CHAPTER 168

WAVE RUNUP ON COMPOSITE-SLOPE AND CONCAVE BEACHES

R. H. Mayer[1] and D. L. Kriebel[1]

ABSTRACT

Laboratory experiments were carried out for regular and irregular wave runup over non-uniform beach profiles, including both bi-linear and concave "equilibrium" beach profiles. Measured runup is shown to be substantially less than that predicted by the Hunt Formula if the exposed beach-face slope is used to define the typical beach slope. More accurate runup estimates are obtained using Saville's hypothetical slope concept; and, Saville's method is integrated with the Hunt Formula to provide simple analytical estimates of wave runup over complex beach topography.

INTRODUCTION

One problem encountered when predicting wave runup on open-coast beaches is the dilemma of how to define the beach slope for use in runup predictive formulas. Most runup design methods have been developed from laboratory tests conducted on uniform beach slopes. For field application, however, where beaches are generally concave in shape, and where offshore sand bars or complex beach berms may exist, the appropriate slope for use in runup calculations is never clear. On natural beaches, different definitions of beach slope can typically differ by a factor of two or three such that runup estimates can vary by a similar amount.

The purpose of this paper is to present a computationally-simple method for calculating wave runup on beaches with composite-slopes or concave shapes. This study consists of two main parts: (1) development of an analytical method for predicting runup on composite-slope or concave beaches and (2) an experimental evaluation of this methodology based on small-scale laboratory tests of wave runup for both regular and random waves.

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[1] Ocean Engineering Program, U.S. Naval Academy, Annapolis, MD 21402

The analytical approach used in this paper integrates two widely-used methods of predicting wave runup: (1) the Hunt Formula, developed by Hunt (1959) for predicting runup of regular waves on coastal structures with a uniform slope and, (2) the "effective" slope method of Saville (1958), which defines an appropriate average slope for use in predicting runup on composite-slope structures. By combining these analytically, a simple quadratic equation is obtained for wave runup that can be solved in closed form for simplified geometries. This solution is then verified by comparison to wave runup measured in over 340 small scale laboratory tests conducted at the U.S. Naval Academy in both regular and irregular waves.

BACKGROUND

Most methods of predicting wave runup on open-coast beaches are variations of the Hunt Formula, proposed by Hunt (1959), and rewritten by Battjes (1974) as

$$\frac{R}{H_o} = \frac{\tan\beta}{\sqrt{H_o/L_o}} = \xi \tag{1}$$

where H_o is the deep water wave height, L_o is the deep water wave length, $\tan\beta$ is the tangent of the beach slope, and where ξ is the surf-similarity parameter which is defined by $\tan\beta/(H_o/L_o)^{1/2}$. Battjes (1974) showed that the dimensionless runup is generally equal to ξ over the range from $0.1 < \xi < 2.3$ for regular waves acting on uniform, smooth, and impermeable laboratory beaches with slopes typical of many natural beach slopes.

For irregular waves, Battjes (1971) also obtained reasonable predictions of the median runup by using the Hunt Formula expressed in terms of the median wave height. Mase (1989) defined the surf similarity parameter in terms of the deep water significant wave height and, based on analysis of laboratory tests on uniform impermeable slopes, developed empirical correlations between ξ and the mean runup, the significant runup, as well as other runup parameters. Holman (1986) developed graphical correlations between runup and the surf similarity parameter, again defined by the significant wave height, based on field measurements over complex barred beach profiles.

A discrepancy can be found, however, between the field results of Holman and the laboratory results of Mase. Mase (1989) found that the significant runup measured in his small-scale laboratory tests on smooth plane slopes was approximately twice as large as values measured in the field by Holman (1986). Mase speculates that his expression provides an upper bound to the scattered field data and further attributes the overprediction to the use of smooth impermeable slopes in the laboratory tests. It is our belief, however, that there is a more fundamental cause of this discrepancy related to the effect of beach profile geometry.

This contradiction between laboratory and field values for runup may be partly explained by considering the bi-linear profile in Figure 1a. As pointed out by Hunt (1959), the Hunt Formula will not accurately predict the runup of regular waves on this profile if either the beach-face slope m or the submerged surf zone slope s is used. Predictions based on the beach-face slope dramatically over-estimate runup while predictions based on the surf zone slope under-estimate the runup. Hunt (1959) suggested an empirical weighted average of the two slopes m and s for bi-linear geometries, but a more general method is required for more complex beach geometries, such as the concave beach profile in Figure 1b.

The most widely-used method for predicting runup over complex geometries was proposed by Saville (1957). Saville suggested that runup predictions over arbitrary geometries should use a hypothetical average or "effective" slope of the entire active surf zone, extending between the wave break point and the runup limit. This approach, however, is cumbersome to apply. It requires a time-consuming iterative solution in which: (1) a runup limit is assumed, (2) an average slope is calculated from the breakpoint to the assumed runup limit, (3) the runup is estimated using this average slope in empirical design curves, (4) the calculated runup is compared to the assumed value. If necessary, the new runup limit is then used to define a new effective slope and the process is repeated until the runup converges on a stable value.

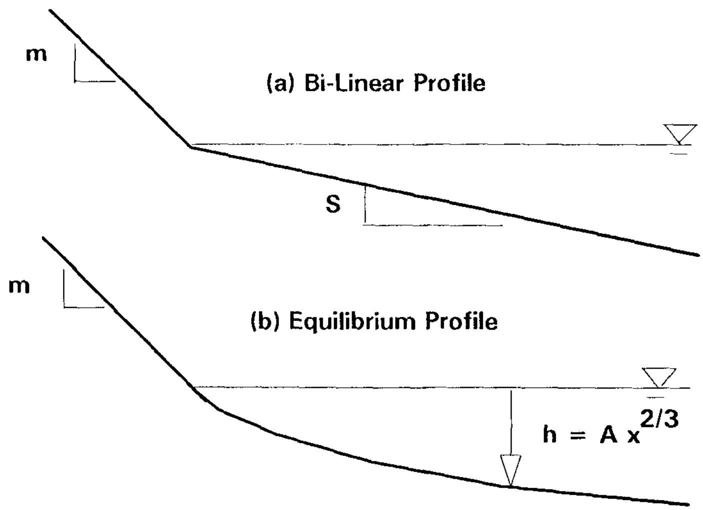

Figure 1. Illustration of idealized beach profiles: (a) bi-linear composite-slope profile and (b) concave equilibrium beach profile.

INTEGRATED RUNUP METHOD

In this study, the Hunt Formula and the Saville effective slope concept are integrated into one general runup equation for application to open-coast beaches. Because the Hunt Formula is known to describe runup for linear beach slopes, the numerous design curves which are traditionally used with Saville's effective slope method are not required. As a result, the iterative solution procedure recommended by Saville can be avoided, and a single expression for runup on composite-slope and concave beaches is found that can be solved analytically or numerically.

Beach geometries considered in both the analytical and experimental phases of this study are depicted in Figure 1. These include: (1) a bi-linear profile and (2) a concave profile given by the "equilibrium" profile form of Dean (1977). In each case, the exposed beach-face slope is assumed to be linear and the tangent of this slope is denoted by m. The surf zone is then either represented by a linear slope, s, or by a concave equilibrium profile form following the theory of Dean (1977) where the depth h is related to the distance offshore by the relationship $h=Ax^{2/3}$ in which A is a parameter relating the general slope of the beach to sediment characteristics.

It is assumed that the physical mechanisms of wave runup are represented by the Hunt Formula in equation (1) which, in dimensional form is given by

$$R = \tan\beta \sqrt{H_o L_o} \tag{2}$$

Following Saville's hypothetical slope concept, $\tan\beta$ is then defined as the average slope between the incipient breakpoint and the runup limit. As illustrated in Figure 2, this may be quantified numerically as

$$\tan\beta = \frac{R + h_b}{X_R + X_b} \tag{3}$$

where h_b is the incipient breaking depth, X_b is the horizontal distance from the shoreline to the breakpoint, and X_R is the horizontal distance to the runup limit. By combining the expression for average slope in equation (3) into the Hunt Formula in equation (2), a single equation for the wave runup is found in the form

$$R = \frac{R + h_b}{X_R + X_b} \sqrt{H_o L_o} \tag{4}$$

Note that the runup appears on both sides of equation (4) and, in fact, it appears in both the numerator and denominator on the right-hand-side since the horizontal runup limit X_R is related to the vertical runup limit R by the slope of the beach face as $X_R=R/m$. As a result, equation (4) can be solved for the runup R in two ways.

For general application over arbitrary beach geometries, equation (4) can be solved iteratively in the form

$$R_{i+1} = \frac{R_i + h_b}{X_{R_i} + X_b} \sqrt{H_o L_o} \tag{5}$$

where the subscripts i and $i+1$ denote old and new values in the iteration process. In this approach, the distances X_R and X_b are determined directly from the known (measured) beach geometry at each step in the iterative solution. Although perhaps not apparent, equation (5) is much simpler to apply than the traditional effective slope method of Saville (1958) because there is no need to look up runup values from design curves. As a result, this method has the advantage that it could be added to any existing numerical model for surf zone hydrodynamics or sediment transport.

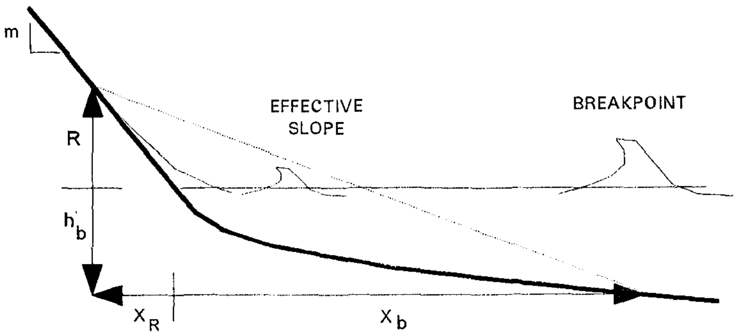

Figure 2. Definition of effective slope for idealized beach profiles.

For idealized open-coast conditions, one way to simplify equation (4) is to assume a uniform beach-face slope, m, as depicted in Figure 1. The horizontal runup excursion is then related to the vertical runup by $X_R=R/m$ and equation (4) gives

$$R = \frac{R + h_b}{R + mX_b} m \sqrt{H_o L_o} \tag{6}$$

This suggests that the runup over a concave beach profile will always be less than runup over a uniform planar beach. For concave beach profiles, the breaking depth h_b is always smaller than the depth of a uniform slope projected out to the breakpoint, mX_b. The ratio $(R+h_b)/(R+X_b)$ is then always less than unity and the runup is less than that predicted by the Hunt Formula using the beach-face slope, m.

Further interpretation indicates that if the breakpoint is near the still water shoreline (shore-break conditions), then as expected

$$R = m \sqrt{H_o L_o} \tag{7}$$

In contrast, if the breakpoint is far offshore so that the surf zone is very wide, then equation (6) simplifies to the Hunt Formula based on the surf zone slope as

$$R = \frac{h_b}{X_b} \sqrt{H_o L_o} \tag{8}$$

For most other conditions however, the solution will lie in between these limits and is may be determined by first rewriting equation (6) as a quadratic equation

$$R^2 + R\, m \left(X_b - \sqrt{H_o L_o} \right) - h_b\, m \sqrt{H_o L_o} = 0 \tag{9}$$

An analytical solution for wave runup over non-uniform beach profiles is then found to be given by the solution of the quadratic equation

$$R = \frac{m}{2} \left(X_b - \sqrt{H_o L_o} \right) \left[-1 + \sqrt{1 + \frac{4\, h_b \sqrt{H_o L_o}}{m \left(X_b - \sqrt{H_o L_o} \right)^2}} \right] \tag{10}$$

Interpretation of equation (10) is somewhat difficult but approximate solutions can be found by applying series expansions or by solving equation (9) subject to assumptions regarding the magnitude of each term. Space limitations prevent a full presentation of these results but, in general, results show that the runup is between the limits suggested in equation (7) and (8). For storm conditions, it can be shown that runup is actually given more closely by equation (8) than equation (7), suggesting that the average surf zone slope, not the average beach-face slope, is most appropriate for runup predictions when the wave conditions are energetic.

EXPERIMENTAL EVALUATION OF INTEGRATED RUNUP METHOD

Over 340 wave tank tests have been conducted in order to verify the integrated approach of combining the Hunt Formula with the Saville effective slope concept in equation (10). Laboratory tests were conducted in the wave basin at the U.S. Naval Academy which is 13 meters long and 5.4 meters wide. Model beach profiles, having shapes depicted in Figure 1, were constructed of smooth impermeable PVC plastic sheets placed on top of an aluminum frame mounted inside a 7.5 meter long and 0.6 meter wide test channel installed in the wave basin. Water depths in the basin were generally kept at 24 centimeters.

Regular and irregular wave tests were conducted on uniform plane slopes and on bi-linear profiles as depicted in Figure 1a. Seven combinations of slopes m and s were tested for regular waves while two combinations were tested for irregular waves. Tests were next conducted on equilibrium profiles, depicted in Figure Ib, for regular waves only. In these tests, water levels were varied since, according to the hypothetical slope concept, runup for given wave conditions should be greater when the water level is elevated due to the steeper average slope of the surf zone.

Waves were generated by a piston-type wavemaker and, because of the lack of reflection compensation in the short wave basin, test durations were kept relatively short. In regular wave tests, typically 10 to 20 waves were generated for each combination of wave height and period. For each beach configuration, 20 combinations of H and T were tested producing 20 values of wave steepness. For irregular waves, tests were conducted in short "bursts" of about 30 waves. Ten such 30-wave sets were used for each combination of significant wave height and peak period tested so that about 300 waves were obtained for each wave spectrum tested. For each structure geometry, ten values of H_S and T_p were used producing 10 values of spectral wave steepness. All random wave tests used JONSWAP wave spectra with a peak enhancement factor of 3.3

Because the goal of this study was to evaluate the integrated runup solution in equation (10), measurements in each laboratory experiment included: (1) incident wave height and period, (2) breaking depth h_b, (3) surf zone width X_b, and (4) wave runup. Input to equation (10) consisted of the first three, with incident waves converted to equivalent deep water values. For irregular waves, measurement of input parameters was rather complicated since the breakpoint changes from one random wave to the next. Both the wave runup and the location of each breaking wave were recorded on video tape. Breaker depth was then determined from the observed surf zone width based on the known geometry.

Regular Waves on Bi-Linear Profile

Results of tests conducted with regular waves on bi-linear beach profiles are shown in Figure 3. Each figure shows the normalized runup plotted as a function of the deep water wave steepness on a log-log scale so that the theoretical solutions plot as a straight line. In each figure, three theoretical predictions are shown: (1) the Hunt Formula based on the beach face slope m from equation (7), (2) the Hunt Formula based on the surf zone slope s from equation (8), and (3) the integrated runup method using the effective slope concept from equation (10). Though not shown, tests conducted on a uniform beach profile confirmed that both equation (10) and the Hunt Formula gave identical results in good agreement with the measured runup values for plane slopes.

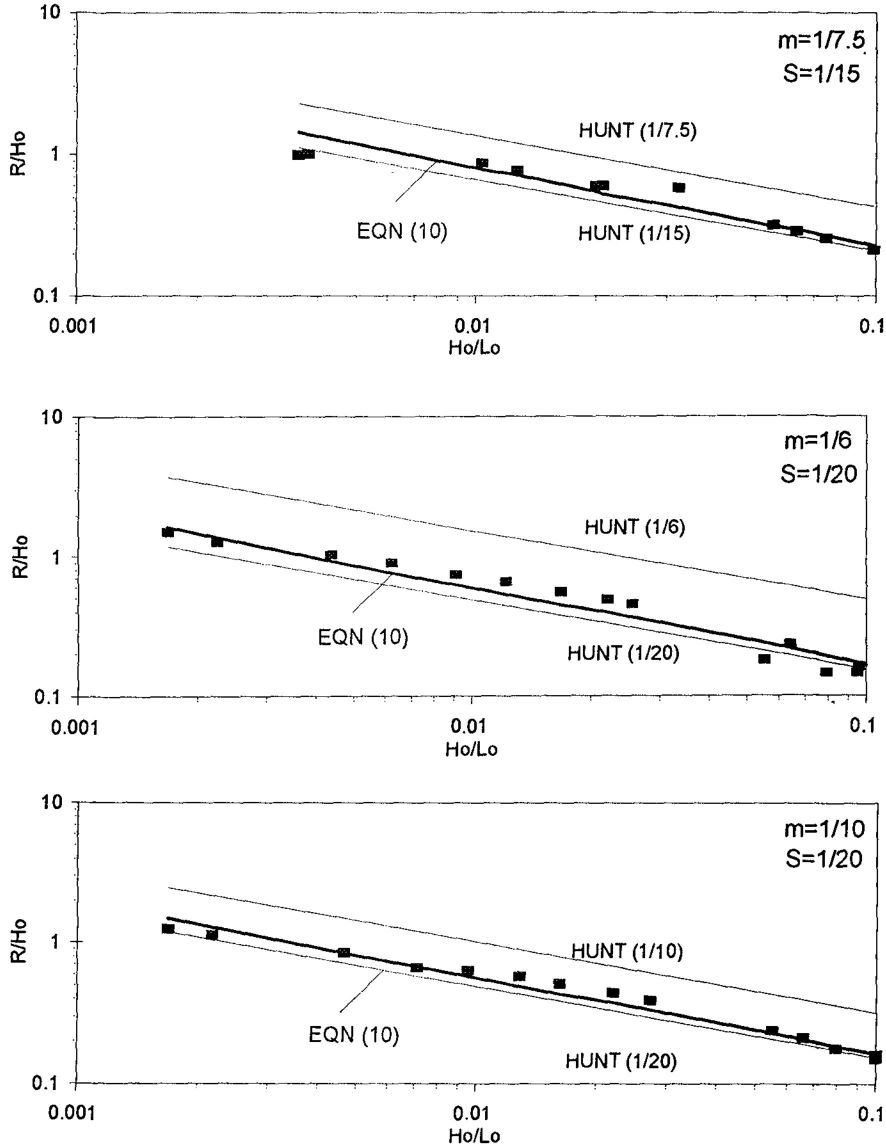

Figure 3. Comparison of measured and predicted runup for bi-linear beach profiles.

Figure 3a shows results for a bi-linear profile with a surf zone slope of 1-on-15 and a beach-face slope of 1-on-7.5. Figures 3b and 3c then show similar results where the surf zone slope was equal to 1-on-20, and where the beach-face slope was 1-on-6 and 1-on-10 respectively. In almost all cases, the measured runup (shown by solid squares) falls between the two limiting predictions based on the Hunt Formula using slopes m and s. It can be seen that the integrated method based on the effective slope concept provides a much better prediction in good agreement with the data.

These results suggest that the proposed method - integrating the Hunt Formula with Saville's effective slope concept - is capable of providing a simple unified description of regular wave runup on bi-linear beach profiles. For all 140 tests conducted on bi-linear profiles in this study, the Hunt Formula, based on the traditional use of the exposed beach-face slope m, over-estimates the runup by an average of 92%. Use of the surf zone slope, s, gives improved predictions but consistently underestimates the runup with an average error of 20%. The integrated method was found to predict runup to within an average error of 13.5%, although it also was somewhat biased toward under-estimating the runup.

Regular Waves on Equilibrium Profiles

Results of tests conducted with regular waves on equilibrium profiles are shown in Figure 4. In these tests, a single equilibrium profile slope parameter was used with A = 0.088 $m^{1/3}$. As depicted in Figure 2b, a linear beach-face slope was then joined to this concave equilibrium profile at a depth where the slopes matched. Two linear beach-face slopes were tested, but results in this paper are only shown for one slope, m=1/8. In Figure 4, three tests results are then shown, corresponding to three different water levels. Figure 4a shows results for a "mean" water level based on a 24.1 cm water depth in the wave basin. Figure 4b then shows a "high" water level condition based on a water depth of 30.4 cm, while Figure 4c shows a "low" water level condition based on a depth of 18.7 cm.

Each figure again shows the normalized runup plotted as a function of the deep water wave steepness on a log-log scale so that the Hunt Formula plots as a straight line. For equilibrium profiles, it is found however that neither the measured runup (solid squares) nor the predicted runup (open squares) based on the integrated method plots as a straight line with the same slope as the Hunt Formula. The reason for this is that, for any given value of wave steepness, the runup depends on the exact values of breaking depth and surf zone width. Because these vary nonlinearly on the concave beach profile based on the equilibrium profile shape $h=Ax^{2/3}$, the solution is then a unique function of h_b and X_b.

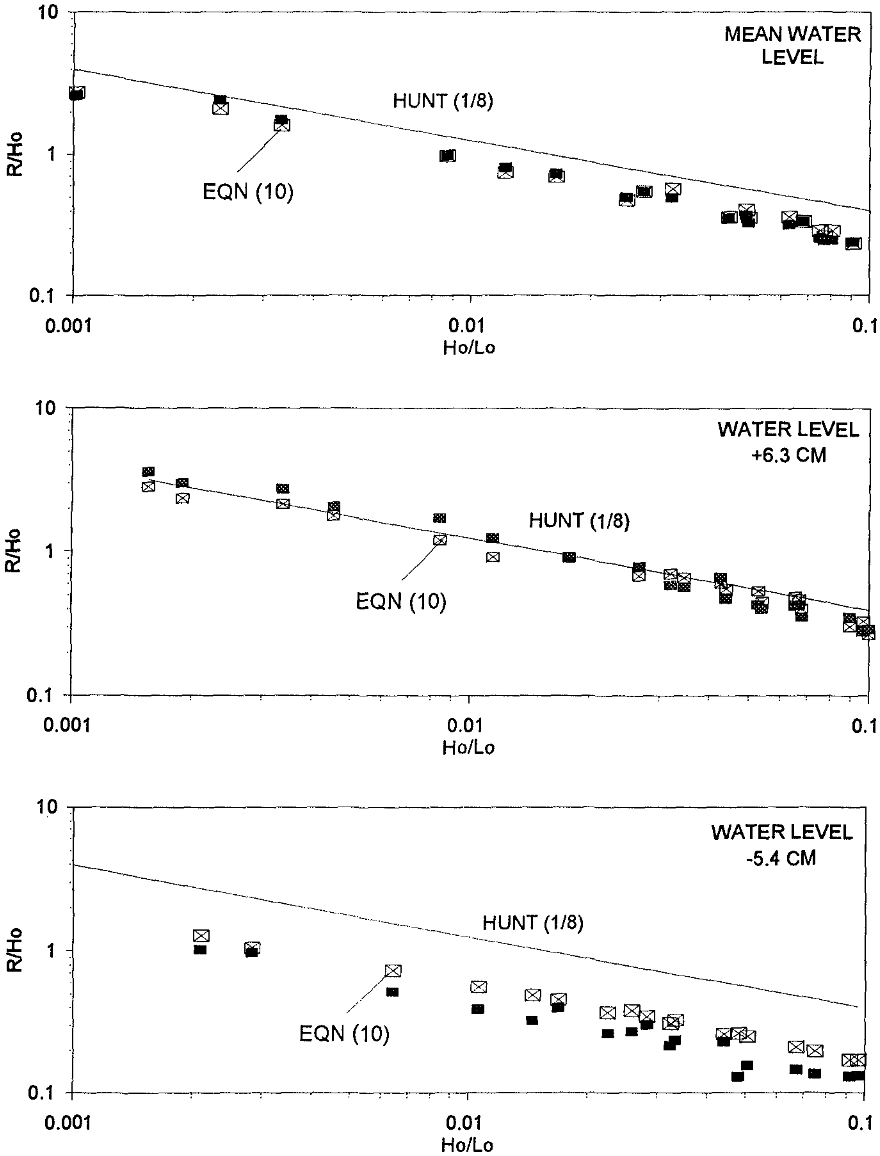

Figure 4. Measured and predicted runup over concave equilibrium profiles with m=1-on-8: (a) normal water level, (b) high water level, and (c) low water level.

For the mean water level condition, in Figure 4a, predictions based on equation (10) are in excellent agreement with measured values. Both are significantly below predictions from the Hunt Formula using the exposed beach-face slope. The difference is most pronounced for high steepness conditions where, in these experiments, the waves are relatively large and break far offshore yet produce relatively small runup so that the effective slope across the surf zone is fairly small.

In Figure 4b, for "high" water level conditions, measured and predicted runup agree much more closely with values predicted by the Hunt Formula using the beach-face slope. With the elevated water level, most waves now break on, or close to, the uniform beach-face slope so that the entire surf and swash zones are confined to the linear beach-face slope. Only in a few cases, with large waves having higher steepness, did waves break offshore on the concave portion of the profile so that runup was less much than expected from the Hunt Formula.

In contrast, Figure 4c for "low" water levels shows a large reduction in runup for essentially the same wave conditions used in the previous two tests. This is the result of the incipient break point being located far offshore over the concave portion of the profile. Measured runup is far below that predicted by the Hunt Formula using the beach-face slope of 1-on-8. Predictions based on the integrated method are in much better agreement but are somewhat higher than measured values. Figures 4b and 4c illustrate a strong dependence of runup on the profile geometry and water level that is rarely discussed in the literature. This should not be overlooked, however, as the measured values in this study show that runup may differ by about a factor of two over the range of water levels tested.

Irregular Waves on Bi-Linear Profiles

The integrated runup method in equation (10) was evaluated for irregular wave runup using statistical wave and runup parameters. Since most previous expressions for random wave runup have correlated statistical runup parameters to the incident significant wave height, a similar approach was used in this study. Incident wave conditions were described by both the mean and significant values for wave height and period. The mean and significant values for breaking depth and surf zone width were also determined from the video tape data of breaking waves. With these values as input, equation (10) was then used to predict the mean and significant runup values for comparison to the measured runup statistics.

Results of this analysis confirmed the results of Battjes (1971) that the mean runup could be predicted quite well using the mean wave conditions. Equation (10) gave quite good results when mean wave height, mean breaking depth, and mean surf zone width were used without use of any empirical coefficients. Use of the significant wave height in equation (10) produced runup estimates that were substantially below the measured significant runup. These results seem to indicate that the mean wave conditions describe the best average slope across the surf zone for use in runup predictions according to the Saville hypothetical slope method. Use of significant wave conditions produced a flatter average slope since the significant breakpoint is located much farther offshore. In the integrated runup method, this smaller effective slope produces smaller runup estimates that do not agree with the measured values.

Examples of runup predictions using the integrated method in equation (10) based on mean wave statistics are shown in Figure 5. Dimensionless mean runup, defined as mean runup normalized by mean deep water wave height, is plotted against the mean wave steepness, defined as the mean wave height divided by deep water wavelength calculated by linear wave theory using the mean wave period of the random sea. In Figure 5a, results are shown for a baseline test on a planar beach profile having a 1-on-15 slope. These results confirm the ability of the Hunt Formula, and the integrated method which is identical to the Hunt Formula for a planar slope, to predict the mean random wave runup on uniform plane slopes. The results are in contrast to the findings of Mase (1989), however, who found that the Hunt Formula did not describe runup on plane slopes.

Figures 5b and 5c then show results for irregular wave runup on bi-linear profiles. It is evident that use of the Hunt Formula based on the exposed beach-face slope gives large over-estimates in the mean wave runup. Use of the integrated method, with the average slope defined by the mean breakpoint and the mean runup limit, then gives much better results. When the results in Figure 5 are analyzed, it is found that the Hunt Formula based on the linear beach-face slope gives an average error of 93% while the use of the Hunt Formula coupled with the Saville effective slope concept gives an average error of just 12%. As with regular waves, the integrated method tends to be slightly biased toward under-estimating the runup.

Additional subjects of concern for design are the distribution of runup about the mean value and the extreme runup statistics. In this study, it has been found that, while incident waves and breaking depths are described quite well by the Rayleigh distribution, the runup distribution is wider than the Rayleigh distribution with both more small runup events and more very large runup events. As a result, the assumption of a Rayleigh distribution for runup is not valid over composite-slope beaches.

Results of this study indicate that the ratio between the significant runup and the mean runup is between 1.5 and 2.0, with an average of about 1.8. In contrast, the Rayleigh distribution would suggest a ratio of 1.6. Similarly, the ratio between the average of the largest ten-percent of the measured runup events to the mean runup varied between about 1.8 and 2.7 with an average of about 2.25. The Rayleigh distribution would suggest a ratio of 2.0. In general, no specific functional dependence was found between these ratios and either wave steepness or the surf similarity parameter, although there was a general trend toward greater ratios for the higher-steepness, more energetic waves..

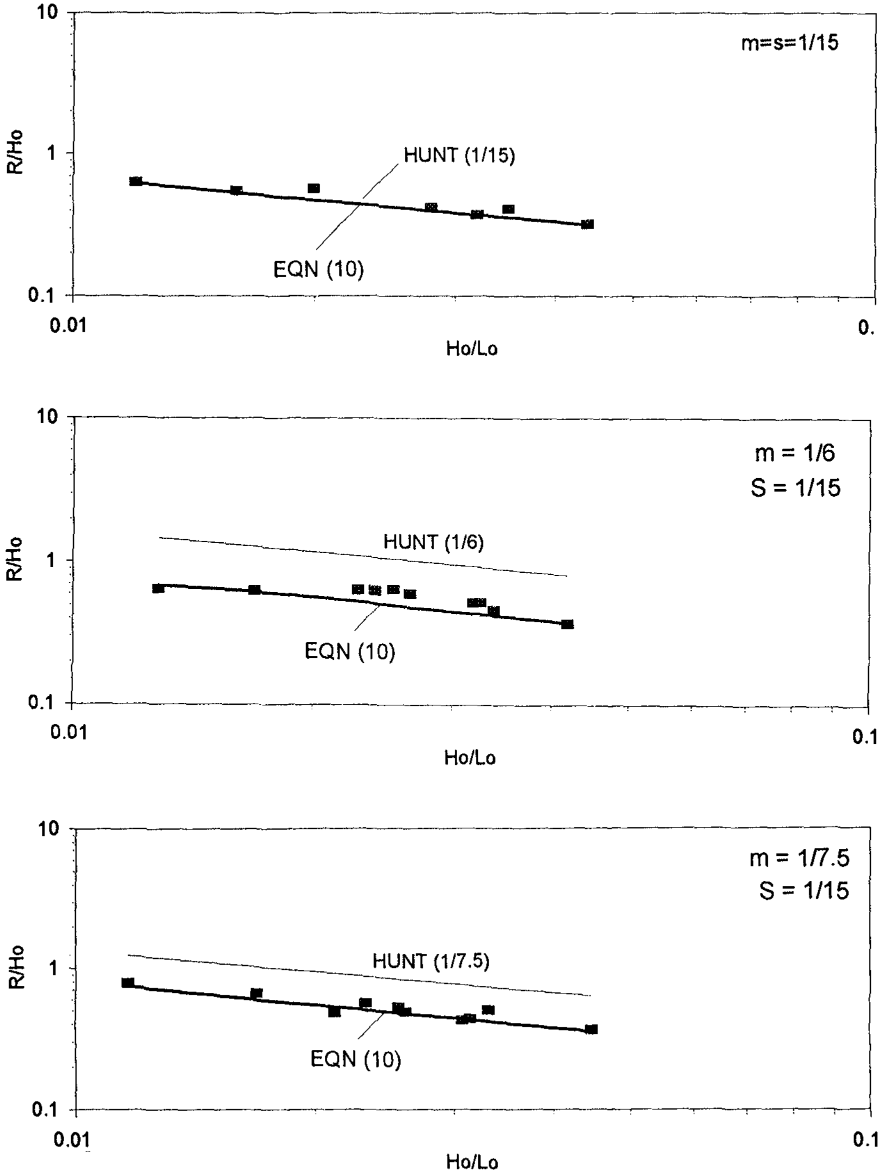

Figure 5. Comparison of measured and predicted runup of irregular waves on (a) linear profile and (b) bi-linear profiles. All wave heights are mean values.

Alternative Integrated Runup Method

The integrated runup method presented above works well but is complicated by the requirements that: (1) the breakpoint of incident waves must be determined *a priori* and (2) the runup limit is initially unknown and enters the description of the effective beach slope. An alternate effective slope concept of de Waal and van der Meer (1992) overcomes these difficulties by defining the effective slope between points on the profile that are a distancc of H_o above and below the still-water level. For a bi-linear beach profile, the effective slope according to de Waal and van der Meer is given by $\tan\beta = 2ms/(m+s)$ and the runup based on the Hunt Formula is then

$$R = \frac{2ms}{m+s}\sqrt{H_o L_o} \tag{11}$$

Results of equation (11) for the mean runup are shown in Figure 6. Also shown are the predictions from the integrated method in equation (10) and from the empirical equations of Mase (1989). As shown, the method of de Waal and van der Meer produces results that have approximately the same average error as obtained using the Saville effective slope concept. The method of Mase over-estimates the runup severely primarily since it is based on planar uniform slopes.

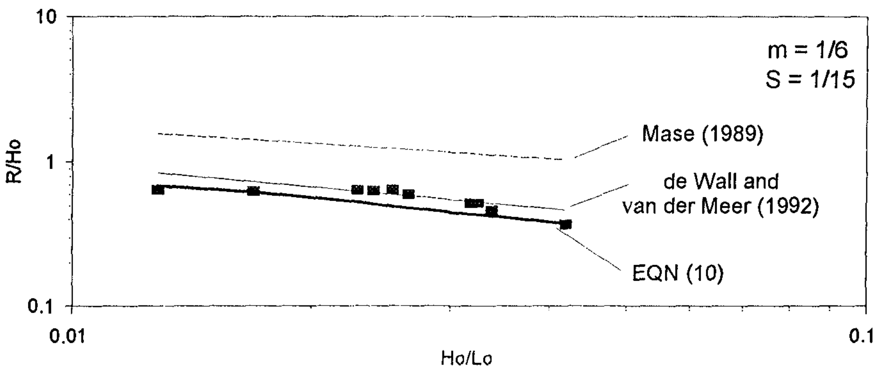

Figure 6. Comparison of measured and predicted mean runup in random waves from various prediction methods.

CONCLUSIONS

This paper has included two main parts: (1) the development of a simple integrated method for predicting runup on composite-slope beaches and (2) the experimental verification of this integrated approach in both regular and irregular waves. It has been shown that by combining the Hunt runup formula with the Saville effective slope concept, an analytical solution is obtained for runup over idealized bi-linear and equilibrium beach profiles. The solution can also be obtained by iterative methods for use in existing numerical models for surf zone hydrodynamics or beach profile change to estimate the runup over more complicated beach geometries.

The integrated method demonstrates that runup over concave and bi-linear beach profiles is less than would be expected from the Hunt Formula based on use of the exposed beach-face slope. Laboratory tests conducted on impermeable smooth slopes tend to verify the analytical solution for both bi-linear beaches and concave equilibrium beaches in both regular and random waves. While additional work on extreme runup statistics in random waves is needed, preliminary results indicate that the integrated method can be applied directly using the mean incident wave height to estimate the mean runup.

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