CHAPTER 15

FALSE WAVES IN WAVE RECORDS AND NUMERICAL SIMULATIONS

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ABSTRACT

It is common practice to consider the random waves as a succession of discrete waves characterized by individual amplitudes and periods. The zero-up-crossing criterion isolates some discrete waves that are not physical waves. The orbital criterion avoids these "false waves". As a result, the orbital criterion proves to be more consistent and robust, and to have a less variability. The selection of the discretization criterion results in some significant differences in the wave statistics, which are analyzed. As an example, while the mean period for the zero-up-crossing criterion is T_{02}, the mean period for the orbital criterion is T_{01}.

INTRODUCTION

Regular waves can be characterized by amplitude and period, and random waves may be described by the energy spectrum. However, it is common practice to consider the random waves as a succession of "discrete waves" characterized by individual amplitude and period.

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Unfortunately, a variety of reasonable criteria for discretizing waves have been proposed by different authors. In fact, any method used to define a discrete wave in regular waves could be extended to the case of random waves.

A number of papers are related to wave statistics and may be affected by the wave discretization procedure. Moreover, a variety of subjective criteria are used for neglecting small waves in the analysis.

Giménez et al. (1994) have proposed an orbital criterion for discretizing waves. Using numerical simulations, the authors have proved that this method is more consistent and robust than the zero-up-crossing criterion. These results are in good agreement with the observations given by Pires-Silva and Medina (1994) analyzing wave records off the coast of Portugal.

This paper describes first the most common wave discretization methods, and summarizes the concepts and properties of "orbital wave" and "false wave". The advantages of the orbital criterion are presented, including consistency, robustness and a less variability. Finally, the influence of the wave discretization criteria on the wave statistics is analyzed using numerical simulations.

WAVE DISCRETIZATION CRITERIA

The more commonly used wave discretization criteria are the following:

* The ZUC criterion

In the zero-up-crossing criterion, a discrete wave is limited by two consecutive up-crossings of the mean level. Following Rice (1954), Longuet-Higgins (1958) showed that for linear random waves the mean period using the ZUC criterion is T_{02}, where T_{ij} is given by:

$$T_{ij} = \sqrt[j-i]{\frac{m_i}{m_j}} \quad (1)$$

where m_n is the *n*th moment of the energy spectrum S(f),

$$m_n = \int_0^\infty f^n S(f) df \quad (2)$$

* The ZDC criterion

In the zero-down-crossing criterion, a discrete wave is limited by two consecutive down-crossings of the mean level. For linear random waves, the ZUC and the ZDC criteria are statistically equivalent. Therefore, the mean period using the ZDC criterion is also T_{02}.

* The crest-to-crest criterion

In the crest-to-crest criterion, a discrete wave is limited by two consecutive maxima of the surface displacement function. From Rice (1954), it can be proved that the mean period using the crest-to-crest criterion is T_{24}.

THE ORBITAL CRITERION

For linear waves, the free surface elevation in a fixed point, $\eta(t)$, can be modeled by:

$$\eta(t)=\sum_{i=1}^{M} c_i \cos(2\pi f_i t+\varphi_i) \tag{3}$$

where the frequencies f_i are $i\Delta f$, the phases φ_i are random variables distributed uniformly over the interval $[0,2\pi[$, and the amplitudes c_i are such that over any frequency interval $[f_i,f_i+\Delta f[$ is:

$$\frac{1}{2}c_i^2=\int_{f_i}^{f_i+\Delta f} S(f)\,df \tag{4}$$

The Hilbert transform of $\eta(t)$ is:

$$\hat{\eta}(t)=\sum_{i=1}^{M} c_i \sin(2\pi f_i t+\varphi_i) \tag{5}$$

The functions $\eta(t)$ and $\hat{\eta}(t)$ can be taken as the real and the imaginary part, respectively, of the analytical function AF(t):

$$AF(t)=\sum_{i=1}^{M} c_i \exp[j(2\pi f_i t+\varphi_i)]=\eta(t)+j\hat{\eta}(t) \tag{6}$$

where $j=\sqrt{-1}$ is the imaginary unit. AF(t) can be expressed in the form:

$$\left.\begin{aligned} &AF(t)=A(t)\exp[j\theta(t)]\\ &A(t)=\sqrt{\eta^2(t)+\hat{\eta}^2(t)}\\ &\theta(t)=\arctan\frac{\hat{\eta}(t)}{\eta(t)} \end{aligned}\right\} \tag{7}$$

where A(t) is the wave envelope and $\theta(t)$ is the phase angle. As noted by Medina and Hudspeth (1987), AF(t) represents the orbital movement of a point floating on the sea surface.

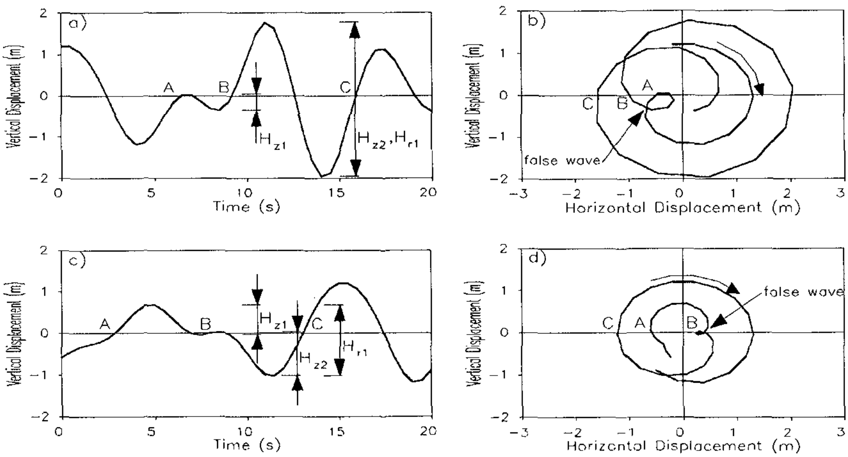

Figure 1.- ZUC criterion vs. orbital criterion: (a) example of time series; (b) orbital analysis of a); (c) example of time series; (d) orbital analysis of c).

According to Giménez et al. (1994), the orbital criterion defines a discrete wave as corresponding to a 2π advance of the phase angle in the complex plane. Figures 1(a) and 1(c) show two pieces of numerical simulation from a JONSWAP spectrum. Figures 1(b) and 1(d) represent the corresponding analytical functions AF(t). The ZUC and the orbital waves are denoted by H_z and H_r respectively. In both pieces two ZUC waves (AB and BC), but only one orbital wave (AC), are present. The first small wave in Figure 1(a) is not considered by the orbital criterion. Furthermore, the two ZUC waves in Figure 1(c) are only one, and higher, wave in the orbital criterion.

Giménez et al. (1994) define false wave as any discrete wave that does not correspond to a 2π advance in the complex plane. Examples of these false waves are shown in Figures 1(b) and 1(d). In the same reference, the authors prove (both, mathematically and numerically) that the mean period using the orbital criterion is T_{01}. Furthermore, the discrepancy between T_{01} and T_{02}, is completely due to the presence of false waves.

ADVANTAGES OF THE ORBITAL CRITERION

Because of its dependence of m_4, the crest-to-crest criterion is very sensitive to the cut-off frequency. On the other hand, the ZUC and the ZDC are statistically equivalent for linear random waves. Therefore, the ZUC criterion has been used as reference for analyzing the advantages of the orbital criterion. Giménez et al. (1994) have carried out that comparison, and have come to the next conclusions:

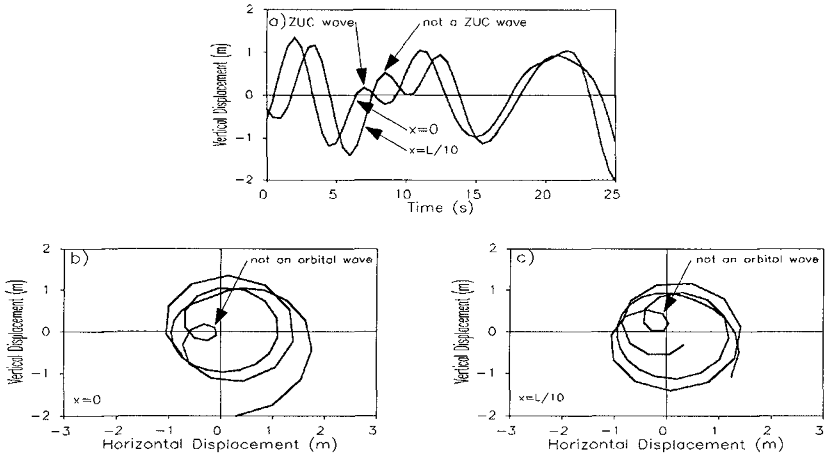

Figure 2.- ZUC criterion vs. orbital criterion: (a) time series of two close points; (b) orbital analysis of the first point; (c) orbital analysis of the second point.

* Consistency

Figure 2(a) shows a numerical simulation of two time series corresponding to two points in the sea surface separated by 10% of the mean wavelength L. There is a perturbation that is a ZUC wave in x=0, but not in x=L/10. This physical inconsistency can be solved using the orbital criterion. Figures 2(b) and 2(c) show that the perturbation is a false wave.

Another problem of the ZUC criterion is the wide variety of subjective thresholds for neglecting small invalid waves (see Rye, 1974; van Vledder, 1983; Thompson and Seeling, 1984; Mansard and Funke, 1984; Mase and Iwagaki, 1986). For the orbital criterion, any wave that does not correspond to a 2π advance of the phase angle is not an actual wave. No additional thresholds are required.

* Robustness

Figure 3(a) shows a piece of simulation, and the same record when a 5% of white noise is added. Additional ZUC waves appear due to the presence of noise. However, Figure 3(b) shows that these waves are not orbital waves. In fact, most of the additional ZUC waves due to noise are false waves. Therefore, the ZUC criterion is more sensitive to noise than the orbital criterion.

Giménez et al. (1994) have analyzed the sensitivity to noise using numerical simulations. The mean period of orbital waves is underestimated about 2% when a 2% of white noise is added. On the contrary, the resulting error in the mean period of ZUC waves is about 10%. The underestimations with a 5% of white noise are about 5% for orbital waves, and 20-25% for ZUC waves.

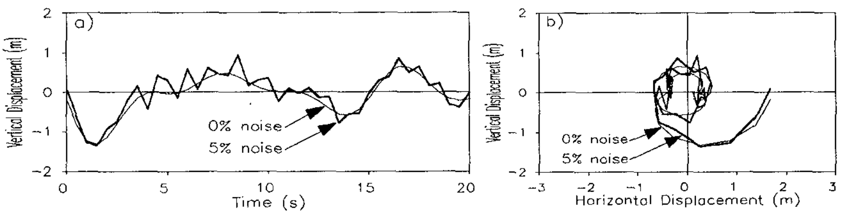

Figure 3.- Influence of white noise: (a) time series; (b) orbital analysis.

* <u>Less variability</u>

As proved in Appendix A, the variabilities of the mean periods T_{01} and T_{02} are:

$$CV^2\left[T_{01}\right]=\frac{\int_0^\infty S^2(f)\left[T_{01}f-1\right]^2 df}{m_0^2 T_R} \tag{8}$$

$$CV^2\left[T_{02}\right]=\frac{\int_0^\infty S^2(f)\left[\left(T_{02}f\right)^2-1\right]^2 df}{4 m_0^2 T_R} \tag{9}$$

where CV[.] is the coefficient of variation and T_R is the length of the record. The result depends on the spectral shape. Table 1 shows the variability for JONSWAP type spectra (see Goda, 1985) with a peak frequency of 0.1 Hz and different values of the peak enhancement parameter γ. The mean period T_{01} proves to have a less variability than T_{02}.

γ	$CV[T_{01}]$	$CV[T_{02}]$
1	$0.677/\sqrt{T_R}$	$0.695/\sqrt{T_R}$
2	$0.686/\sqrt{T_R}$	$0.744/\sqrt{T_R}$
3.3	$0.678/\sqrt{T_R}$	$0.771/\sqrt{T_R}$
5	$0.655/\sqrt{T_R}$	$0.776/\sqrt{T_R}$
7	$0.623/\sqrt{T_R}$	$0.762/\sqrt{T_R}$
10	$0.577/\sqrt{T_R}$	$0.728/\sqrt{T_R}$

Table 1.- Variability of T_{01} and T_{02}.

DISTRIBUTIONS OF WAVE HEIGHTS AND PERIODS

The mean period is not the only statistical parameter that is affected by the selected wave discretization criterion. The parameters related with discrete waves are altered in two ways. First of all, the presence of false waves varies significantly the number and characteristics of small waves. As a second effect, the total number of orbital waves in a record is less than the number of ZUC waves. Therefore, the relative number of higher waves is slightly greater in the orbital criterion than in the ZUC criterion.

A comparison between the statistics for orbital and ZUC waves has been developed using numerical simulations. These simulations were obtained using a DSA-FFT algorithm (see Tuah and Hudspeth, 1982) and a JONSWAP type spectrum (see Goda, 1985) with N=8192 points and a time interval $\Delta t=0.2$ sec. 1000 simulations were carried out for different values of the peak enhancement parameter ($\gamma=1,2,3.3,5,7,10$). The number of simulated ZUC waves varies from about 180000 (for $\gamma=10$) to 210000 (for $\gamma=1$). The results for all the indicated values of γ have been analyzed, and figures corresponding to the extremal values $\gamma=1$ and $\gamma=10$ are included in this paper.

The methodology developed by Sobey (1992) was used for the statistical analysis of the simulations. Therefore, the wave periods were obtained by linear interpolation in the neighborhood of the zero-up-crossings. The wave heights were determined from quadratic interpolation at the crest and the trough. The wave heights were normalized by H_{rms}, and the wave periods by T_{01}. Finally, the waves were aggregated, according to their normalized height and period, into a joint histogram in 0.025x0.025 dimensionless bins.

Figures 4(a) and 4(b) show the joint distributions of wave heights and periods, for $\gamma=1$, corresponding to the orbital and the ZUC criteria respectively. Figures 5(a) and 5(b) show the same distributions for $\gamma=10$. For both criteria, the distribution is bimodal. The external contour corresponds to points with a probability of 0.01. The other contours correspond to probabilities of 0.20, 0.40, 0.60 and so on. The values in the thicker lines are 0.01, 1.00 and 2.00.

A maximum is located near the point corresponding to H_{rms} and T_{01}. This maximum becomes higher for narrower spectra, and is slightly greater in the orbital criterion. On the other hand, the maximum corresponding to small waves has a less value for orbital waves.

Figures 6(a) and 6(b) show the distributions of wave heights for $\gamma=1$ and $\gamma=10$ respectively. The probability of both the orbital and the ZUC waves is underestimated by the Rayleigh distribution near its mode, and overestimated in the ranges of small and high waves. As expected according to the narrow band assumption, the fitting is slightly better for narrower spectra.

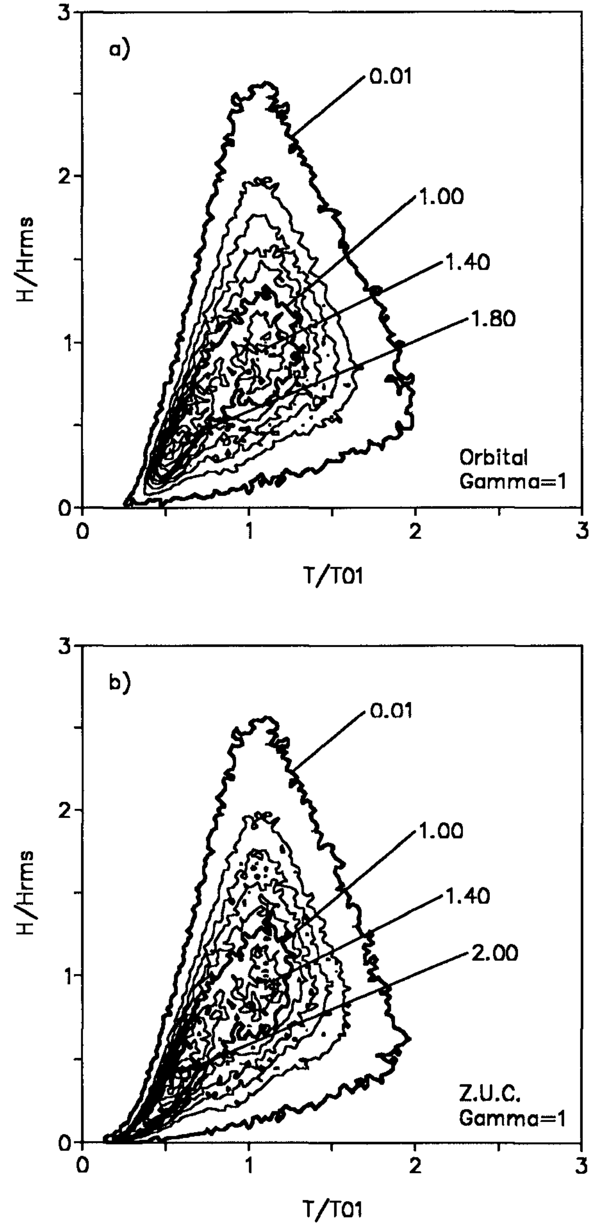

Figure 4.- Joint distributions of wave heights and periods for $\gamma=1$: (a) orbital waves; (b) ZUC waves.

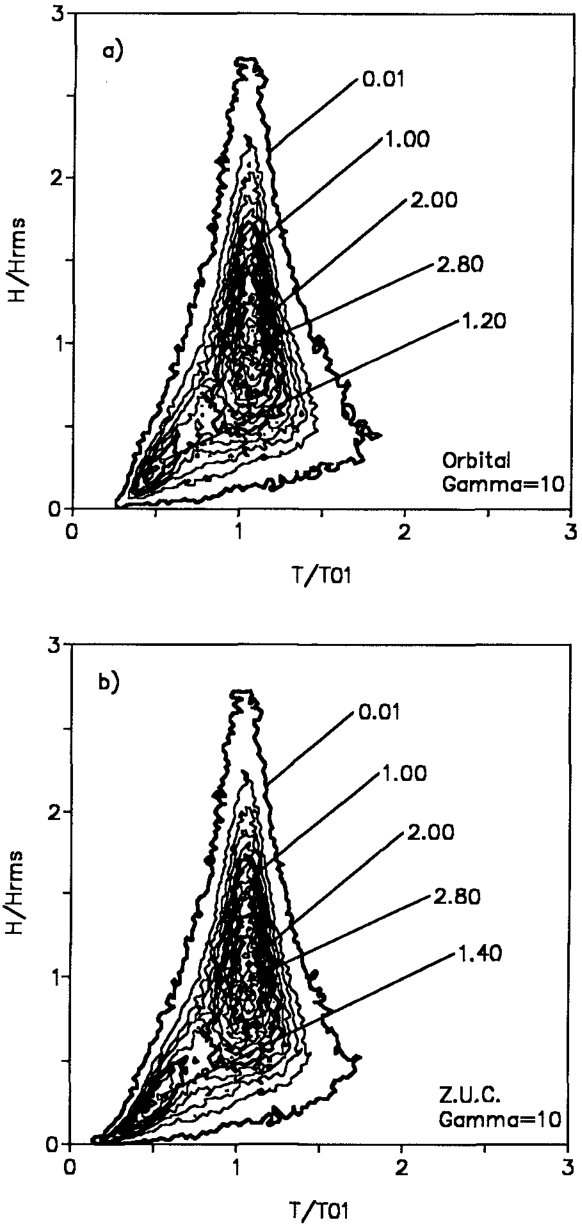

Figure 5.- Joint distributions of wave heights and periods for $\gamma=10$: (a) orbital waves; (b) ZUC waves.

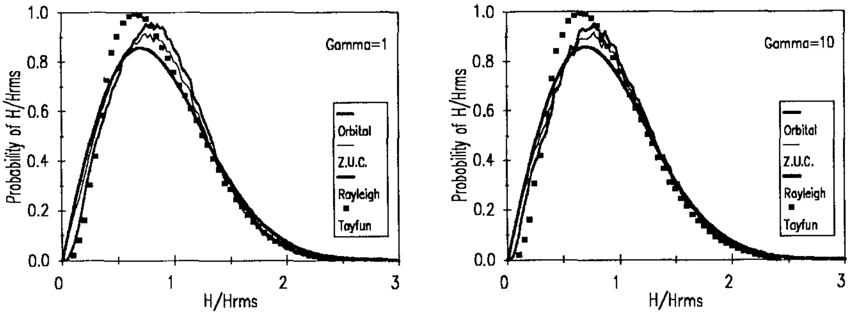

Figure 6.- Distributions of wave heights: (a) $\gamma=1$; (b) $\gamma=10$.

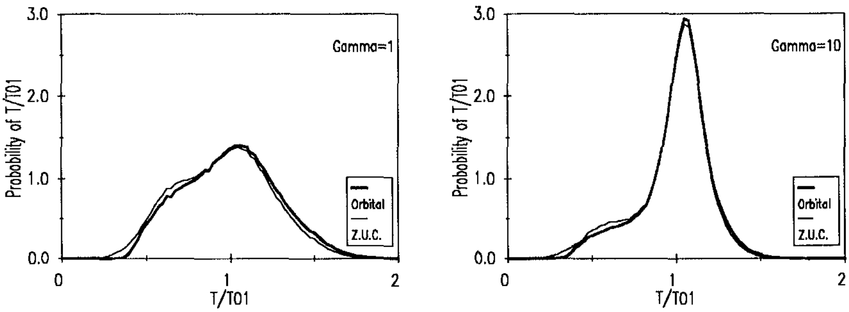

Figure 7.- Distributions of wave periods: (a) $\gamma=1$; (b) $\gamma=10$.

A comparison between the distributions of orbital and ZUC wave heights leads to the next conclusions: in the orbital criterion, the probability is less in the range of small waves, greater around the mode, and nearly the same in the range of hight waves. This behaviour could be expected due to the presence of false waves in the ZUC criterion.

Figures 6(a) and 6(b) also show the distribution of Tayfun (1981) corresponding to $\nu=0.2$. This distribution is based on the values of the envelope with a lag of $T_{01}/2$, and therefore is supposed to be more appropriate for orbital waves than for ZUC waves.

Figures 7(a) and 7(b) show the distributions of wave periods for $\gamma=1$ and $\gamma=10$ respectively. As noted above, the mean period is T_{01} for orbital waves and T_{02} for ZUC waves. It can be concluded from these distributions that the orbital waves have higher periods than the ZUC waves, as expected.

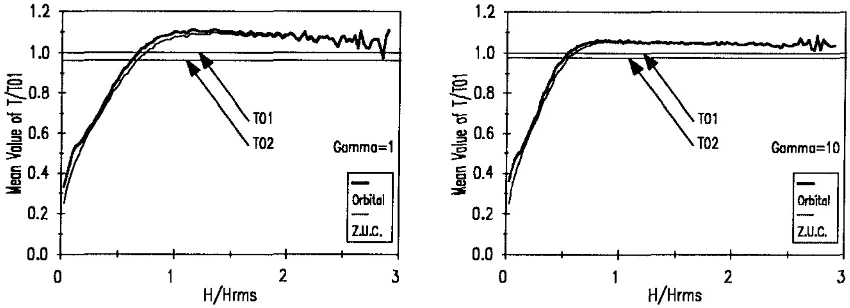

Figure 8.- Variation of the mean wave period as a function of the wave height: (a) $\gamma=1$; (b) $\gamma=10$.

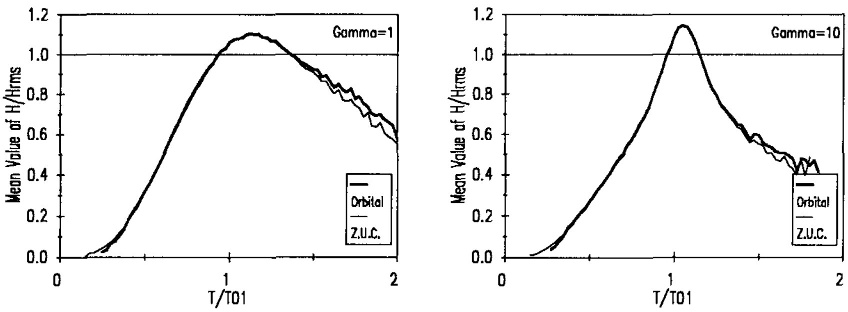

Figure 9.- Variation of the mean wave height as a function of the wave period: (a) $\gamma=1$; (b) $\gamma=10$.

Figures 8(a) and 8(b) show the variation of the mean wave period for $\gamma=1$ and $\gamma=10$, respectively, as a function of the wave height. The mean value of the orbital waves is greater for every wave height, especially in the range of small waves. Waves with $H/H_{rms}>0.5$-0.7 have a mean period over T_{01}, with a certain tendency to this value for higher waves.

Figures 9(a) and 9(b) show the variation of the mean wave height for $\gamma=1$ and $\gamma=10$, respectively, as a function of the wave period. The greatest mean wave heights correspond to periods around T_{01}. The orbital waves have a larger mean wave height than the ZUC waves in the range of high periods.

CONCLUSIONS

The ZUC criterion commonly used to discretize wave records generates discrete waves that do not correspond to physical waves. The orbital criterion avoids these "false waves", and proves to be more consistent and robust, and to have a less variability. Finally, the wave statistics are altered by the selection of the wave discretization criteria. The differences are basically located in the range of small waves, but some slight variations are also found for larger waves.

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APPENDIX A: VARIABILITY OF THE MEAN PERIODS

Random waves, taken as a Gaussian ergodic stochastic process, are described by the energy spectrum S(f). When this spectrum is estimated from records of finite length T_R, the resulting M components are independent random variables. If T_R is large enough, then every component is distributed as a chi-squared law with two degrees of freedom. The mean and variance of this distribution are:

$$\left.\begin{aligned} E[S_{m,R}] &= S_m \\ \sigma^2[S_{m,R}] &= S_m^2 \end{aligned}\right\} \tag{A.1}$$

where $S_m = S(m\Delta f)$ with $\Delta f = 1/T_R$, and $S_{m,R}$ is the corresponding estimation. The *i*th moment of the estimated spectrum is:

$$m_{i,R} = \sum_{m=1}^{M} f_m^i S_{m,R} \Delta f \tag{A.2}$$

From (A.1) and (A.2), it can be proved that the mean of the random variable $m_{i,R}$ is:

$$E[m_{i,R}] = m_i \tag{A.3}$$

and the covariance between the moments $m_{i,R}$ and $m_{j,R}$ is:

$$C[m_{i,R}, m_{j,R}] = \frac{1}{T_R} \int_0^\infty f^{i+j} S^2(f) df \tag{A.4}$$

The square of the coefficient of variation of $m_{i,R}$ is:

$$CV^2[m_{i,R}] = \frac{C[m_{i,R},m_{i,R}]}{E^2[m_{i,R}]} = \frac{1}{m_i^2 T_R}\int_0^\infty f^{2i} S^2(f)df \tag{A.5}$$

The period $T_{ij}=(m_i/m_j)^k$, where $k=1/(j-i)$, cannot be determined when S(f) is unknown. On the other hand, $T_{ij,R}=(m_{i,R}/m_{j,R})^k$ can be obtained from the estimated spectrum. However, $T_{ij,R}$ is a random variable, and presents a certain variability. The objective of this appendix is to obtain an expression for that variability.

A Taylor expansion of $T_{ij,R}$ and an integration lead to:

$$\begin{aligned} E[T_{ij,R}] &= \int_0^\infty\int_0^\infty T_{ij,R}\,p(m_{i,R},m_{j,R})dm_{i,R}dm_{j,R} \simeq \\ &\simeq T_{ij}\left[1+\frac{k(k-1)}{2}CV^2[m_{i,R}]+\frac{k(k+1)}{2}CV^2[m_{j,R}]-k^2\frac{C[m_{i,R},m_{j,R}]}{m_{i,R}m_{j,R}}\right] \end{aligned} \tag{A.6}$$

The same method leads to:

$$\begin{aligned} E[T_{ij,R}^2] &= \int_0^\infty\int_0^\infty T_{ij,R}^2\,p(m_{i,R},m_{j,R})dm_{i,R}dm_{j,R} \simeq \\ &\simeq T_{ij}^2\left[1+k(2k-1)CV^2[m_{i,R}]+k(2k+1)CV^2[m_{j,R}]-4k^2\frac{C[m_{i,R},m_{j,R}]}{m_{i,R}m_{j,R}}\right] \end{aligned} \tag{A.7}$$

Therefore:

$$CV^2[T_{ij,R}] = \frac{E[T_{ij,R}^2]-E^2[T_{ij,R}]}{E^2[T_{ij,R}]} \simeq k^2\left[CV^2[m_{i,R}]+CV^2[m_{j,R}]-2\frac{C[m_{i,R},m_{j,R}]}{m_{i,R}m_{j,R}}\right] \tag{A.8}$$

From (A.4) and (A.5), if T_R is large enough, then $CV[m_{i,R}]$, $CV[m_{j,R}]$ and $C[m_{i,R},m_{j,R}]$ are much smaller than one, and, according to (A.6):

$$E[T_{ij,R}] = T_{ij} \tag{A.9}$$

Therefore, the random variable $T_{ij,R}$ can be used for estimating T_{ij}, and the expression (A.8) gives the variability of the estimation. For T_{02} the result is:

$$CV^2[T_{02,R}] = \frac{1}{4}\left[CV^2[m_{0,R}] + CV^2[m_{2,R}] - 2\frac{C[m_{0,R}, m_{2,R}]}{m_{0,R}m_{2,R}}\right] =$$

$$= \frac{1}{4}\left[\frac{1}{m_0^2 T_R}\int_0^\infty S^2(f)df + \frac{1}{m_2^2 T_R}\int_0^\infty f^4 S^2(f)df - \frac{2}{m_0 m_2 T_R}\int_0^\infty f^2 S^2(f)df\right] = \quad (A.10)$$

$$= \frac{1}{4m_0^2 T_R}\int_0^\infty S^2(f)\left[1 + \frac{m_0^2}{m_2^2}f^4 - 2\frac{m_0}{m_2}f^2\right]df = \frac{1}{4m_0^2 T_R}\int_0^\infty S^2(f)\left[(T_{02}f)^2 - 1\right]^2 df$$

This expression was obtained by Cavanié (1979). On the other hand, the variability of the estimation of T_{01} is:

$$CV^2[T_{01,R}] = CV^2[m_{0,R}] + CV^2[m_{1,R}] - 2\frac{C[m_{0,R}, m_{1,R}]}{m_{0,R}m_{1,R}} =$$

$$= \frac{1}{m_0^2 T_R}\int_0^\infty S^2(f)df + \frac{1}{m_1^2 T_R}\int_0^\infty f^2 S^2(f)df - \frac{2}{m_0 m_1 T_R}\int_0^\infty f S^2(f)df = \quad (A.11)$$

$$= \frac{1}{m_0^2 T_R}\int_0^\infty S^2(f)\left[1 + \frac{m_0^2}{m_1^2}f^2 - 2\frac{m_0}{m_1}f\right]df = \frac{1}{m_0^2 T_R}\int_0^\infty S^2(f)\left[T_{01}f - 1\right]^2 df$$

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