CHAPTER 44

SHEAR STRESSES AND MEAN FLOW IN SHOALING AND BREAKING WAVES

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ABSTRACT

We investigate the vertical, wave averaged distributions of shear stresses and Eulerian flow in normally incident, shoaling and breaking waves. It is found that shear stresses are solely due to wave amplitude variations, which can be caused by shoaling, boundary layer dissipation and/or breaking wave dissipation. The resulting shear stress and mean flow distributions for these cases are derived, and compared with earlier work.

The attractive, now frequently used modelling choice of specifying a shear stress at the mean surface level is discussed in the context of the constituent equations and related boundary conditions and constraints. A derivation of the shear stress at the mean surface level is given both by using the momentum balance and energy balance equations, which is shown to lead to the same result, if the effects of a changing roller are incorporated correctly(3).

Finally, matching solutions for the shoaling and breaking wave cases between the boundary layer and the middle layer for the shear stresses and the wave averaged flow are derived.

INTRODUCTION

The intentions of the present paper are to present a conceptual view on the constituent equations and related boundary conditions and constraints for the vertical distributions of wave averaged shear stresses and Eulerian flow for the cases of

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shoaling and breaking waves, including boundary layer dissipation effects. While the relevant physical phenomena are indicated, it is not intended to suggest optimal closure hypotheses for all the phenomena, but rather to indicate their physical consequences in a transparent way. Examples are the eddy viscosity assumptions and the roller effects, for which simplified choices are made, which are, to a certain degree, not essential but only made for simplicity and transparency of the concept.

It is found that the sloping bottom does not introduce additional terms in the shear stress distributions found by Deigaard and Fredsoe (1989). The separate cases of no-breaking and breaking are discussed.

Furthermore, it is shown how a general expression for the mean Eulerian flow may be derived, which incorporates the effects of sloping bottom and wave amplitude variations, either due to boundary layer dissipation, shoaling or breaking. In the special cases of horizontal bottom and no-breaking and of sloping bottom and no-breaking, the expression reduces to Longuet-Higgins' conduction solution (1953) and to Bijker et al's shoaling solution (1974), respectively.

CONSTITUENT EQUATIONS AND CONDITIONS

If we neglect the effects of advective acceleration for the time-mean flow and assume a wave-averaged eddy viscosity approximation for the Reynolds' stresses, the local momentum equation most commonly used to solve the wave averaged Eulerian flow reads:

$$\frac{\partial}{\partial z} \nu_t \frac{\partial U}{\partial z} = g\overline{\eta_x} + (\overline{u^2})_x - (\overline{w^2})_x + (\overline{uw})_z \tag{1}$$

where:

U is the wave averaged Eulerian flow,
ν_t is the eddy viscosity,
η_x is the mean water level set-up, and
u, w are the horizontal and vertical wave-orbital velocities.

This equation is assumed to be valid in the full vertical domain, except for the region near the free surface. Somewhat heuristicly, we assume the equation to be valid unto mean water level, since a wave averaged situation is considered, and we let the effects of the near surface layer (such as the roller) be effectuated in a shearstress acting at the mean water level.

Let us introduce the common assumption that the wave terms (orbital velocity moments) can be derived independently from the mean flow, i.e. we assume that there exists a sufficiently accurate wave theory to describe the wave terms which neglects the wave-current interaction. This is of course a simplification, and we may state that a true break-through here would be achieved if we could tackle the problem with a Lagrangian approach in which waves and currents would be considered simultaneously, which at the same time would allow us to deal consistently with the near surface layer (NSL). However, until such an approach is developed we rely on the experience that the suggested approach is shown to yield sufficiently accurate results for the moment.

Let us further assume that the turbulence viscosity is constant over depth. Note that this is only for the clarity of the argumentation. The rationale which follows would also allow for a depth-varying viscosity, as long as it is time invariant and does not depend on the undertow solution itself.

The above implies that we have one equation to solve for the undertow and the mean water level set-up. Integrating the equation twice yields:

$$\nu_t U = \frac{1}{2} g\overline{\eta}_x z'^2 + \int_0^{z'} \int_0^{z'} (\overline{u^2})_x dz dz - \int_0^{z'}\int_0^{z'} (\overline{w^2})_x dz dz + \int_0^{z'} (\overline{uw}) dz + C_1 z' + C_2 \qquad (2)$$

This expression contains three unknowns, viz. the two constants of integration, C_1 and C_2 remaining in this expression, and the set-up gradient. We therefore need, in addition to this equation, three boundary conditions and/or constraints:

(1) no-slip condition at the bottom ($U = 0$), from which we can find C_2;
(2) shear stress condition at the transition from the middle layer (ML) to the NSL (τ_t given), from which we can find C_1;
(3) mass balance constraint (total mass flux in the lower layers balances that in the NSL), from which we can derive the set-up gradient.

The essential information which we need here is related to the NSL. In fact, condition (2) is based on the assumption that we know the shear stress at the lower end of the NSL from the momentum balance for this layer. Similarly, when formulating the constraint (3), we assume the total mass flux in the NSL to be known.

This route is suggested by Stive and De Vriend (1987) and also followed by Deigaard et al. (1991). The former derive a formal expression through a third depth integration, while the latter use an iteration procedure which sees to it that a set-up gradient is created such that the correct depth-average mass flux is created. The result should be the same.

Alternatively, one could use the depth-averaged horizontal mass and momentum equations, with the wave-induced radiation stresses and mass fluxes properly modelled, instead of giving τ_t and imposing constraint (3). Note also that imposing constraint (3) is not correct in 3-D situations, where the mass flux in the NSL is not necessarily compensated in the lower parts of the same water column (cf. De Vriend and Kitou, 1990). In fact, later we shall use the depth-averaged momentum equation, with the usual expressions for the radiation stresses, to derive an expression for τ_t. Note that in either case we face the problem of describing the NSL, via τ_t and the mass flux, or via the wave-related terms in the depth-averaged mass and momentum equations.

DISTRIBUTION OF ORBITAL VELOCITY MOMENTS AND SHEAR STRESSES ON A SLOPING BOTTOM IN THE MIDDLE LAYER

In order to derive the shear stress distribution the orbital velocities outside and a little away from the bottom boundary layer need to be known. The idea is to use these results to look at the shear stress distribution in the middle layer for the cases of sloping bottom and boundary layer dissipation and of sloping bottom and breaking wave dissipation. Furtheron, we will look at these cases with boundary layer effects on the shear stress distribution included.

Since we are interested in the case of spatially varying waves on a sloping bottom, we must at least rely on a non-uniform depth approximation. This has been done by De Vriend and Kitou (1990) and recently also by Rivero and Arcilla (1994) for the shallow water approximation to shoaling waves. The result as obtained for the orbital velocity moments reads:

$$\overline{u^2} = \frac{1}{2} A^2 + O\left(A^2 \frac{\lambda^2}{L^2}\right)$$

$$\overline{w^2} = O\left(A^2 \frac{h^2}{L^2}\right) \qquad (3)$$

$$\overline{uw} = \frac{1}{2} A^2 h_x - \frac{1}{4} \left(A^2\right)_x z'$$

where

λ = wave length,
L = scale of horizontal variations, and
A = (aω)/(kh).

Based on these results we can investigate our two cases.

(1) The case of a sloping bottom and boundary layer dissipation

Using the above results the shear stress becomes

$$\tau(z')/\rho = \nu_t U_z = g\overline{\eta}_x z' + \frac{1}{4} (A^2)_x z' + \frac{1}{2} A^2 h_x + C_1 \qquad (4)$$

Because of the absence of dissipation in the NSL there is no shear stress at the mean water level, sothat

$$\tau(z')/\rho = g\overline{\eta}_x (z' - d_m) + \frac{1}{4} (A^2)_x (z' - d_m) \qquad (5)$$

In fact, no shear stresses will exist in the whole of the middle layer for (Deigaard and Fredsoe, 1989):

$$\rho g \overline{\eta}_x d_m = -\frac{1}{4}\rho (A^2)_x d_m = -\frac{1}{2} E_x \tag{6}$$

which complies with the fact that no shear stresses can be maintained in the middle layer due to the existence of irrotational flow. However, we will show later that due to the constraint of depth averaged zero mean flow, the existence of an additional mean water level gradient is required.

(2) The case of a sloping bottom and wave breaking dissipation

Again using the above results the shear stress in this case becomes:

$$\tau(z')/\rho = g\overline{\eta}_x (z' - d_m) + \frac{1}{4}(A^2)_x (z' - d_m) + \tau_t \tag{7}$$

Now a shear stress at mean water level exists due to the presence of the roller. The mean water level gradient again follows from the mean flow constraint.

DERIVATION OF THE SHEAR STRESS AT MEAN WATER LEVEL

Here derivations of the shear stress at mean water level are presented, and we show that the result is consistent between using either momentum flux or energy flux considerations.

From the above Equation (7) we find for the bottom shear stress:

$$\tau_b = -\rho g d_m \overline{\eta}_x + \tau_t - \frac{1}{4}\rho (A^2)_x d_m \tag{8}$$

Since we consider wave breaking dissipation only, the mean bottom shear stress τ_b should be zero.

The shear stress τ_t may be resolved between the τ_b equation and the depth mean horizontal momentum balance equation. In order to do this we need to introduce an expression for the radiation stress which at least needs to be extended with the roller effect. Following Svendsen (1984), Deigaard and Fredsoe (1989) suggest the shallow water approximation

$$S_{xx} = S_{xx,p} + S_{xx,u} = \frac{1}{2}E + \left[E + \rho \frac{Rc}{T}\right] \tag{9}$$

where R is the roller area.

The above implies that the set of equations available to resolve the shear stress τ_t reads:

$$\frac{dS_{xx}}{dx} + \rho g d_m \overline{\eta}_x = 0 \tag{10}$$

$$\frac{dS_{xx}}{dx} = \frac{3}{2} E_x + \frac{\rho}{T} (Rc)_x \tag{11}$$

$$\tau_b = -\rho g \overline{\eta}_x d_m + \tau_t - \frac{1}{4} \rho (A^2)_x d_m = -\rho g \overline{\eta}_x d_m + \tau_t - \frac{1}{2} E_x = 0 \tag{12}$$

which yields:

$$\tau_t = -E_x - \frac{\rho}{T} (Rc)_x \tag{13}$$

and

$$\tau(z') = -E_x \left(1 + \frac{d_m - z'}{2d_m} \right) - \rho g \overline{\eta}_x (d_m - z') - \frac{\rho}{T} (Rc)_x \tag{14}$$

These equations are equivalent to equations (56) and (58) of Deigaard and Fredsøe (1989), when introducing the relation

$$E_x = \frac{1}{c} \left(E_f \right)_x = -\frac{D}{c} \tag{15}$$

where E_f is the energy flux and D is the dissipation due to wave breaking.

In slightly different notation Equation (13) reads:

$$\tau_t = -\frac{1}{c} \left(E_f \right)_x - \left(2E_r \right)_x \tag{16}$$

where E_f is the energy flux, now using the expression of kinetic roller energy E_r (introduced by Svendsen, 1984, and used by Nairn et al., 1990):

$$2E_r = S_{xx,roller} = \rho Rc^2/L = \rho Rc/T \tag{17}$$

The presence of the roller leads to the additional term, which is due to the nonnegligible velocity contribution to the momentum flux, since the velocities are of magnitude c. The contribution due to the roller enhanced pressure is probably not only small, but may also be already included in $(E_f)_x$ since this property is quantified using the surface elevation variance.

One may, however, observe that the shear stress at NSL also appears in the energy balance equation, which according to Nairn et al. (1990) reads:

$$(E_f)_x + (E_r c)_x + \tau_t c = 0 \tag{18}$$

where they have introduced Svendsen's (1984) suggestion for the energy flux due to the roller and the result given in Deigaard and Fredsoe (1989) that the dissipation D is due to the work done by the shear stress due to the roller acting on the fluid right below it. Again, the roller related contribution is due to the mean transfer of kinetic energy (proportional to c^2) with the roller velocity (equal to c).

Rearranging the last equation for τ_t yields

$$\tau_t = -\frac{1}{c}(E_f)_x - \frac{1}{c}(E_r c)_x \tag{19}$$

which indicates that when we accept that the spatial variations in c are small relative to those in E_r we are faced with a factor 2 difference. Note that here also we have assumed that the horizontal component of the shear stress is negligibly different from the shear stress along the roller-wave interface.

It appears that the apparent inconsistency is caused by the complicated situation that occurs when the volume of the roller is changing in the wave propagation direction (see Appendix). Beside the shear layer between the roller and the wave there is a net transfer of water from the wave to the roller. If the roller is losing water ($dR/dx < 0$) the horizontal momentum transfer from the roller to the wave is not only that due to the shear layer, but also the momentum of the water leaving the roller. If the roller gains water ($dR/dx > 0$), the water leaving the wave has negligible horizontal velocity and does not change the momentum of the water remaining in the wave. In both cases, however, there is an additional energy dissipation of

$$\rho \frac{1}{2} c^2 \left|\frac{dR}{dt}\right| \tag{20}$$

The corrections that these considerations give to the NSL shear stress and to the energy dissipation rate were derived in Deigaard (1993), and it appears that the energy balance equations in both cases yield:

$$(E_f)_x + (2E_r c)_x + \tau_t c = 0 \tag{21}$$

which removes the discrepancy and implies a correction to Nairn et al. (1990) and Stive and De Vriend (1994).

MATCHING SOLUTION BETWEEN BOUNDARY LAYER AND MIDDLE LAYER

Where in the foregoing we have neglected the bottom boundary layer effects on the time- and depth-averaged momentum equation since it is a second-order effect in the overall momentum balance, we present in this section a matching solution to derive the mean Eulerian flow over the vertical, with accuracy near the bottom. Due to the existence of the bottom boundary layer the near-bottom horizontal orbital velocity may be shown to include a phase difference compared to the orbital velocity in the middle layer (cf. Longuet-Higgins, 1953). Because of the sloping bottom and/or due to wave breaking a variation of the wave amplitudes results which may be shown to yield the following matched result for the horizontal orbital velocity (cf. Bijker et al., 1974):

$$u = A(x)\left[\cos\chi - e^{-\phi}(\chi - \phi)\right] \tag{22}$$

where

$\phi = z'/\delta$
$\delta^2 = 2\nu_t/\omega$
$\chi = \omega t - \psi$
$k = d\psi/dx.$

By using continuity

$$w_z = -u_x \tag{23}$$

and depth integration

$$w = -\int_0^{z'} u_x dz' \tag{24}$$

we may derive the following expressions for the time-averaged values of the wave terms needed in Equation (1):

$$\overline{u^2} = \frac{1}{2}A^2\left[1 + e^{-2\phi} - 2e^{-\phi}\cos\phi\right] \tag{25}$$

$$\begin{aligned}\overline{uw} = {} & A^2k\delta\left[-\frac{1}{4} + \frac{\phi}{2}e^{-\phi}\sin\phi - \frac{e^{-\phi}}{4} + \frac{e^{-\phi}}{2}\cos\phi\right] - \\ & -\left(\frac{A^2}{2}\right)_x \frac{\delta}{2}\left[-\frac{1}{2} + \phi + e^{-\phi}\cos\phi - \phi e^{-\phi}\cos\phi - \frac{e^{-2\phi}}{2}\right]\end{aligned} \tag{26}$$

These equations were derived by Bijker et al. (1974) for the shoaling wave case, but they are equally valid for the case of wave breaking, since in the shoaling case it is the amplitude variation only that impacts on these terms. The difference being solely that during shoaling the amplitudes increase, while due to breaking they decrease.

Using the above expressions for the wave-averaged orbital terms we may resolve Equation (2) and the set of three boundary conditions and constraints to yield:

$$
\begin{aligned}
\nu_t U = {} & \left(g\overline{\eta_x} + \frac{1}{4}(A^2)_x\right)\left[\frac{1}{2}z^2 - z'd_m\right] + \frac{\tau_t}{\rho}[z'] + \\
& + (A^2)_x \frac{\delta^2}{4}\left[2e^{-\phi}\sin\phi + \frac{1}{2}\, e^{-\phi}\cos\phi + \frac{1}{2}\,\phi e^{-\phi}(\sin\phi - \cos\phi) + \frac{1}{4}e^{-2\phi} - \frac{3}{4}\right] + \\
& + A^2 k \frac{\delta^2}{2}\left[-\frac{1}{2}e^{-\phi}\phi(\sin\phi + \cos\phi) - e^{-\phi}\cos\phi + \frac{1}{2}e^{-\phi}\sin\phi + \frac{1}{4}e^{-2\phi} + \frac{3}{4}\right]
\end{aligned}
\tag{27}
$$

Note that for transparency we have assumed an eddy viscosity invariant over depth, which is not necessary: it only yields an attractive, analytical expression over the total depth, which allows us to point out the respective approximations for the cases derived before. Longuet-Higgins' solution (1953) contains the fourth term of Equation (19) only. In the shoaling case of Bijker et al. (1974) we have

$$
\begin{aligned}
g\overline{\eta_x} &= -\frac{1}{4}(A^2)_x \text{ and} \\
\tau_t &= 0
\end{aligned}
\tag{28}
$$

so that only the last two terms of Equation (27) result. For breaking waves on a sloping bottom we need all four terms. Note that in all cases a small mean water level gradient may be necessary to comply with a depth-averaged zero Eulerian flow.

Here, we refrain from a deeper analysis of the surfzone situation with strong, breaking induced wave amplitude variations. Clearly, the mean water level gradient and the NSL shear stress (as represented by the first two terms in Equation (27)) will exert a major influence on the mean Eulerian flow distribution. However, we may expect that especially near the bottom the boundary layer term and the amplitude variation term will show their significance. An insight into their qualitative influence is given below.

For the shoaling wave case the mean Eulerian flow distribution reads:

$$
U = \frac{A^2 k}{\omega} f'(\phi) + \frac{A}{\omega}\frac{dA}{dx}\, g'(\phi) \tag{29}
$$

where the vertical form functions are given by

$$
f'(\phi) = \left[-\frac{1}{2}e^{-\phi}\phi(\sin\phi + \cos\phi) - e^{-\phi}\cos\phi + \frac{1}{2}e^{-\phi}\sin\phi + \frac{1}{4}e^{-2\phi} + \frac{3}{4}\right] \tag{30}
$$

$$g'(\phi) = \left[2e^{-\phi}\sin\phi + \frac{1}{2}e^{-\phi}\cos\phi + \frac{1}{2}\phi e^{-\phi}(\sin\phi - \cos\phi) + \frac{1}{4}e^{-2\phi} - \frac{3}{4}\right] \tag{31}$$

Their behaviour is graphically illustrated in Figure 1, which teaches us that close to the bottom the amplitude variation effect strengthens the streaming effect in the shoaling zone, but weakens it in the breaker zone. Whether this actually occurs is depending on the relative magnitude of these terms, which may be estimated as follows. At the edge of the boundary layer we find:

$$\begin{aligned} U_\infty &= \frac{3}{4}\frac{A^2k}{\omega} - \frac{3}{4}\frac{A}{\omega}\frac{dA}{dx} \\ &= \frac{3}{4}\frac{A^2k}{\omega}\left[1 - \frac{1}{Ak}\frac{dA}{dh}\frac{dh}{dx}\right] \end{aligned} \tag{32}$$

Using linear shoaling it may be shown (Bijker et al., 1974) that 1/(Ak) dA/dh = O(1) for kh ≥ 1, sothat only in the breaker zone and with relative steep slopes the amplitude variation effect becomes important.

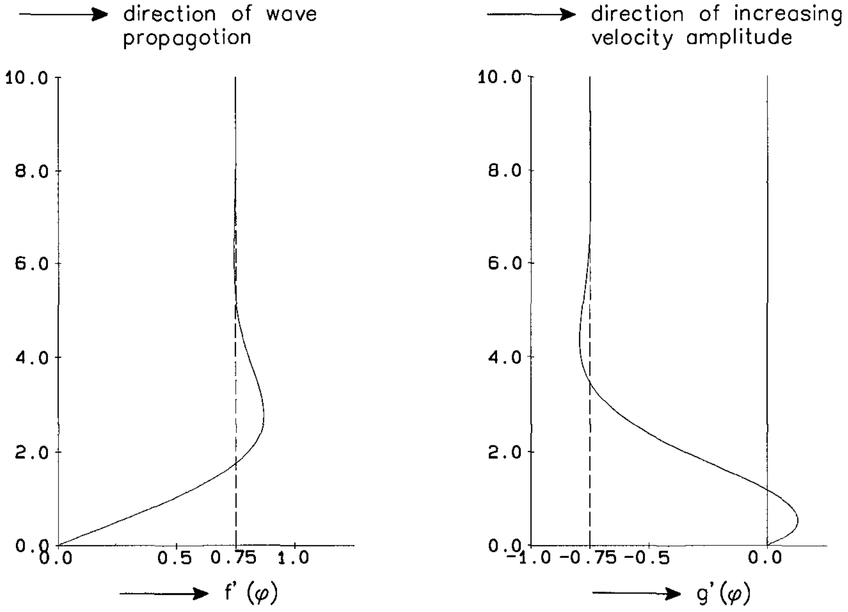

Figure 1 Form functions of streaming effect ($f'(\varphi)$) and amplitude variation effect ($g'(\varphi)$)

CONCLUSIONS AND DISCUSSION

A conceptual view is presented on the constituent equations and related boundary conditions and constraints for the vertical distributions of wave averaged shear stresses and Eulerian flow for the cases of shoaling and breaking waves, including boundary layer dissipation effects. We have refrained from suggesting optimal closure hypotheses for the relevant processes, but we have tried to indicate their physical consequences in a transparent way. It is found that shear stresses are solely due to wave amplitude variations, which can be caused by shoaling, boundary layer dissipation and/or breaking wave dissipation. The resulting shear stress and mean flow distributions for these cases are derived, and compared with earlier work.

The specification of the shear stress at the mean surface level is discussed in the context of the constituent equations and related boundary conditions and constraints. A derivation of the shear stress at the mean surface level is given both by using the momentum balance and energy balance equations, which is shown to lead to the same result, if the effects of a changing roller are incorporated correctly (see Appendix). Finally, matching solutions for the shoaling and breaking wave cases between the boundary layer and the middle layer for the shear stresses and the wave averaged flow are derived.

We conclude by noting that the foregoing considerations concern the 2-D wave-induced water motion in a vertical plane parallel to the direction of wave propagation. The currents are weak and boundary-layer processes are primarily wave-induced. The wave orbital motion and the mean current higher up in the vertical can be treated separately, the former as inviscid, the latter as viscous flow. Advection plays no significant role in the mean current field.

Although this situation must occur now and then in nature, it rather concerns a wave flume. The common situation on a natural beach is more complicated, because

- the wave field is directionally spread, so whatever definition is chosen for "the" direction of wave propagation, there are always wave components in other directions;
- the waves are not monochromatic, there is always energy in the low-frequency bands (edge waves, surfbeat, shear waves, etc.);
- there can be strong currents, tidal, wave-driven or otherwise, and not necessarily in the alongshore direction; in general, these currents make a non-zero angle with the direction of wave propagation;
- strong spatial gradients of the bed topography and wave or current parameters can occur in any direction.

As a consequence, one would have to abandon a number of simplifying assumptions which underlie the present 2-D vertical (2-DV) model. One of these for instance is that in 3-D situations the mass flux in the NSL is not necessarily compensated in the lower parts of the same water column (cf. De Vriend and Kitou, 1990).

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APPENDIX

MEAN SURFACE SHEAR STRESS DUE TO A CHANGING ROLLER

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The apparent inconsistency, mentioned in Section 4 of the above paper, is caused by the complicated situation that arises when the volume of the roller is changing in the propagation direction. In this case the formulation of the energy dissipation and of the surface shear stress will have to be modified. In the following, the notation of the paper is used.

Due to the existence of a shear layer between the roller and the wave a shear stress (averaged over a wave length) τ_t exists which is assumed to apply at the mean water level. In addition there is a net transfer of water from the wave to the roller of:

$$\frac{1}{L}\frac{dR}{dt} = \frac{c}{L}\frac{dR}{dx} = \frac{1}{T}\frac{dR}{dx} \tag{A.1}$$

We are interested in the horizontal momentum transferred from the roller to the wave. If the roller is losing water ($dR/dx < 0$) this momentum is not just τ_t, but also the momentum of the water leaving the roller, which has a horizontal velocity of c. If the roller gains water ($dR/dx > 0$), the water leaving the wave has a negligible horizontal velocity, and it does not change the momentum of the water remaining in the wave.

If there is a net transfer of water to or from the roller this water will be mixed with the water in the roller ($dR/dx > 0$) or in the wave ($dR/dx < 0$). In both situations there is an additional energy dissipation, just as energy is lost when a rain drop hits the wind screen of a fast car, or if the car loses a drop of water on the road. The additional energy dissipation is:

$$\rho \frac{1}{2} c^2 \left|\frac{dR}{dt}\right| \tag{A.2}$$

The force between the roller and wave is split into a (symmetric) shear stress and a contribution from the net transfer of water makes the analysis complex. In Deigaard (1993) the shear stress in the shear layer was modelled as a Reynolds' stress with an exchange of water between the roller and the wave. A variation in the roller volume is then represented by a difference between the rate of fluid moving up and down.

With the modified energy dissipation and momentum transfer to the wave the analysis can proceed for the two cases, a decreasing roller and a growing roller:

Case 1: $\frac{dR}{dx} < 0$

Dissipation rate:

$$D = \tau_t c - \rho \frac{1}{2} \frac{c^2}{L} \frac{dR}{dt} = \tau_t c - \rho \frac{1}{2} c^2 \frac{c}{L} \frac{dR}{dx} = \tau_t c - \frac{\rho}{T} \frac{c^2}{2} \frac{dR}{dx} \tag{A.3}$$

Energy dissipation expressed as a gradient in the energy flux:

$$D = - \frac{dE_f}{dx} - \frac{\rho}{T} \frac{d(\frac{1}{2}Rc^2)}{(dx)} \tag{A.4}$$

Horizontal momentum transferred to the wave surface (averaged over a wave length):

$$\tau_s = \tau_t - \rho \frac{c}{L} \frac{dR}{dt} = \tau_t - \rho \frac{c}{L} c \frac{dR}{dx} = .$$

$$\tau_t - \rho \frac{c}{T} \frac{dR}{dx} = \left(\frac{D}{c} + \frac{\rho}{T} \frac{c}{2} \frac{dR}{dx} \right) - \rho \frac{c}{T} \frac{dR}{dx} =$$

$$- \frac{1}{c} \frac{dE_f}{dx} - \frac{\rho}{2Tc} \frac{d(Rc^2)}{dx} - \frac{\rho}{2T} \frac{cdR}{dx} = \tag{A.5}$$

$$- \frac{1}{c} \frac{dE_f}{dx} - \frac{\rho}{2T} \frac{dRc}{dx} - \frac{\rho}{2T} \frac{Rdc}{dx} - \frac{\rho}{2T} \frac{cdR}{dx} =$$

$$- \frac{1}{c} \frac{dE_f}{dx} - \frac{\rho}{T} \frac{dRc}{dx} = - \frac{1}{c} \frac{dE_f}{dx} - \frac{d(2E_r)}{dx}$$

Case 2: $\frac{dR}{dx} > 0$

Dissipation rate:

$$D = \tau_t c + \rho \frac{1}{2} \frac{c^2}{L} \frac{dR}{dt} = \tau_t c + \frac{\rho}{T} \frac{c^2}{2} \frac{dR}{dx} \tag{A.6}$$

Horizontal momentum transferred to the wave surface:

$$\tau_s = \tau_t = \frac{1}{c}D - \frac{\rho}{T}\frac{c}{2}\frac{dR}{dx} =$$

$$-\frac{1}{c}\frac{dE_f}{dx} - \frac{\rho}{2Tc}\frac{d(Rc^2)}{dx} - \frac{\rho}{2T}\frac{cdR}{dx} = \tag{A.7}$$

$$-\frac{1}{c}\frac{dE_f}{dx} - \frac{\rho}{T}\frac{dRc}{dx} \approx -\frac{1}{c}\frac{dE_f}{dx} - \frac{d(2E_r)}{dx}$$

As appears from the above derivations, an additional energy dissipation occurs due to a growing as well as a decreasing roller volume which causes additional work done, and consequently increases the shear stress, expressed by τ_s, which we have to apply at the mean water level. This removes the apparent inconsistency as noted in Section 4.