INITIAL GAP IN BREAKOUT OF HALF-BURIED SUBMARINE PIPE DUE TO WAVE ACTION

Adrian W.K. Law[1] and Mostafa A. Foda[2]

ABSTRACT

An analytical formulation is presented for the analysis of the breakout of a half-buried submarine pipe due to wave action. The formulation accounts for the contact between the pipe and the soil due to the horizontal hydrodynamic force. An asymmetric gap geometry is described that incorporates a second order expansion in the gap opening due to the pipe rise. Results with this formulation demonstrate the existence of an initial gap of the order 1 mm in the breakout experiments. With this initial gap, the gap flux dominated the influx of water into the gap from the beginning of the breakout process.

1.0 INTRODUCTION

Breakout of a partially buried pipe under severe wave action is one of the possible mechanisms for the pipe to detach from the sea bed (Foda, 1985). The configuration is schematically sketched in Figure 1. Experiments were performed by the authors to investigate the breakout of a half-buried pipe in the laboratory, and the results were previously reported in Foda et al. (1990).

[1] Bechtel Corporation, 50 Beale St., P.O. Box 193965, San Francisco, CA 94119, USA

[2] Hydraulic and Coastal Engineering Division, Department of Civil Engineering, University of California at Berkeley, Berkeley, CA 94720, USA

The bulk of the Foda et al. (1990)'s paper dealt with the analysis of the pressure and displacement measurements from the breakout tests. An uniform lift analytical model using the poro-rigid bed assumption was also presented in that paper. This uniform lift model is based on the consideration of the action of the net lift force on the pipe. This net lift force induces an uniform liftoff of the pipe in the vertical direction, thus creating a symmetric gap geometry as well as symmetric gap suction pressures about the vertical axis of the semi-circular trench. However, further considerations of the effect of the oscillatory horizontal hydrodynamic force reveal that this uniform liftoff conceptualization may not be fully representative of the physical breakout process. These considerations are discussed in the followings.

Through analysis of the measurements on the hydrodynamic forces and the resultant forces by the gap suction pressures, we demonstrated in Foda et al. (1990) that the pipe was always in contact with the soil. In essence the pipe was bouncing back and forth against the sides of the soil's semi-circular interface. In doing so, the gap distribution would become asymmetric with respect to the vertical axis. There would be some finite opening on one end of the semi-circular trench whereas on the other end the gap would be closed by the contact between the pipe and the soil. This asymmetric gap distribution, however, is not reflected in the symmetric gap assumption in the uniform lift model.

The asymmetric gap distribution also poses differences in the pressure boundary conditions on both ends. On the opening edge the gap pressure is subjected to the positive pressure buildup due to the stagnation point in front of the pipe, whereas on the other side the contact between the pipe and the soil isolates the suction pressures inside the gap to the ambient pressure above. The situation in terms of the pressure boundary conditions is thus closer to the tilted lift case than the uniform lift case in Mei et al. (1985). With the differences in the gap pressure distribution about the vertical axis, the resulting horizontal force will be reinforcing the horizontal hydrodynamic force in the first harmonic. This is indeed what was observed in our analysis of the laboratory measurements on the horizontal force balance. The above discussion demonstrates that an asymmetric gap model can better represent the physical situation than the uniform lift model.

With these considerations, we set out to develop an asymmetric gap formulation that incorporates the effect of the oscillatory horizontal hydrodynamic force. Subsequently, based on this formulation, we will address the existence of some initial gap opening in the beginning of the breakout process. However a brief summary of the laboratory experiments is first presented in the next section.

2.0 EXPERIMENTAL SETUP

The experimental setup consisted of a 0.22 m diameter PVC pipe half-buried in a sand basin inside a wave flume under 0.61 m water depth. The sand was a well-sorted medium sand with d_{50} of 0.32 mm and with permeability approximately equal to 0.85×10^{-11} m^2. The half-buried pipe was subjected to monochromatic waves of approximately 3 second wave period and 0.15 m wave height until breakout from the sea bed occurred.

Submerged weight of the pipe was used as a control parameter to generate different net lift forces on the pipe thus producing different breakout times. The vertical displacement of the pipe was monitored using a hermetically sealed sensor installed inside the pipe. Pressures around the pipe were also recorded. Additional descriptions of the experimental setup can be found in Foda et al. (1990).

A typical vertical movement history of the half-buried pipe in the experiments is shown in Figure 2. As can be observed in the figure, after the half-buried pipe was exposed to the monochromatic waves, the pipe started to rise with a steady but very slow rate (of the order of 2.5 mm/min in the experiments). No significant soil erosion around the pipe was observed. Minor oscillatory motion of the pipe was also recorded superimposing on the net vertical movement. The net rising rate gradually increased near the breakout time. Subsequently, at the breakout time a sudden and rapid release of the pipe from the sea bed occurred.

3.0 PORO-RIGID MODEL WITH ASYMMETRICAL GAP DISTRIBUTION

In this section, an analytical formulation is presented for the analysis of the breakout of a half-buried pipe from sea bed due to wave action. First, the movement of the pipe during one wave cycle is examined as shown in Figure 3. The pipe oscillates horizontally with respect to the underlying trench due to the wave-induced horizontal force. This horizontal oscillatory motion of the pipe produces a mirror image setting between the two halves of a wave period. With this mirror image and in a quasi-steady manner, it is postulated that the situation can be analyzed as if the pipe is subjected to a steady lift force F_L and an unidirectional steady horizontal force F_h (Figure 4) of the following magnitudes based on the Morrison equation:

horizontal
$$F_h = \frac{2}{T}\int_0^{T/2} \rho \left| C_M \frac{\pi D^2}{4}\frac{\partial u}{\partial t} + \frac{1}{2} C_D D u|u| \right| dt \qquad (1a)$$

vertical $$F_L = \frac{1}{T}\int_0^T \frac{1}{2}\,\rho\, C_L D u^2 dt \tag{1b}$$

where T is the wave period, ρ the water density, D the pipe diameter, u the wave-induced horizontal velocity at the mud line, and C_L, C_M and C_D the lift, added mass and drag coefficients for a half-buried pipe respectively. In our experiments the value of C_L was measured to be 2.55.

At this point, to facilitate further analysis we introduce a conformal mapping to transform the geometry of the seabed to a full half plane without indentation:

$$\varsigma = \frac{1}{2} - \frac{1}{2D}\left(z + \frac{D^2}{4z}\right) \tag{2}$$

where $z = (x,y)$ = the physical plane with coordinate orientation as shown in Figure 4; $\varsigma = (\xi,\eta)$ = the mapped domain. From a geometrical standpoint, the asymmetric gap induces by the rise of the pipe while maintaining contact with the soil can be approximated as:

$$\Delta(\theta,t) = \Delta_o(t)\left(\sin\theta - \frac{\delta}{2}\cos\theta(1-\cos\theta)\right); \quad \delta = \frac{\Delta_o(t)}{R} \tag{3}$$

where Δ is the gap thickness, Δ_o the midgap thickness, and R the pipe radius. The gap distribution is shown in Figure 5. The asymmetry of the gap is thus only in the second order, or $O(\delta^2)$. However this second order asymmetry properly accounts for the expansion in the gap opening due to the pipe rise, and thus is necessary for computing the growth of the gap flux which is the supply of ambient water through the gap opening at $\xi = 1$. In the mapped plane, this gap distribution becomes:

$$\Delta(\xi,t) = \Delta_o(t) f(\xi); \quad f(\xi) = \left(2\sqrt{\xi - \xi^2} + \delta(t)\xi(2\xi - 1)\right) \tag{4}$$

We will now follow the poro-rigid bed approach in Mei et al. (1985) to formulate the governing equation for the flow inside the gap between the pipe and the soil. In this approach, the soil is assumed to be poro-rigid and Darcy's law is assumed for the pore water flow. Lubrication theory is used to analyze the flow along the gap between the pipe and the soil. Following the analysis of Mei et al. (1985) and with an further assumption of a no-slip boundary condition at the surface of the porous bed, we obtain the following water budget equation for the expanding gap:

$$\xi D\frac{d\Delta_o}{dt} = \frac{k}{\mu\pi}\int_0^1 \frac{p}{\eta - \xi}d\xi + \frac{\partial p}{\partial \xi}\frac{\Delta^3\sqrt{\xi - \xi^2}}{6\mu D} \tag{5}$$

where μ is the viscosity, k the intrinsic permeability of the porous bed, and p the dynamic pressure in the gap. Taking $\xi = 1$, the left-hand side of Eq. 5 then represents the first order, or $O(\delta)$, expansion rate of the gap volume. The first term on the right-hand side gives the contribution to such expansion from the pumping of pore water into the gap, which is given in terms of a Cauchy principle-value integral. The last term of Eq. 5 gives the gap flux. It should be noted that the gap flux term will not vanish at $\xi = 1$ despite its appearance. This is because both $\Delta(\xi = 1)$ and $\left.\frac{\partial p}{\partial \xi}\sqrt{\xi - \xi^2}\right|_{\xi=1}$ have finite non-zero values at this edge of the semi-circular trench where the ambient water is drawn into the gap.

Next, the balance of forces before the breakout is considered. With the asymmetric gap distribution, the resultant force from the gap suction pressures in the horizontal direction will be reinforcing the horizontal hydrodynamic force. Together they will then be resisted by a reaction force from the soil. In the vertical direction, the balance of forces yields:

$$F + D\int_0^1 p\, d\xi = 0 \tag{6}$$

where F is the net lift force = time-averaged hydrodynamic lift F_L minus the pipe submerged weight F_s. The horizontal force balance in the first hamornic and the vertical force balance in the zero harmonic were both observed in the experimental results.

At this point, we need to address the pressure boundary condition at the gap opening $\xi = 1$. Due to Bernoulli's effect, there will be positive pressure buildup with the stagnation point in front of the pipe. This positive pressure can be calculated in a quasi-steady manner as:

$$p_o = \frac{1}{T}\int_0^T \frac{1}{2}\rho\, u^2 dt = \tag{7}$$

The formulation of the asymmetric gap model is now completed. We will further convert the governing equations into nondimensional form by introducing the following variables:

$$P = \frac{pD}{F};\ \sigma = (Dk)^{-1/3}\Delta_o;\ \tau = \frac{Fk^{2/3}}{\mu D^{7/3}}t \tag{8}$$

The two governing equations, Eqs. 5 and 6, become:

$$\xi \frac{d\sigma}{d\tau} = \frac{1}{\pi}\int_0^1 \frac{P}{\eta - \xi} d\xi + \sigma^3 \frac{\partial P}{\partial \xi} \frac{\sqrt{\xi - \xi^2}}{6} f^3(\xi) \tag{9}$$

$$\int_0^1 P d\xi = -1 \tag{10}$$

Eqs. 9 and 10 permit numerical computation in a manner as suggested in Mei et al. (1985). However it should be emphasized that closely spaced numerical grid will be required near ξ=1. This is to avoid introducing arbitrary large opening for the computation of the gap flux in the numerical algorithm. For example, following the mid-point computation approach in Mei et al. (1985) with N+1 nodes (resolution $\varepsilon = N^{-1}$), the volume of the gap flux in Eq. 9 will be computed as $\frac{P_{at\xi=11} - P_{at\xi=1-\varepsilon}}{6\sqrt{\varepsilon/2}} \sigma^3 \left(\sqrt{2\varepsilon} + \delta\right)^3$. Since this flux term should be proportional to $\sigma^3\delta^3$, it is thus required that $\varepsilon < \frac{\delta^2}{2}$. Typically $\delta \sim O(10^{-2})$ when the gap flux becomes significant, and thus the requirement would then be $\varepsilon \sim O(10^{-4})$ or $N \sim O(10^4)$. This is a stringent requirement since the computation will then involve finding the inverse of a large NxN matrix, a task that is both time-consuming and error-prone.

Note that if indeed the finite gap opening at the edge of the trench is due only to the second order expansion due to the pipe rise, it would be more appropriate to use another time scale, $T_b = \frac{\mu D^{8/3}}{F k^{5/6}}$, based on the balance of terms in Eq. 5 at $\xi = 1$ of $\frac{D^2\delta}{T_b} \sim \frac{kF}{\mu D} \sim \frac{FD\delta^6}{\mu}$. However whether the gap opening is realistically second order in nature will be discussed in the following section on the existence of an initial gap.

4.0 Initial Gap

As shown in Figure 2, the pipe typically rose with a net constant rate during the initial stage in the breakout process. These initial rates of rise, $R\frac{d\delta}{dt}$, were of the order of 2.5 mm/min (0.04 mm/s) in the laboratory experiments. The gap volume expansion due to this vertical rise of the pipe should be balanced by the combined pore water and gap fluxes. In the following section we will discuss the implication of these measured rates on the initial gap geometry base on the derived governing equations.

Numerical computation using Eqs. 9 and 10 without the gap flux term shows that the initial rate of rise, $\frac{d\sigma}{d\tau}$, due to the pore water flux alone is approximately 0.85. This is slower than the corresponding rate with the uniform lift model which can be determined analytically to be equal to $8/\pi$ (see Mei et al. (1985)) but in the same order of magnitude.

With D = 0.22m and k = 0.85×10^{-11} m^2 in our experiments, the non-dimensional length scale will then be $(Dk)^{1/3} = 0.12$ mm. The breakout time scale, T_b, was of the order of $O(10^2)$ in the breakout tests. The measure rate of rise of approximately 0.04 mm/s thus represents a nondimensional rate of rise of around 30. This measured rate is an order of magnitude higher than the predicted rate supported by the pore water flux. Despite the fact that there were possible variations in some of the experimental parameters such as the soil permeability, the differences were judged to be too large to be explained by these variations. Clearly this points to the existence of an initial gap in the beginning of the breakout process. In addition, it also shows that in our experiments the pore water flux, though always important in relieving the negative pressures near the contact ($\theta \cong 0$) where the gap width is always small, was insignificant in terms of expanding the gap volume and allowing the pipe to rise.

Physically, this initial gap can be due to the imperfect contact between the pipe and the soil. Another mechanism of generating this initial gap in the soil region around the two ends of the semi-circular trench would be through plastic deformations. In the first halve of the wave cycle, one end of the soil trench will be compressed as the horizontal force and the resultant force from the gap pressures together push the pipe against the soil. This compression will subsequently be released in the second halve of the wave cycle when the pipe moves to rest against the opposite side. It is possible that the soil skeleton in these areas is weakened by the periodic compression and relaxation, allowing plastic deformations to take place.

We will first use a scaling analysis to examine the magnitude of this initial gap. Let $\overline{\Delta}$ be a characteristic length scale for the initial gap. Comparing the two terms in the RHS of Eq. 5 for $\xi = 1$ suggests that the balance between the pore water and the gap fluxes will be of the order of $O\left(\frac{kD}{\overline{\Delta}^3}\right)$. This implies that $\overline{\Delta}$ should be much larger than $\sqrt[3]{kD}$, or 0.1 mm in this case.

The initial gap geometry in reality can be very complicated, and will also depend to some extent on the processes how the pipe was half-buried. To get an general assessment on the order of magnitude of the initial gap in our experiments, we assume a simple constant gap geometry as follows:

$$\Delta = \Delta_i \qquad for \qquad 0 < \theta \leq \pi \tag{11}$$

where Δ_i represents an initial constant gap. Ignoring the pore water flux term, Eq. 5 can be simplified in the physical plane to be:

$$R^2(1-\cos\theta)\frac{d\delta}{dt} = \frac{\Delta^3}{12\mu R}\frac{\partial p}{\partial\theta} \tag{12}$$

Substituting the constant gap geometry into Eq. 12 and integrating yield:

$$p = p_o - \frac{12\mu R}{\Delta_i^3}\frac{d\delta}{dt}(\pi - \theta + \sin\theta) \tag{13}$$

Finally, substituting Eq. 13 into 6 in the physical plane, the initial rate of rise can then be estimated as:

$$R\frac{d\delta}{dt} = \frac{(F + 2p_o R)\Delta_i^3}{18\pi\,\mu R^3} \tag{14}$$

With R = 0.11 m, μ = 10^{-3} kg/ms, $p_o \sim O\left(\frac{F}{R}\right)$ and $F \sim O(10)$ in our experiments, the measured initial rate of rise of the order of 0.04 mm/s would represent an initial gap Δ_i of the order of 1 mm in the beginning of the breakout process.

5.0 CONCLUSION

An asymmetric gap model is presented for the analysis of the breakout of a half-buried pipe from sea bed due to wave action. The model accounts for the contact between the pipe and the soil due to the horizontal hydrodynamic force. An asymmetric gap geometry is described that incorporates the second order expansion in the gap opening due to the pipe rise. Quasi-steady vertical force balance is assumed. The resultant horizontal force from the distribution of the gap pressures reinforces the horizontal hydrodynamic force in the first hamornic which is consistent with the experimental observations. Calculations with this asymmetric gap model demonstrate the existence of an initial gap of the order of 1 mm between the pipe and the soil in the breakout experiments. This initial gap is postulated to be due to imperfect contact between the pipe and the soil and possible plastic soil deformations due to periodic compression and relaxation of the soil skeleton around the pipe. With this initial gap, the gap flux is found to dominate the influx of water from the beginning of the breakout process. The pore water flux, though important in relieving the negative pressures near the contact where the gap width is always small, was insignificant in terms of expanding the gap volume and allowing the pipe to rise. The role of the initial gap and the soil deformation around the pipe to the breakout process will be further examined in the future.

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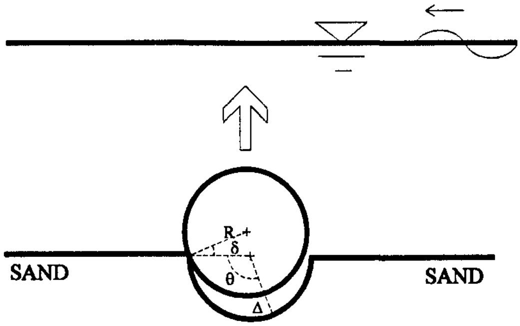

Figure 1. Configuration of the breakout of a half-buried pipe from sea bed due to wave action.

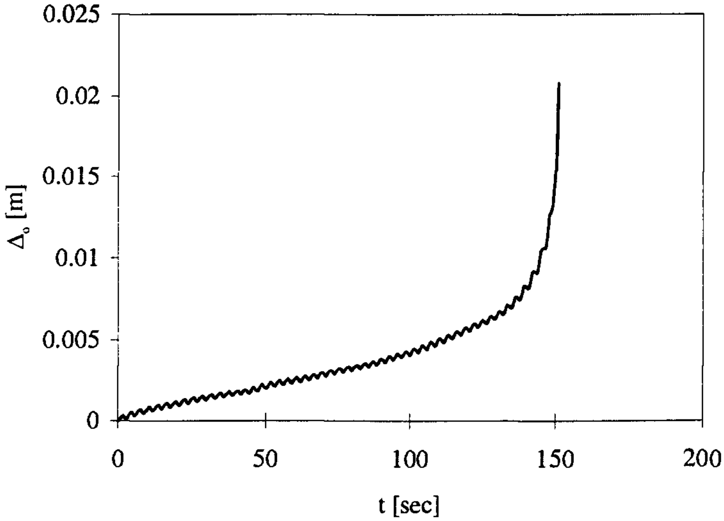

Figure 2. Measured vertical displacement history in Experiment F4.

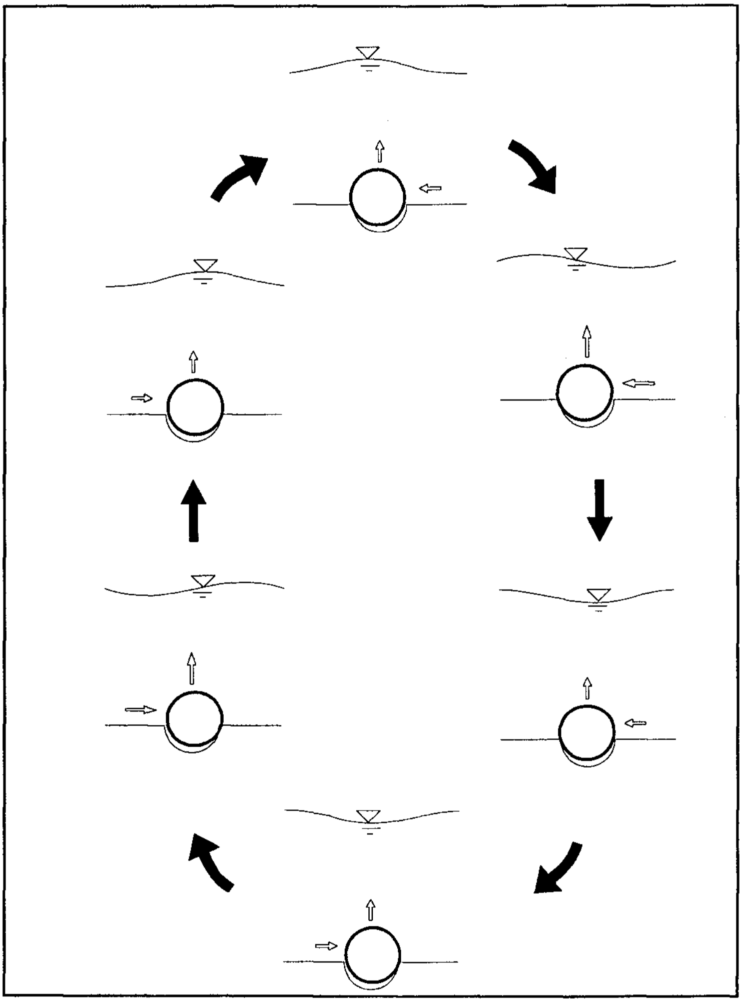

Figure 3. Movement of the pipe during one wave cycle.

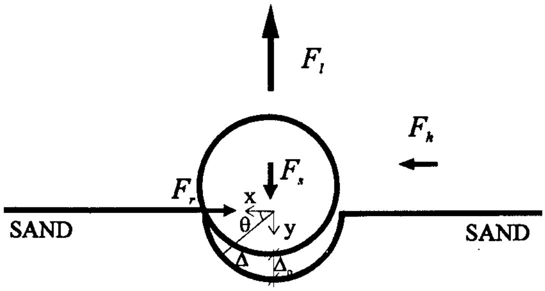

Figure 4. Conceptual model for the quasi-steady analysis of the breakout of a half-buried pipe from sea bed due to wave action.

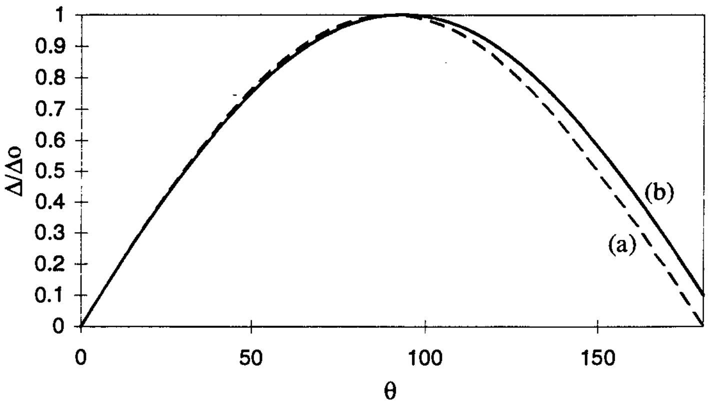

Figure 5. Geometry for (a) a symmetric gap $\Delta = \Delta_o \sin\theta$ and (b) an asymmetric gap $\Delta = \Delta_o (\sin\theta - \delta\cos\theta\,(1 + \cos\theta)/2\,)$ with $\delta = 0.1$.