CHAPTER 30

TIME DOMAIN MODELLING OF WAVE BREAKING, RUNUP, AND SURF BEATS

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Abstract

In this paper we study wave breaking and runup of regular and irregular waves, and the generation of surf beats. These phenomena are investigated numerically by using a time-domain primary-wave resolving model based on Boussinesq type equations. As compared with the classical Boussinesq equations the ones adopted here allow for improved linear dispersion characteristics, and wave breaking is incorporated by using a roller concept for spilling breakers. The swash zone is represented by incorporating a moving shoreline boundary condition and radiation of short and long period waves from the offshore boundary is allowed by the use of absorbing sponge layers. The model results presented include wave height decay, mean water setup, depth-averaged undertow, shoreline oscillations and the generation and release of low frequency waves.

Introduction

Previous work on the modelling of surf beats and low frequency waves in the surf zone has mainly been based on the wave-averaged approach for the primary waves combined with linear or nonlinear equations for the long waves (Symonds et al., 1982; Schäffer, 1993; Roelvink, 1993).

In the analytical work of Symonds et al. (1982) a sinusoidal variation of the break point was assumed corresponding to weakly modulated short waves. Inside the surf zone the modulation or groupiness of these waves was assumed to vanish and the wave height was taken to be a fixed proportion of the local water depth. The low frequency waves were described by the linear shallow water equations driven by radiation stresses according to linear wave theory. Incoming bound long waves and frictional effects were neglected.

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Schäffer (1990, 1993) improved this analytical work in a number of ways of which the inclusion of the incoming bound long waves and their transformation on a sloping beach turned out to be most important for the results. He also considered different types of surf zone models for the incident modulated short waves: The first one was the saturation approach by Symonds et al. (1982), neglecting groupiness inside the surf zone. The second one assumed a fixed break point position but allowed for full transmission of groupiness into the surf zone. The model finally adopted was a hybrid between the two allowing the transmission of groupiness to be zero, partial, full or even to be reversed.

Roelvink (1993) developed a numerical model using a nonlinear description of the low frequency waves based on the nonlinear shallow water equations including bottom friction and radiation stress terms. The primary waves were described by a wave energy balance equation including dissipation terms similar to the formulation by Battjes and Janssen (1978).

As an alternative to the wave-averaged approaches Watson and Peregrine (1992) used the nonlinear shallow water (NSW) equations to resolve the short wave motion as well as the long wave motion. A shock-capturing method allowed for an automatic treatment of bores and shocks without the need for any special tracking algorithm. The drawback of this method is, however, the lack of frequency dispersion, which forces the high frequency waves to move with shallow water celerities and to steepen into bores at a certain distance from the seaward boundary. Another problem is the prediction of the fluctuating surf zone width. It is well known that the NSW equations are not able to predict the break point position in regular waves. Hence it is unlikely that they should be able to give a good estimate of the time varying break point in wave groups and irregular waves.

In the present work we have studied and modelled the nonlinear interaction processes described above on the basis of a special type of Boussinesq equations. The equations were derived by Madsen et al. (1991) and Madsen and Sørensen (1992) and have proved to incorporate improved linear dispersion characteristics and shoaling properties, which are important for a correct representation of the nonlinear energy transfer (Madsen and Sørensen, 1993). Incorporation of wave breaking is an essential part of the model complex and it is based on the concept of surface rollers following the formulation by Schäffer et al. (1992,1993). The model has proved to be able to represent a variety of processes such as the initiation and cessation of spilling wave breaking, and the evolution of wave profiles before, during and after wave breaking.

The present paper presents a further extension of the Boussinesq model by including the swash zone and the moving shoreline. The new extension of the model makes it possible to simulate wave breaking and runup of irregular waves and to study the generation of surf beats and low frequency waves induced by short-wave groups.

Only 1D results will be presented in this paper, while the extention to 2D horizontal problems is described by Sørensen et al. (1994).

A Simple Model for Spilling Breakers

The incorporation of wave breaking for spilling breakers in the Boussinesq model is based on the concept of surface rollers as described by Schäffer et al. (1992,1993). A brief summary of the concept will be given in the following. First of all it can be split up into two parts:

- Determination of the spatial and temporal variation of surface rollers.
- Determination of the effect of the rollers on the wave motion.

The determination of the rollers is based on the heuristic geometrical approach by Deigaard (1989). First of all wave breaking is initiated when the local slope of the surface elevation exceeds a critical value, $\tan\phi$. Due to the transition from initial breaking to a bore-like stage in the inner surf zone this angle, ϕ, is assumed to vary in time. Breaking is assumed to be initiated for an angle of 20 Degrees, which then gradually changes (with an exponential decay) to a smaller terminal angle of 10 Degrees. Locally, the roller is defined as the water above the tangent of $\tan\phi$ and wave breaking is assumed to cease when the maximum of the local slope becomes less than $\tan\phi$.

The determination of the effect of the surface rollers on the wave motion is inspired by the simple model suggested by Svendsen (1984). The basic principle is that the surface roller is considered as a volume of water being carried by the wave with the wave celerity and this is assumed to result in the vertical distribution of the horizontal particle velocity shown in Fig 1. Although this is a very simple approximation it allows for the description of some important physical phenomena:

- It leads to additional convective terms in the depth-integrated momentum equations. This results in a conversion of potential energy into forward momentum flux immediately after breaking and while the wave height starts to decay the radiation stress may keep constant for a while. This leads to the well known horizontal shift between the break point and the point where the setup in the mean water level is initiated.
- The inclusion in the mass balance of the net mass transport due to the roller has a significant effect on the time-average of the depth-averaged particle velocities and improves the prediction of the depth-averaged undertow in the surf zone.

These two features are illustrated in the following by Fig 2, dealing with wave height decay and setup, and by Fig 3 dealing with wave height decay and undertow.

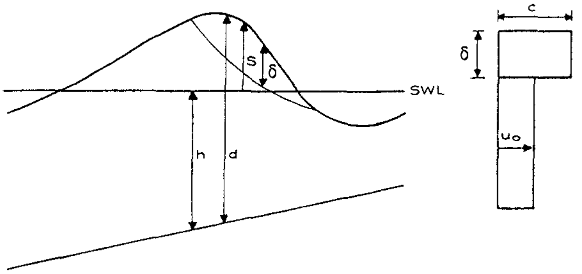
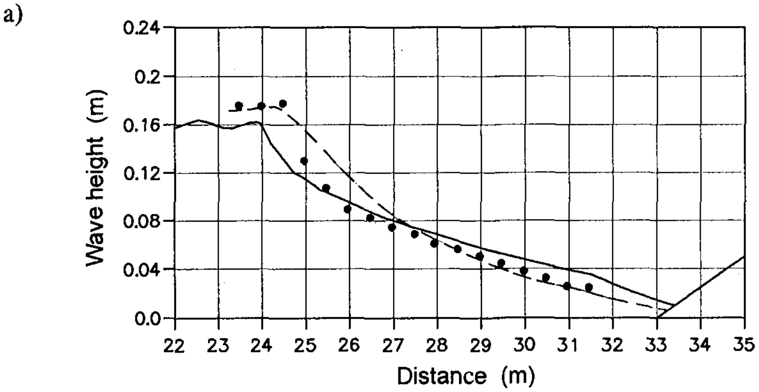


Figure 1 Cross-section and assumed velocity profile of a breaking wave with a surface roller.
Definitions: S=elevation, d=total water depth, δ=roller thickness, c=wave celerity, Uo=particle velocity.

Fig 2 shows a comparison with the measurements of test 1 presented by Stive (1980). In this test incident monochromatic waves shoal and break as spilling breakers on a plane slope of 1:40. The wave period is 1.79 s and at the seaward boundary the depth is 0.70 m and the wave height is 0.145 m. From the variation in wave heights wave breaking is seen to occur at a distance of 24.5 m from the wavemaker, while the setup in mean water level starts at 25.5 m. The numerical results obtained by the present Boussinesq model are slightly underestimating the last part of the shoaling, breaking occurs at 24.0 m and setup starts at 24.7 m. Except for the discrepancy near the break point the overall agreement with the measurements of wave heigth and setup is acceptable.



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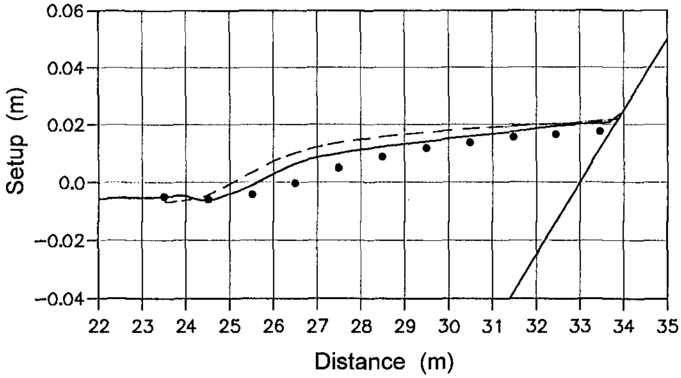

Figure 2 Monochromatic waves on a mild slope
a) Computed and measured variation of the wave height
b) Computed and measured variation of MWL
—— Present model ---- Kobayashi et al. (1989)
• Measurements by Stive (1980)

Kobayashi et al. (1989) solved the nonlinear shallow water NSW equations by using a dissipative shock-capturing method in which bores or shock fronts are frozen to cover only a few grid points. Due to the lack of dispersion this type of model will predict breaking to occur very near the seaward boundary. For this reason the NSW calculations were started very near the observed breaking point at a depth of 0.2375 m with an incoming wave heigth of 0.172 m. From Fig 2 we notice that for the computed results the positions of the break point and the start of setup coincide. Furthermore, the wave height decay is clearly underestimated initially. However, the overall performance of the NSW model is very good.

Fig 3 shows a comparison with the measurements by Hansen and Svendsen (1984). In this test incident monochromatic waves shoal and break as spilling breakers on a plane slope of 1:34.25. The wave period is 2.0 s and at the seaward boundary the depth is 0.36 m and the wave height is 0.12 m. The Boussinesq model was started at the toe of the slope with a cnoidal input. The variation of the wave height is shown in Fig 3a. We notice that the position of the break point is quite well predicted, and also the rate of energy decay after breaking is satisfactory. However, again the maximum wave heights occurring just before breaking are underestimated in the simulation. The depth-averaged undertow is obtained by determining the time-average of the particle velocity Uo, defined in Fig. 1. In Fig. 3b this is compared to the measurements of Hansen and Svendsen (1984). The agreement is seen to be much improved as compared with the NSW results by Kobayashi et al. (1989).

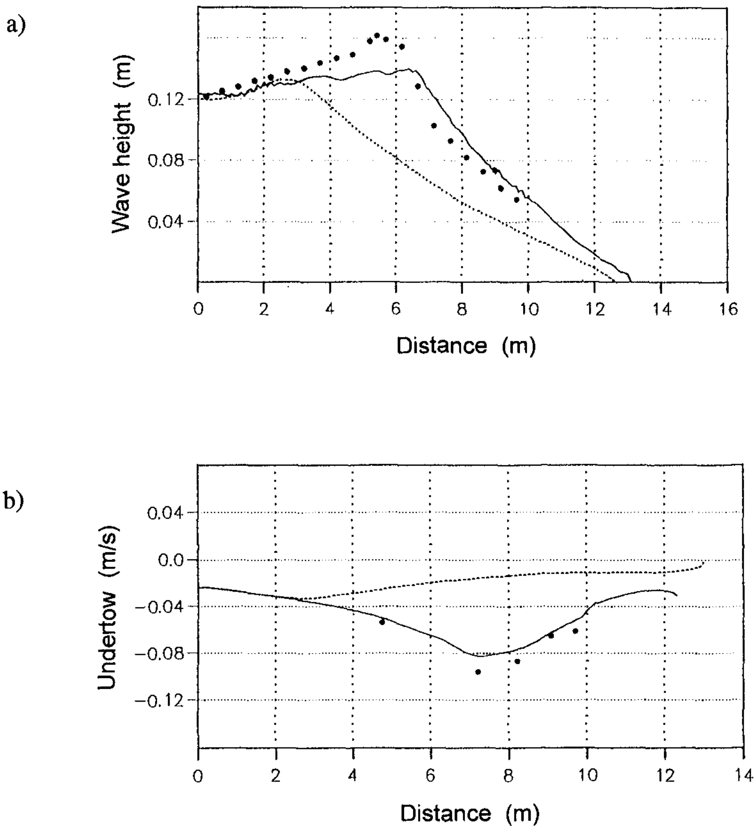

Figure 3 Monochromatic waves on a mild slope
a) Computed and measured variation of the wave height
b) Computed and measured depth-averaged undertow
—— Present model ---- Kobayashi et al. (1989)
• Measurements by Hansen and Svendsen (1984)

For this case measured wave profiles immediately seaward of the observed break point were not available, and consequently the NSW model was started at the toe of the slope. This position being quite far from the break point makes it a very difficult test for the NSW model. Clearly, the NSW model fails to predict the break point and the wave height starts decaying much too early. This generally leads to a significant underestimation of the wave height. Also, the undertow is clearly underestimated, partly due to the discrepancy in the wave height and partly due to the fact that the NSW equations do not include the important net mass transport in the rollers.

Swash Oscillations and Runup

One of the difficult points in simulating runup of regular and irregular waves is the treatment of the moving shoreline. In the present work we have adopted a modified version of the slot-technique described by Tao (1983). A brief summary of this concept will be given in the following.

First of all the computational domain is extended artificially to cover the solid beach by introducing narrow channels or slots in which the waves can propagate. The width of the slot will enter the depth-integrated mass and momentum equations in two ways:

- Generally the water depth will be replaced by a cross sectional area which will include the slot area.
- The time derivative of the surface elevation will be multiplied by the slot width.

The artificial slots have to be very narrow to avoid a distortion of the mass balance and a disturbance of the flow in the physical domain. On the other hand, the numerical solution will break down when the width becomes too small. In practice the width is chosen in the interval between 0.01 and 0.001 times the grid size. An alternative way to interpret this technique is to consider the solid beach to be replaced by a porous beach with a very low porosity factor (the relative slot width).

In order to make this technique operational in connection with Boussinesq type models a couple of problems call for special attention:

- The Boussinesq terms are switched off at the still water shoreline where their relative importance is extremely small anyway.
- The convective terms are treated by central differences in the physical flow domain and by upwind differences inside the slots.
- An explicit filter is introduced near the still water shoreline to remove short wave instabilities during uprush and downrush.

Some of the advantages of the method is that it works very well in combination with implicit numerical schemes and it is quite easy to generalize and implement in two horizontal dimensions. The drawback is that the method will always introduce minor errors in the mass balance and these will generally lead to an underestimation of the maximum uprush and downrush on impermeable slopes.

To check the accuracy of the method we have tested the numerical model against the analytical solution for non-breaking shallow water waves on a sloping beach (Carrier and Greenspan, 1958). The analytical solution is based on the nonlinear shallow water equations, hence for this test case we have switched off the Boussinesq terms everywhere.

The test case considered is an initial water depth of 0.5 m, a wave period of 10 s, a slope of 1/25, and a wave height of 50% of the limiting value (giving breaking at rundown). Fig 4 shows the horizontal motion of the shoreline computed with three different values of the relative slot width (0.01, 0.005 & 0.001). The numerical solution is clearly converging towards the analytical solution, but even for a value of 0.001 the maximum runup is still underestimated by 8%. The reason is that the upper part of the swash zone is containing only a thin film of water and this is quite sensitive to even small portions of water entering the porous beach.

Further testing and a more complete description of the technique will appear in Madsen et al. (1995).

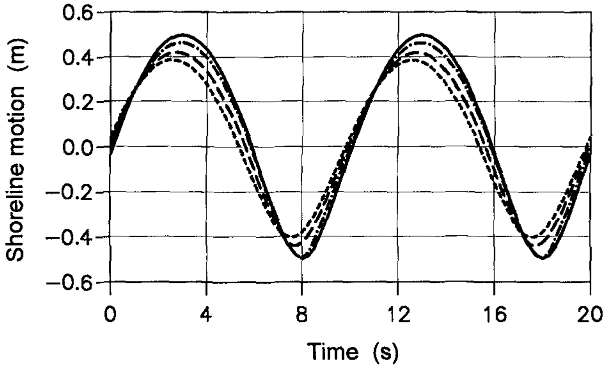

Figure 4 Horizontal motion of the shoreline

——— Analytical solution by Carrier & Greenspan (1958)

-------- Present model with a slot width of 0.01

– – – – -"- slot width of 0.005

—·—·— -"- slot width of 0.001

Low Frequency Waves in the Surf Zone

The extension of the Boussinesq model to include wave breaking of spilling breakers and a moving shoreline formulation makes it possible to simulate the generation of surf beats due to shoaling, breaking and runup of irregular waves. The simplest possible study of this phenomenon can be made with a bichromatic wave group composed of a superposition of two sine waves with slightly different frequencies.

For simplicity we consider a test case where linear boundary conditions can be applied at the seaward boundary. The bathymetry consists of a horizontal section of 40 m with a water depth of 2.0 m and a section of 75 m with a constant slope of 1/34.25 (Fig 5). Waves are generated internally and re-reflection of waves from the seaward boundary is avoided by using a 5 m wide sponge layer. As input we apply linear bichromatic waves with frequencies of 0.4 hz and 0.5 hz and with amplitudes of 0.03 m on both frequencies. This leads to a *linear* wave group in the horizontal section of the flume, and it has several advantages: One is that higher order boundary conditions are not neccessary and the other is that the determination of the amplitude of the outgoing free low frequency wave due to the surf beat can be determined accurately by simple means. This is not the case if the seaward boundary is placed in shallow water. In order to resolve the superharmonics in shallow water we use a grid size of 0.1 m and a time step of 0.02 s.

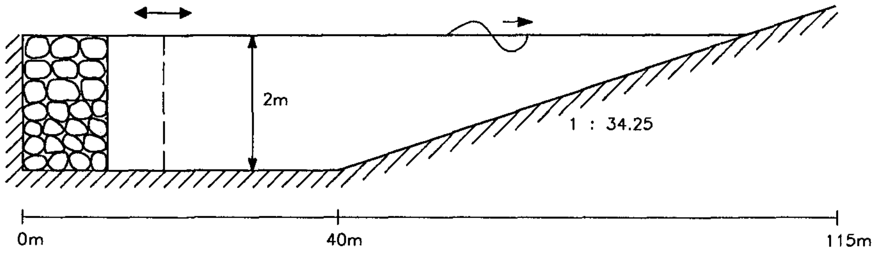

Figure 5 Bichromatic waves on a mild slope. Sketch of model setup.

Fig 6 shows the instantaneous surface elevation as a function of the distance from the seaward boundary. The individual waves can be seen to steepen and break in shallow water. The figure also contains two curves, showing the maximum and minimum elevation obtained during the last two wave groups of the simulation. The maximum curve has three spikes indicating three different break points. The outermost point is seen to be at 101 m and the envelope drops steeply landwards of this point due to the roller dissipation. The second break point occurs at 102.5 m and a new steep descent follows. This pattern indicates that the highest waves decay to become smaller than the second highest waves before the latter start decaying. In other words, the modulation of the primary waves or the groupiness is reversed inside the surf zone. This phenomenon was also discussed by Schäffer (1993).

Fig 7 shows the resulting surf beat obtained by low-pass filtering the surface elevation time series in each grid point with a cutoff at 0.1 hz. The envelope plot shows a combination of bound and free long waves running in the onshore direction and reflected free waves running in the offshore direction. Furthermore, the stationary setup is included in the plot. An almost perfect nodal point is occurring at 94 m, but further offshore the nodes become more and more open indicating that the outward free long waves become dominant compared to the inward bound long waves. The overall picture for the long waves shows the same behaviour as the analytical solution by Schäffer (1993) for weakly modulated primary waves.

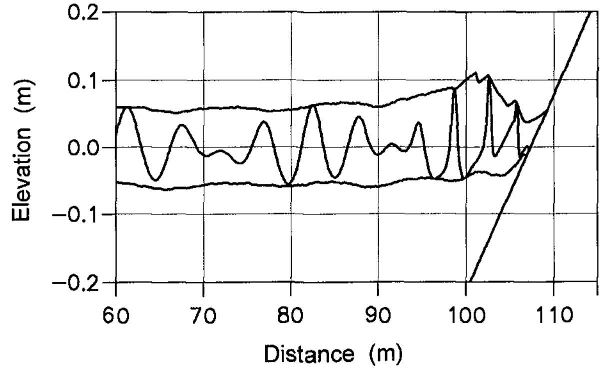

Figure 6 Bichromatic waves on a mild slope. Spatial variation of the surface elevation.

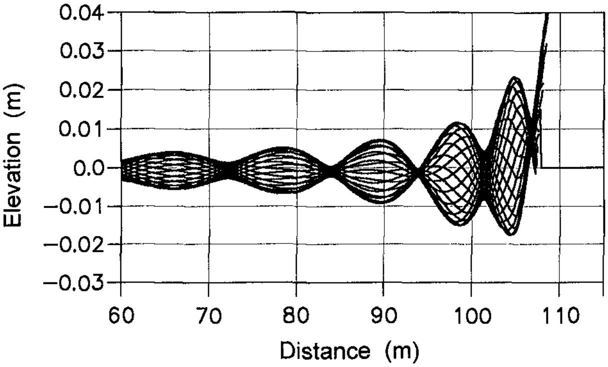

Figure 7 Bichromatic waves on a mild slope. Envelope of lowpass filtered surface elevations.

Fig 8 shows the temporal variation of the shoreline and the spatial and temporal evolution of the individual rollers identified by the model. The outermost break point is again spotted at 101 m and we clearly see how the break point changes in time. The slope of the trajectories of the rollers indicate the local speed of the breaking waves and especially close to the shoreline this is clearly influenced by the swash oscillations. The temporal variation of the shoreline shows a low frequency variation superimposed by individual swash oscillations. Unfortunately, the use of the slot-technique for handling the moving boundary has the undesired effect, that the individual wave runups disappear too rapidly due to the thin film of water penetrating the porous beach. In reality we expect the shoreline motion to be less spiky because the thin film of water will stay on the slope until the next wave arrives. This aspect requires further investigation.

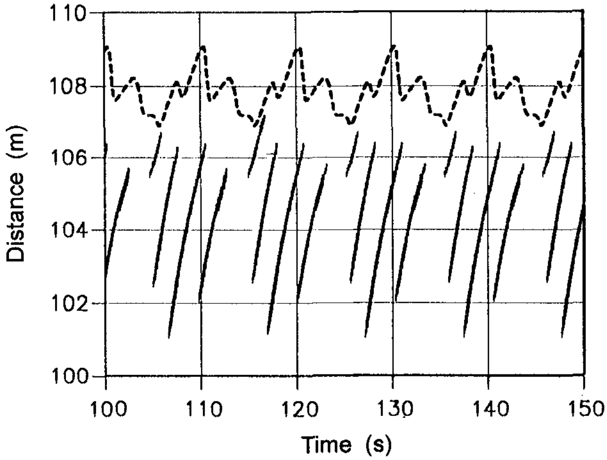

Figure 8 A time-space plot of the horizontal motion of the shoreline (---) and the tracks of detected surface rollers (—).

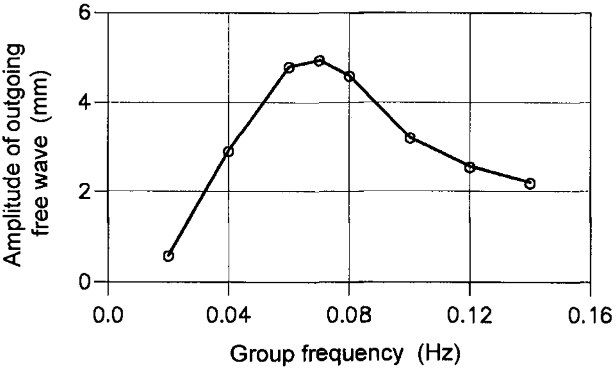

Figure 9 The amplitude of the outgoing free long wave as a function of the group frequency, Δf.

To investigate the sensitivity of the surf beat to the variation of the group frequency a number of simulations were made with different fully modulated bichromatic wave inputs: The first frequency was fixed to be 0.5 hz while the second was varied in the range of 0.36 hz - 0.48 hz, leading to group frequencies in the interval of 0.14 hz to 0.02 hz. Input amplitudes of 0.03 m on both frequencies were used in all cases. Fig 9 shows the amplitude of the resulting outgoing free long wave as a function of the group frequency, Δf. A clear local maximum is obtained for a value of 0.07 hz indicating that the surf beat mechanism is indeed sensitive to the ratio between the surf zone width and the wave length of the low frequency waves. The local maximum in the resulting amplitude occurs when the long waves reflected from the shoreline are in phase with the long waves generated to move in the offshore direction by the oscillating break point. This confirms the mechanism first described by Symonds et al. (1982).

Further investigations and comparisons with e.g. the measurements by Kostense (1984) will appear in Madsen et al. (1995).

Conclusion

A Boussinesq type model with improved linear dispersion characteristics and with a roller concept for spilling breakers is extended to include the swash zone and the moving shoreline. The model is capable of representing a variety of processes such as the initiation and cessation of breaking, and the evolution of wave profiles before, during and after wave breaking. In this work we have concentrated on resulting wave-averaged quantities such as wave height decay, mean water setup and depth-averaged undertow.

The new extension of the model to include the swash zone makes it possible to simulate wave breaking and runup of irregular waves and to study the generation of surf beats and low frequency waves induced by short-wave groups. The test cases presented is this paper are only first examples of what can be achieved by the use of the Boussinesq model. The results obtained so far are qualitatively correct and most promising. However, further work is necessary to quantify the accuracy of the results.

Acknowledgement

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