Second-order wave interaction with arrays of vertical cylinders of arbitrary cross section

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ABSTRACT

This paper proposes an approximate calculation method for the second-order wave interactions with arrays of vertical cylinders of arbitrary cross section. In mathematical formulations, the first- and the second-order boundary value problems are derived by perturbation method, and Green's Identity Formula is used to express the distribution of the velocity potentials on horizontal plane. Second-order water surface elevations near the cylinders and wave forces acting on the cylinders are computed, and the results are verified by comparing with wave tank experiments in the valid range of the Stokes second-order wave theory.

1. INTRODUCTION

Nonlinear wave interactions with structures are important under severe wave conditions, and the developments of a numerical method for calculating nonlinear wave forces and wave deformations are needed. A numerical calculation method of the second-order wave forces utilizing Hskind's reciprocal relationship has been proposed(e.g.,Molin,1979), and this method has been further applied to the case of plural vertical cylinders(e.g.,Masuda et. al.,1986). Nonlinear wave field near the structures, however, can not be calculated by this method because the second-order velocity potential in a fluid region is not determined. On the other hand, great efforts have been made to predict

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the nonlinear wave field near the structure by solving the second-order boundary value problem. For examples, Yamaguchi and Tsuchiya(1974) derived a second-order solution of closed form for a vertical cylinder, Garrison(1979) proposed a numerical solution based on a source distribution method using Green's function. Recently, Kriebel(1987) proposed an analytical solution of the second-order diffraction problem for a single circular cylinder and Kim and Yue(1989) showed the complete second-order diffraction solution for an axisymmetric body. However, these methods are difficult to be applied to the case of plural cylinders of arbitrary cross section.

The main purpose of this study is to develop an approximate calculation method for the second-order interaction between water waves and the arrays of vertical cylinders of arbitrary cross section. The validity of the method are confirmed through comparison with experiments and other numerical results. In mathematical formulations, perturbation method is used to derive the first- and second-order boundary value problems, and Green's Identity Formula is also used to express the distribution of the velocity potentials on a horizontal plane(Ijima et al.,1974). A particular solution for scattered waves of the second-order problem is approximately expressed on the assumption that it has the same form of an eigenfunction in the vertical direction as that of the second-order Stokes solution.

2. FORMULATION OF THE BOUNDARY VALUE PROBLEM

2.1 General formulation

Fig.1 shows a coordinate system, x and y being the horizontal axes and z the vertical axis taken upward from the undisturbed still water surface. Fixed vertical cylinders of arbitrary cross section in water of uniform depth h, are subject to the incident waves with first-order wave amplitude ζ_0, and angular frequency σ, propagating with an incident angle θ, measured from positive x direction. It is assumed that the fluid is both inviscid and incompressible, and its motion is irrotational. The velocity potential $\Phi(x, y, z, t)$ satisfy following Laplace equation in the whole fluid region.

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{1}$$

The kinematic boundary condition and the dynamic boundary condition on the free water surface, and bottom boundary conditions are written as:

$$\frac{\partial\zeta}{\partial t} - \frac{\partial\Phi}{\partial z} + \frac{\partial\Phi}{\partial x}\frac{\partial\zeta}{\partial x} + \frac{\partial\Phi}{\partial y}\frac{\partial\zeta}{\partial y} = 0 \qquad (z = \zeta(x, y, t)) \tag{2}$$

$$\frac{\partial\Phi}{\partial t} + g\zeta + \frac{1}{2}\left\{\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right\} = Q \qquad (z = \zeta(x, y, t)) \tag{3}$$

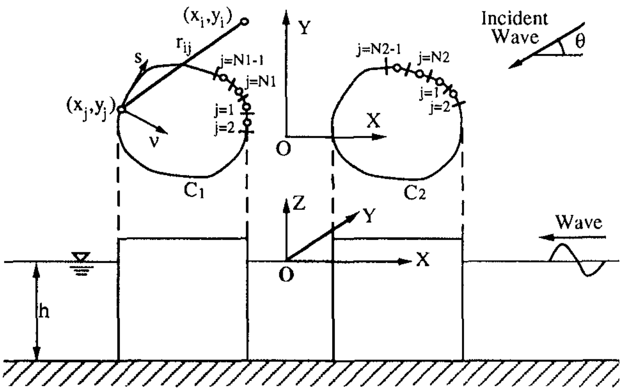

Fig.1 Definition sketch.

$$\frac{\partial \Phi}{\partial z} = 0 \qquad (z = -h) \tag{4}$$

where Q is a constant and g is a gravity acceleration.

In order to obtain the first- and the second-order boundary value problems, perturbation method is employed. The velocity potential $\Phi(x, y, z, t)$, the water surface elevation $\zeta(x, y, t)$, and the constant value Q are expressed in power series by introducing a parameter ϵ ($\epsilon = k\zeta_0$, where k is the incident wave number).

$$\left.\begin{aligned}
\Phi(x,y,z,t) &= \frac{g}{k\sigma} Re\left[\epsilon\phi_1^{(1)}(x,y,z)\exp(-i\sigma t)\right. \\
&\qquad \left. +\epsilon^2\left\{\phi_2^{(0)}(x,y,z) + \phi_2^{(2)}(x,y,z)\exp(-i2\sigma t)\right\}\right] \\
\zeta(x,y,t) &= \frac{1}{k} Re\left[\epsilon\eta_1^{(1)}(x,y)\exp(-i\sigma t)\right. \\
&\qquad \left. +\epsilon^2\left\{\eta_2^{(0)}(x,y) + \eta_2^{(2)}(x,y)\exp(-i2\sigma t)\right\}\right] \\
Q &= \frac{g}{k} Re\left[\epsilon Q_1 + \epsilon^2 Q_2\right]
\end{aligned}\right\} \tag{5}$$

The time independent component $\phi_2^{(0)}$ in the second-order potential function does not contributes to the second-order wave forces and water surface elevation. Since the main purpose of this study is to develop the numerical calculation method of the second-order wave forces and wave height, we consider the time dependent component $\phi_2^{(2)}$ only in the following formulations.

By expanding both Eqs.(2) and (3) in Taylor series about z=0, and substituting Eq.(5) into them, the first- and the second-order combined free surface boundary conditions are obtained.

2.2 First-order problem

The first-order boundary value problem is given as follows:

$$\nabla^2 \phi_1^{(1)}(x, y, z) = 0 \tag{6a}$$

$$\frac{\partial \phi_1^{(1)}}{\partial z} - \frac{\sigma^2}{g}\phi_1^{(1)} = 0 \qquad (z = 0) \tag{6b}$$

$$\frac{\partial \phi_1^{(1)}}{\partial z} = 0 \qquad (z = -h) \tag{6c}$$

The solution is obtained by separating the potential function $\phi_1^{(1)}$ into the incident plane waves and the scattered waves. A general solution of Eq.(6a), which satisfies free surface boundary condition and bottom boundary condition simultaneously, is given as(Ijima et al.,1974)

$$\phi_1^{(1)}(x, y, z) = \{\varphi_{1I}(x, y) + \varphi_{1S}(x, y)\} \frac{\cosh k(z + h)}{\cosh kh} \tag{7}$$

In Eq.(7), φ_{1I} and φ_{1S} represent the potential of the first-order incident plane waves and the scattered waves respectively, and φ_{1I} is given as

$$\varphi_{1I}(x, y) = -i \exp\{-ik(x\cos\theta + y\sin\theta)\} \tag{8}$$

where $i = \sqrt{-1}$ and k satisfies the following dispersion relationship.

$$\frac{\sigma^2 h}{g} = kh \tanh kh \tag{9}$$

φ_{1S} must satisfy the following Helmholtz equation since $\phi_1^{(1)}$ satisfies Laplace equation.

$$\nabla^2 \varphi_{1S}(x, y) + k^2 \varphi_{1S}(x, y) = 0 \tag{10}$$

2.3 Second-order problem

The second-order boundary value problem is given as follows:

$$\nabla^2 \phi_2^{(2)}(x, y, z) = 0 \tag{11a}$$

$$\frac{\partial \phi_2^{(2)}}{\partial z} - 4\frac{\sigma^2}{g}\phi_2^{(2)} = \frac{i}{k}\left\{\left(\frac{\partial \phi_1^{(1)}}{\partial x}\right)^2 + \left(\frac{\partial \phi_1^{(1)}}{\partial y}\right)^2 + \left(\frac{\partial \phi_1^{(1)}}{\partial z}\right)^2\right\}$$
$$+ \frac{\eta_1^{(1)}}{2k}\left\{\frac{\sigma^2}{g}\frac{\partial \phi_1^{(1)}}{\partial z} - \frac{\partial^2 \phi_1^{(1)}}{\partial z^2}\right\} \qquad (z = 0) \tag{11b}$$

$$\frac{\partial \phi_2^{(2)}}{\partial z} = 0 \qquad (z = -h) \tag{11c}$$

The second-order free surface boundary condition ,Eq.(11b), is nonhomogeneous because the quadratic forcing terms appear in the right hand side. The solution may be obtained by the separation of potential function $\phi_2^{(2)}$ as

$$\phi_2^{(2)}(x,y,z) = \phi_{2I}(x,y,z) + \phi_{2L}(x,y,z) + \phi_{2F}(x,y,z) \tag{12}$$

where ϕ_{2I} represents the second-order incident plane waves expressed as

$$\left.\begin{aligned} \phi_{2I} &= \varphi_{2I}(x,y)\frac{\cosh 2k(z+h)}{\cosh 2kh} \\ \varphi_{2I}(x,y) &= -\frac{3i}{8}\frac{\Gamma}{k}\frac{\cosh 2kh}{\sinh^4 kh}e^{-i2k(x\cos\theta + y\sin\theta)} \end{aligned}\right\} \tag{13}$$

where $\Gamma = \sigma^2 h/g$.

In Eq.(12), ϕ_{2L} represents a phase-locked wave which satisfies the nonhomogeneous free surface boundary condition and ϕ_{2F} represents a free wave which satisfies the homogeneous free surface boundary condition. The boundary value problems of ϕ_{2L}, and ϕ_{2F} are obtained by a substitution of Eqs.(12) and (13) into Eqs.(11a),(11b) and (11c).

Firstly, the boundary value problem for ϕ_{2L} is given as follows:

$$\nabla^2\phi_{2L}(x,y,z) = 0 \tag{14a}$$

$$\begin{aligned} \frac{\partial\phi_{2L}}{\partial z} - 4\frac{\sigma^2}{g}\phi_{2L} = \frac{i}{k}\Bigg\{&\frac{3\Gamma^2 - k^2}{2}\varphi_{1s}(\varphi_{1s} + 2\varphi_{1I}) + \\ &\frac{\partial\varphi_{1s}}{\partial x}\left(\frac{\partial\varphi_{1s}}{\partial x} + 2\frac{\partial\varphi_{1I}}{\partial x}\right) + \frac{\partial\varphi_{1s}}{\partial y}\left(\frac{\partial\varphi_{1s}}{\partial y} + 2\frac{\partial\varphi_{1I}}{\partial y}\right)\Bigg\} \quad (z=0) \end{aligned} \tag{14b}$$

$$\frac{\partial\phi_{2L}}{\partial z} = 0 \quad (z=-h) \tag{14c}$$

By assuming that the ϕ_{2L} has the same form of an eigenfunction for z direction as the Stokes second-order solution has(Sabuncu et al.,1985), the particular solution of ϕ_{2L} is approximately expressed as follows:

$$\phi_{2L}(x,y,z) = \gamma f(x,y)\frac{\cosh 2k(z+h)}{\cosh 2kh} \tag{15}$$

where γ and $f(x,y)$ are given as follows:

$$\gamma = \frac{i}{2k^2}\frac{1}{\tanh 2kh - 2\tanh kh} \tag{16}$$

$$f(x,y) = \frac{3\Gamma^2 - k^2}{2}\varphi_{1s}(\varphi_{1s} + 2\varphi_{1I}) + \frac{\partial\varphi_{1s}}{\partial x}\left(\frac{\partial\varphi_{1s}}{\partial x} + 2\frac{\partial\varphi_{1I}}{\partial x}\right) + \frac{\partial\varphi_{1s}}{\partial y}\left(\frac{\partial\varphi_{1s}}{\partial y} + 2\frac{\partial\varphi_{1I}}{\partial y}\right) \quad (17)$$

Secondary, the boundary value problem for ϕ_{2F} is given as follows:

$$\nabla^2\phi_{2F}(x,y,z) = 0 \quad (18a)$$

$$\frac{\partial\phi_{2F}}{\partial z} - 4\frac{\sigma^2}{g}\phi_{2F} = 0 \qquad (z=0) \quad (18b)$$

$$\frac{\partial\phi_{2F}}{\partial z} = 0 \qquad (z=-h) \quad (18c)$$

The solution of Eq.(18a), which satisfies the homogeneous free surface boundary condition and bottom boundary condition simultaneously, can be expressed in much the same way as the first-order problem as

$$\phi_{2F}(x,y,z) = \varphi_{2F}(x,y)\frac{\cosh k^{(2)}(z+h)}{\cosh k^{(2)}h} \quad (19)$$

where $k^{(2)}$ satisfies the following dispersion relationship.

$$\frac{(2\sigma)^2 h}{g} = k^{(2)}h\tanh k^{(2)}h \quad (20)$$

φ_{2F} must satisfy the following Helmholtz equation since ϕ_{2F} satisfies Laplace equation.

$$\nabla^2\varphi_{2F}(x,y) + (k^{(2)})^2\varphi_{2F}(x,y) = 0 \quad (21)$$

3. NUMERICAL CALCULATION METHOD

As shown in **Fig.1**, the boundaries C_1 and C_2 are defined as the intersection of still water surface and the vertical structures, and ν as an unit vector normal to the boundary. $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ are the coordinates taken in the fluid region and on the boundary respectively. Since the potentials of the first-order scattered waves and the second-order free waves satisfy the Sommerfeld's radiation condition respectively, each potential at $\boldsymbol{X}_i$ can be expressed by means of Green's Identity Formula as follows (Ijima et al.,1974):

$$\varphi_{1S}(\boldsymbol{X}_i) = \alpha \oint_{C_1+C_2} \left\{ \varphi_{1S}(\boldsymbol{X}_j) \frac{\partial}{\partial \nu} H_0^{(1)}(kr_{ij}) - H_0^{(1)}(kr_{ij}) \frac{\partial}{\partial \nu} \varphi_{1S}(\boldsymbol{X}_j) \right\} ds \tag{22}$$

$$\varphi_{2F}(\boldsymbol{X}_i) = \alpha \oint_{C_1+C_2} \left\{ \varphi_{2F}(\boldsymbol{X}_j) \frac{\partial}{\partial \nu} H_0^{(1)}(k^{(2)}r_{ij}) - H_0^{(1)}(k^{(2)}r_{ij}) \frac{\partial}{\partial \nu} \varphi_{2F}(\boldsymbol{X}_j) \right\} ds \tag{23}$$

where r_{ij} represents a distance between $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ and $H_0^{(1)}$ represent the first kind of Hankel function, and $\alpha = -i/2(\boldsymbol{X}_i$ is on the boundary) and $\alpha = -i/4(\boldsymbol{X}_i$ is in the fluid region)

For the first- and the second-order potential functions, no-flow conditions around the cylinders are written as follows:

$$\frac{\partial \phi_1^{(1)}}{\partial \nu} = 0 \qquad (-h \le z \le 0) \tag{24}$$

$$\frac{\partial \phi_2^{(2)}}{\partial \nu} = 0 \qquad (-h \le z \le 0) \tag{25}$$

By substituting Eq.(7) into Eq.(24) and Eq.(12) into Eq.(25), and by integrating Eq.(24) and Eq.(25) over the water depth after multiplying by eigenfunctions $\cosh k(z+h)$ and $\cosh 2k(z+h)$ respectively, the boundary conditions of φ_{1S} and φ_{2F} on C_1 and C_2 are derived as follows:

$$\frac{\partial \varphi_{1s}}{\partial \nu} = -\frac{\partial \varphi_{1I}}{\partial \nu} \tag{26}$$

$$\frac{\partial \varphi_{2F}}{\partial \nu} = -\frac{i}{4k\Gamma} \frac{1}{M_0} \frac{1}{\gamma} \frac{(k^{(2)})^2}{(2k)^2 - (k^{(2)})^2} \left(\frac{\partial \varphi_{2I}}{\partial \nu} + \gamma \frac{\partial f}{\partial \nu} \right) \tag{27}$$

where $M_0 = (1 + 2k^{(2)}h/\sinh 2k^{(2)}h)/2$

In order to obtain the numerical solutions of the integral equations written as Eqs.(22) and (23), the boundaries C_1 and C_2 are discretized into elements ΔS_j of N_1 and N_2 numbers, and the coordinate $\boldsymbol{X}_i$ is set on them. On each element, the potential and its normal derivative are assumed to be constant. By applying Eqs.(26) and (27) to Eqs.(22) and (23) respectively, the linear matrix equations for φ_{1s} and φ_{2F} are obtained as follows:

$$\sum_{j=1}^{N_1+N_2} \left(\overline{G_{ij}^{(1)}} - \delta_{ij}\right) \varphi_{1S}(\boldsymbol{X}_j) = - \sum_{j=1}^{N_1+N_2} G_{ij}^{(1)} \overline{\varphi_{1I}}(\boldsymbol{X}_j)$$
$$(i = 1 \sim N_1 + N_2) \tag{28}$$

$$\sum_{j=1}^{N} \left(\overline{G_{ij}^{(2)}} - \delta_{ij}\right) \varphi_{2F}(\boldsymbol{X}_j) = -\beta \sum_{j=1}^{N} G_{ij}^{(2)} \left\{\overline{\varphi_{2I}}(\boldsymbol{X}_j) + \gamma \overline{f}(\boldsymbol{X}_j)\right\}$$
$$(i = 1 \sim N_1 + N_2) \tag{29}$$

$$\left.\begin{array}{ll} G_{ij}^{(1)} = -\dfrac{i}{2}\displaystyle\int_{\Delta S_j} H_0^{(1)}(kr_{ij}) & \overline{G_{ij}^{(1)}} = -\dfrac{i}{2}\displaystyle\int_{\Delta S_j} \dfrac{\partial}{\partial \nu} H_0^{(1)}(kr_{ij}) ds \\ G_{ij}^{(2)} = -\dfrac{i}{2}\displaystyle\int_{\Delta S_j} H_0^{(1)}(k^{(2)}r_{ij}) & \overline{G_{ij}^{(2)}} = -\dfrac{i}{2}\displaystyle\int_{\Delta S_j} \dfrac{\partial}{\partial \nu} H_0^{(1)}(k^{(2)}r_{ij}) ds \end{array}\right\} \tag{30}$$

where $\overline{\phi_{1I}}$, $\overline{\varphi_{2I}}$ and $\overline{f}$ mean a normal derivative of ϕ_{1I}, φ_{2I} and f on the boundaries.

From Eq.(15), $f(\boldsymbol{X}_i)$ must satisfy the following Helmholtz equation.

$$\nabla^2 f + (2k)^2 f = 0 \tag{31}$$

By assuming that ϕ_{2L} satisfies the same radiation condition as φ_{1s} and φ_{2F}, $f(\boldsymbol{X}_i)$ can be also expressed by means of Green's Identity Formula as follows:

$$f(\boldsymbol{X}_i) = \alpha \oint_{C_1+C_2} \left\{ f(\boldsymbol{X}_j) \frac{\partial}{\partial \nu} H_0^{(1)}(2kr_{ij}) - H_0^{(1)}(2kr_{ij}) \frac{\partial}{\partial \nu} f(\boldsymbol{X}_j) \right\} ds \tag{32}$$

From Eq.(32), $\overline{f}(\boldsymbol{X}_j)$ can be computed without numerical differentiations of $f(\boldsymbol{X}_j)$ on the boundaries.

The first-order potential function $\phi_1^{(1)}$ expressed in Eq.(7) is obtained by solving the linear matrix equation. After solving the first-order equation, ϕ_{2L} is determined from the first-order solution and thus ϕ_{2F} is obtained by solving the linear matrix equation given as Eq.(29). By applying these second-order solutions to Eq.(12), the second-order potential function $\phi_2^{(2)}$ is determined.

3. RESULTS OF CALCULATIONS

To verify the numerical calculation method, we conducted the wave tank experiments for single and double vertical circular cylinders. The vertical circular cylinders of radius $a = 18.5cm$ are placed in the center of the wave tank of $18m$ long and $10m$ wide, and the water depth is maintained at $h = 40cm$.

Table-1 Incident wave conditions for single and double circular cylinders

Single Circular Cylinder			Double Circular Cylinder		
kh	$T(sec.)$	ζ_0/h	kh	$T(sec.)$	ζ_0/h
1.0	1.46	0.081	0.8	1.74	0.178
1.4	1.14	0.109	1.0	1.46	0.202
			1.4	1.14	0.179
			1.6	1.05	0.175

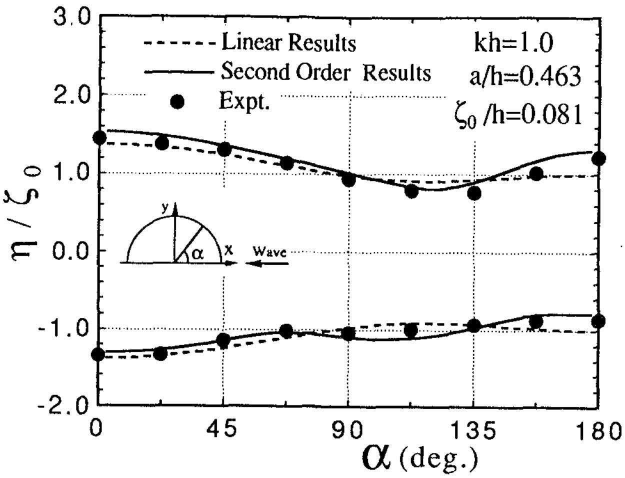

Fig.2 Wave runup and rundown around single circular cylinder.

Table-1 shows the incident wave conditions. The water surface elevations around the cylinders are measured by the wave gages. In the case of the double circular cylinders, the cylinders are placed in a row with spacing $B = 1m$ normal to the incident wave direction.

Fig.2 and **Fig.3** show the wave runup and rundown around the single cylinder for $kh = 1.0$ and $kh = 1.4$. The incident wave propagates from positive x direction and α is measured counterclockwise from positive x direction. In the experimental results, the secondary effects are observed at front side and rear side of the vertical cylinder and the second-order results show good agreements with experimental ones.

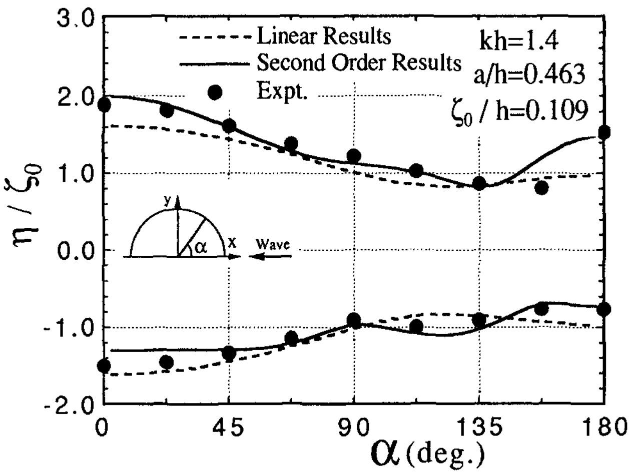

Fig.3 Wave runup and rundown around single circular cylinder.

Fig.4 and **Fig.5** show the wave runup and rundown around the double cylinder for kh = 0.8 and 1.0. In the experiments, the waves become very steep between the cylinders and the wave patterns around the cylinders are more complicate than the case of single cylinder. The linear results are largely different from experimental ones especially at the front side(α = 0), rear side(α = 180) and inner side(α = 270) of the cylinders. On the other hand, the second-order results show good agreements with experimental ones.

Fig.6 and **Fig.7** show the computed maximum wave amplitude distribution around the double and the triple circular cylinders. The wave with kh = 1.0 and ζ_0/h = 0.1 propagates from positive y direction normal to the row of the cylinders(a/h =0.5, B/h =2; a is the cylinder radius and B is the space between cylinders) The second-order results show more complicated pattern than linear ones and secondary effects are clearly observed around the cylinders. The maximum differences of wave amplitude between linear results and second-order ones around the cylinders are about 20%.

Fig.8 shows the maximum wave forces acting on the single and double circular cylinders, and **Fig.9** also shows the maximum wave forces acting on the center of the triple circular cylinders. The cylinders are placed in a row normal to the incident wave direction. The differences between linear results and second-order ones become small as the ka increase.

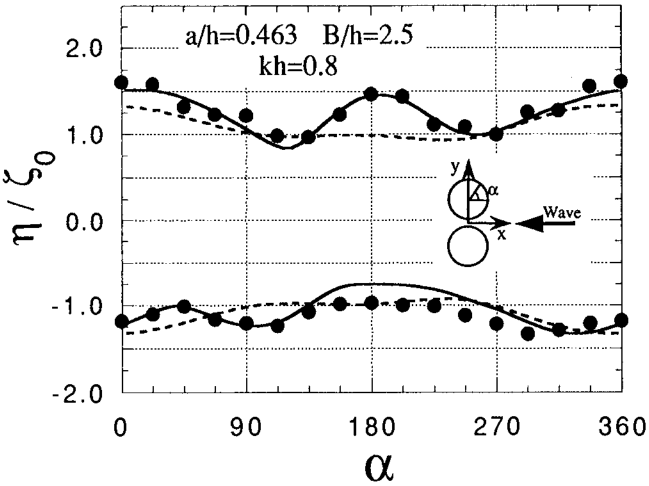

Fig.4 Wave runup and rundown around double circular cylinder.
---- Linear Results, ——— Second-Order Results, ● Expt.

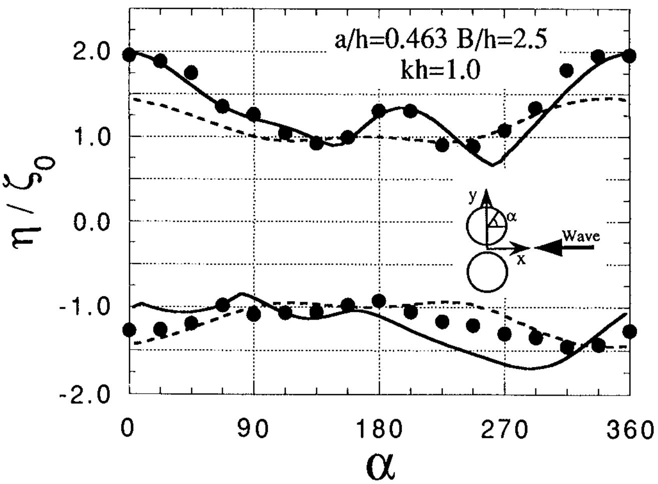

Fig.5 Wave runup and rundown around double circular cylinder.
---- Linear Results, ——— Second-Order Results, ● Expt.

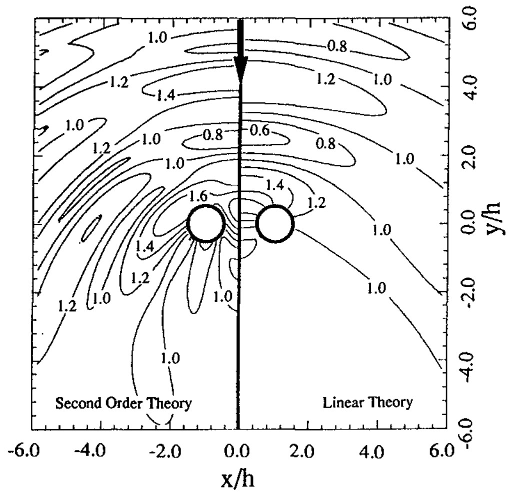

Fig.6 Maximum wave amplitude distribution near double circular cylinders.
$[B/h = 2, a/h = 0.5, \zeta_0/h = 0.1, kh = 1.0]$

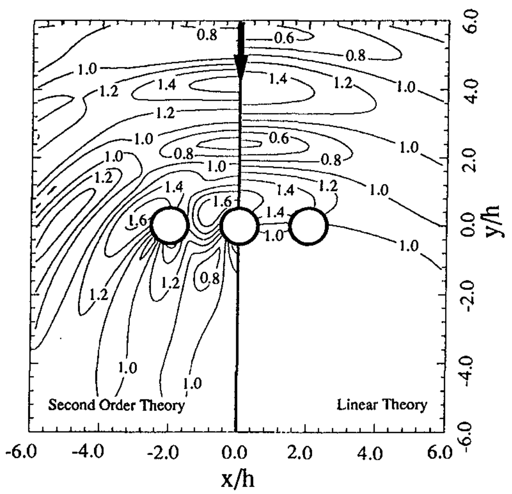

Fig.7 Maximum wave amplitude distribution near triple circular cylinders.
$[B/h = 2, a/h = 0.5, \zeta_0/h = 0.1, kh = 1.0]$

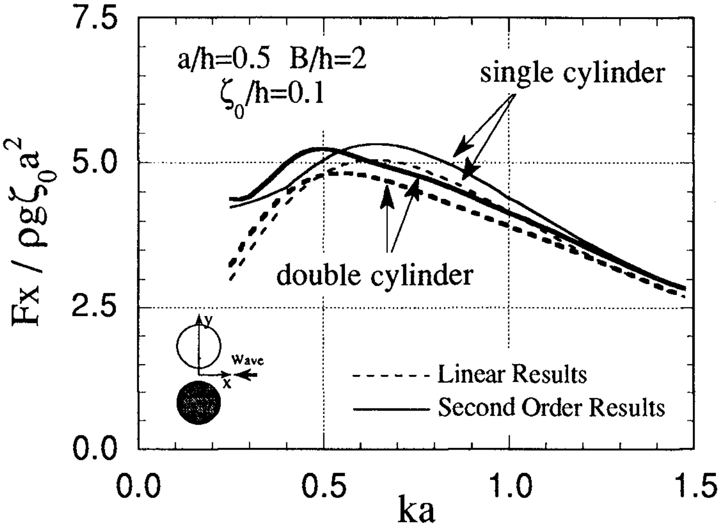

Fig.8 Maximum wave forces acting on double circular cylinders

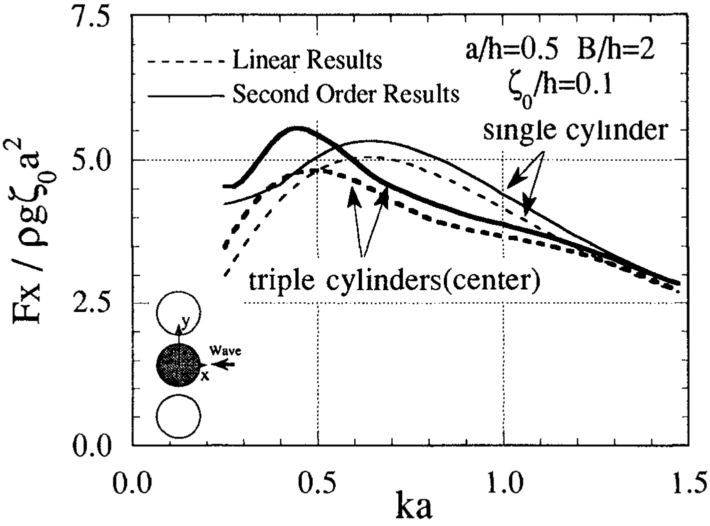

Fig.9 Maximum wave forces acting on triple circular cylinders

On the other hand, the differences become large as the ka decrease for $ka < 0.5$. The divergence of the present method in small value of $ka(ka < 0.3)$, small value of ka means large Ursell parameter in this case, is caused by the failure of the Stokes second-order theory(Isaacson,1978). In $0.6 < ka < 1.3$, the wave forces acting on the single cylinder are always larger than those on the double cylinders and the center cylinder of the triple ones, but with increase of ka, the wave forces acting on those plural cylinders are converged to the value of single cylinder.

5. CONCLUSIONS

An approximate calculation method for the second-order wave interactions with array of vertical cylinders of arbitrary cross section is proposed by using both Green's Identity Formula and perturbation method in combination. The validity of the present method is confirmed by comparing the computed results with experimental ones. Though the present method includes approximation that the second-order potential function for phase-locked wave has the same form of an eigenfunction of the second-order Stokes solution, the nonlinear wave field and wave forces are estimated with good accuracy in the valid range of the Stokes second-order wave theory.

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