_gt)$, free long waves, whose magnitude is -n times of the bound one, are generated and propagate with the phase speed $\sqrt{gh}$.

To control the free long waves, in particular, to suppress the free long waves, one put f=0 in Eq.(8) so that the following extra board motion is added to the motion for the group of primary waves.

$$u|_{x=0}=(c_g/h)\eta_* \tag{10}$$

To introduce arbitrary free long waves $f(x-ct)$ at the wave maker, one should further add

$$u|_{x=0}=(c/h)f. \tag{11}$$

Generation of free long waves for an initial value problem is also discussed in Mizuguchi(1995).

3. NUMERICAL AND EXPERIMENTAL CONFIRMATION

Numerical as well as physical experiments are conducted to confirm the theoretical analysis.

For numerical simulation, Boussinesq equation is employed. Boussinesq equation can describe weakly nonlinear and weakly dispersive waves and is known to be able to simulate well the water waves in shallow water up to the second-order phenomena. We follow the normal procedure in the numerical coding, that is, the staggered mesh in space and the leap-frog method in the time stepping. One advantage of the numerical simulation is that the boundary condition at the wave maker is exactly specified by the horizontal velocity as is done in the analysis.

Laboratory experiment is conducted in a flume of 40m long, 30 cm wide with a piston-type wave maker. The displacement of the wave maker board is calculated by numerically integrating the corresponding velocity at x=0. The displacement of the wave maker board is assumed to be negligibly small.

A group of waves are generated by introducing the velocity

$$u_p=A(t)\cos(2\pi t/T) \qquad (12)$$

at $x=0$, where u_p is the vertically uniform horizontal velocity. $A(t)$ is a slowly-varying amplitude function and is given by

$$A(t)=(a_{max}/2)[1-\cos(2\pi t/T_g)] \quad 0<t<T_g \text{ otherwise } A=0. \qquad (13)$$

Here T the period of the primary waves, T_g the duration of the wave group, and a_{max} is the maximum amplitude of primary waves. In the experiments, T, T_g and a_{max} are chosen to be 1.0 s, 8.0 s and 0.5 cm respectively. The small value of the amplitude a_{max} assures the small amplitude assumption for the primary waves and also justify the way to convert the velocity to the motion of a wave maker. Water depth is 10 cm so that Boussinesq equation is applicable.

First we generate the group of waves in a traditional way, or without any consideration on second-order phenomena. Figure 1 shows comparison of the surface profiles among measured, simulated and of analysis. Overall agreement is very good. The amplitudes measured in the physical experiment show a little decay while propagating. Linear modulation of the wave group, which is not long enough, might be responsible for the decay, although frictional loss may not be negligible. Figure 2 shows long waves obtained by low-pass filtering the data in Fig. 1. All three envelopes agree very well, revealing the separation process of the free long wave, generated at the wave maker, from the bound long wave. Near the wave maker they almost cancel each other to give zero velocity at the wave maker. Some distance from the wave maker the free long wave, which is a positive hump in this case, starts to emerge as it leads the bound long waves with faster phase velocity. The free and bound long waves may be completely separated after travelling long distance, though nonlinear effects may come in to play non-negligible role there.

Next we generate a group of waves with the extra velocity of Eq.(10) or the corresponding paddle motion, which is shown in Fig. 3. Figure 4 shows comparison of the three envelope profiles. As is in Fig. 1, they show good agreement. The measured maximum amplitude in the laboratory experiment is a little smaller than 0.5 cm even very near the wave maker. This may be caused by some mechanical loss and/or the displacement of the paddle to account for Eq.(10). It is noted that there is little difference between the data plotted in Fig. 1 and in Fig. 4.

Fig. 5 shows long waves obtained from the data in Fig. 3. Again three profiles are in good agreement, though they are quite different from those in Fig. 2. The analytical result in Fig. 5 is the Longuet-Higgins and Stewart solution (hereafter abbreviated LHS solution) or Eq.(4). This shows that one can realize the LHS solution by introducing the velocity of bound waves

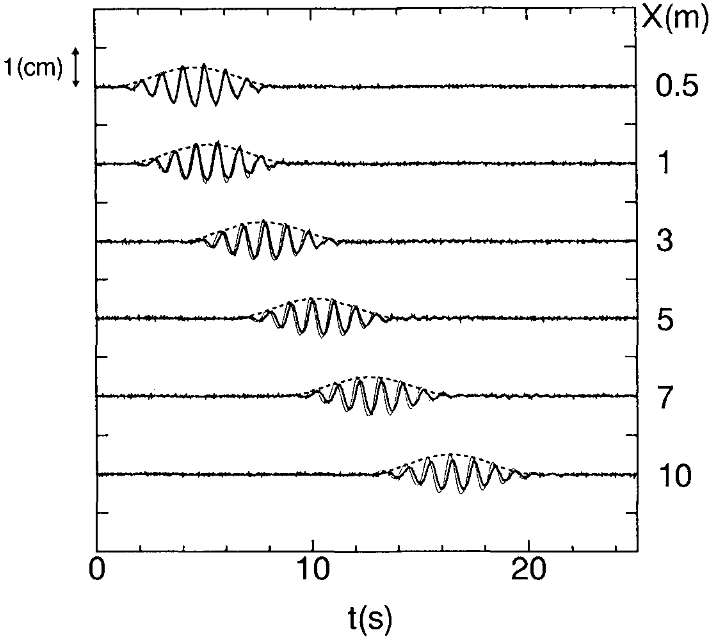

Fig. 1 Surface profiles of normally generated grouping waves
Thick solid lines : laboratory experiment
Thin lines : numerical simulation (Boussibesq equation)
Broken lines : theory (a permanent wave group)

to the wave making. The long waves measured in the physical experiment decrease their magnitude while propagating, as the amplitude of the primary waves also decrease their amplitude. The long waves in Boussinesq model also show a slightly larager difference from the analytical one than those in Fig. 2. The difference may result from the fact that modulation of the wave group is more significant when the free long waves are suppressed. The total magnitude of long waves is larger than that with the free waves and resultant modulation stronger.

It is worthwhile to be stated here that the LHS solution is correct as a solution of the problem and can be realized in an ideal situation. However free long waves, generated at the wave making process as shown in Fig. 2 and/or free long waves generated while wave groups shoals on a sloping bottom as discussed in Nagase and Mizuguchi(1996), contribute to canceling the LHS bound waves which tend to be infinitely large in the very shallow water.

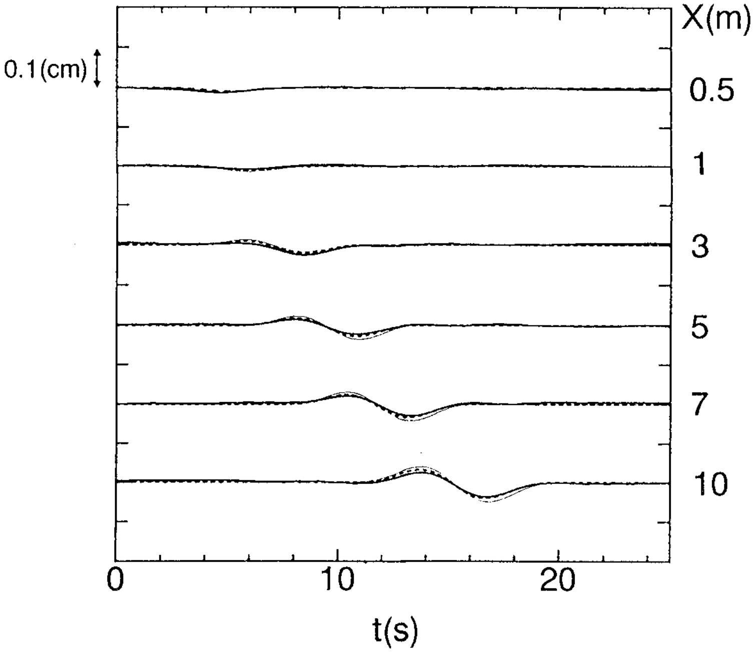

Fig. 2 Long waves obtained by low-pass filtering the data in Fig. 1. Vertical scale is ten-times larger than that in Fig. 1.

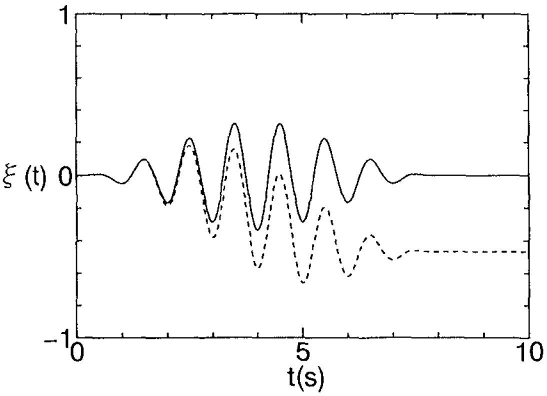

Fig. 3 Displacement of the wave paddle
Solid line ... traditional
Broken line ... free long waves suppressed

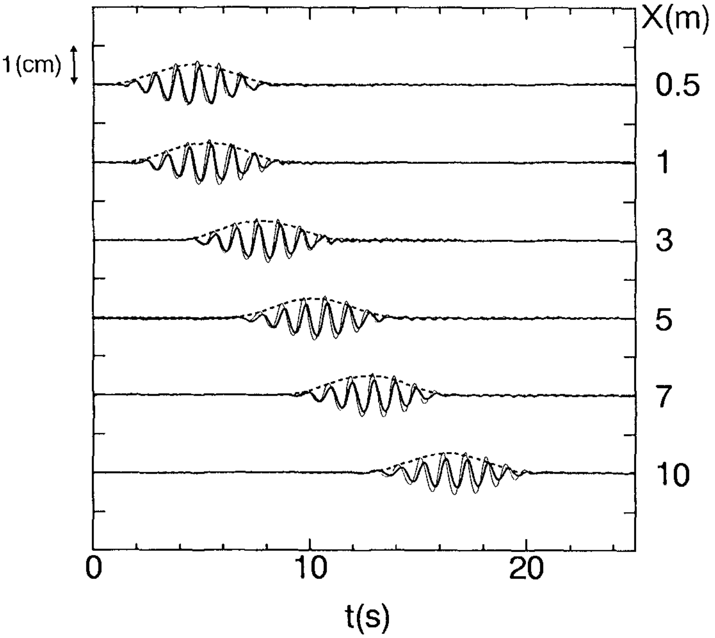

Fig. 4 Surface profiles of grouping waves with no free long waves

4. CONCLUSIONS

From the results stated above, we can conclude

1) A time-domain analysis to understand the generation of second-order long waves in a laboratory flume is presented.

2) Analytical results are successfully confirmed both numerically and experimentally, showing a way to control the generation of second-order long waves for non-periodic wave group.

3) Even in very shallow water bounded long waves is described by the solution of Longuet-Higgins and Stewart(1962). However the observed long waves may not be so large as predicted by the LHS solution as accompanying free long waves nearly cancels it in reality.

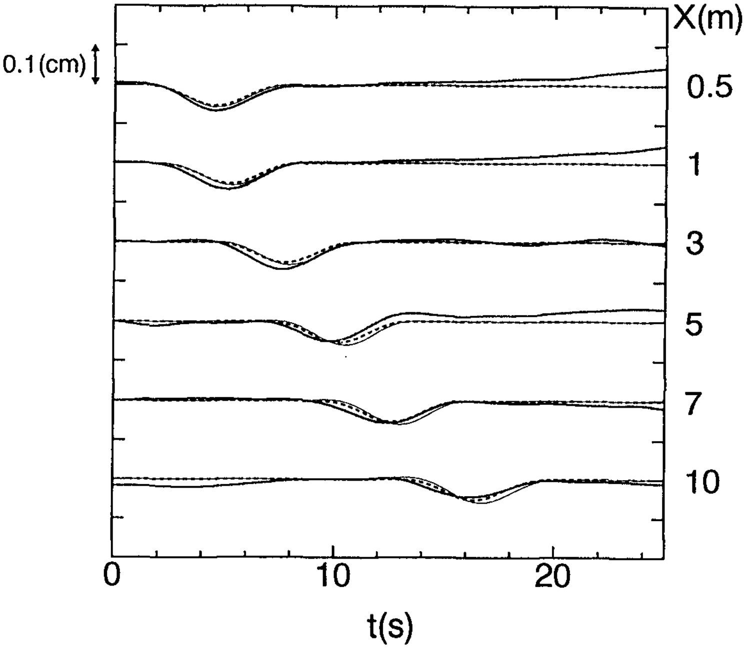

Fig. 5 Bound long waves in the data shown in Fig. 4 (Longuet-Higgins and Stewart solution)

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