CHAPTER 242

Vortex Train in an Offshore Zone

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Abstract

A row of vortices forms in an offshore zone when two-dimensional regular surface waves run up on a sloping flat bed. It is called the offshore vortex train. The vortices begin to appear near a breaking point. Moving in thc offshore direction, they develop and increase their horizontal length scale through the vortex merging. The vortex train forms due to the shear instability between onshore and offshore drift-currents. After reaching a location of the offshore side, however, they decay rapidly because of the decrease of shear strain rate between the drift-currents. The formation region has been investigated on the basis of visual experiments for three bed slopes. The formation does not depend on the type of wave breaking but is observed when the steepness of deep water waves is smaller than 4.2×10^{-2}. The horizontal length scale of the vortices and the velocity of the vortex movement have also been evaluated empirically.

Introduction

A coastal region has attracted considerable research attention because of the central role which it plays in the budget of sediment, the generation of nearshore currents, the diffusion of pollutant, etc. This region is usually divided into the

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nearshore zone and the offshore zone (e. g. Komar 1976). Typical features of the nearshore zone are breaking waves and turbulent fluid motion. Miller (1976) pointed out the formation of breaker vortices from the behavior of air bubbles entrained due to wave breaking. Since his work, the turbulence structure in this zone has been investigated vigorously through field observations and laboratory experiments. Most of these studies, which have been reviewed in detail by Peregrine (1983) and Battjes (1988), are mainly concerned with coherent structures near the free surface. Matsunaga & Honji (1980) and Matsunaga et. al. (1988) showed that the Stokes layer separates periodically near a breaking point when two-dimensional regular waves climb up a sloping flat bed, and that the separated flow forms a vortical pattern and lifts up a large amount of sediment.

On the other hand, the fluid motion is considered usually to be irrotational in the offshore zone. The Lagrangian mass transport of fluid in waves is one of interesting problems in this zone. Bagnold (1947) carried out the experiments in which two-dimensional, regular waves propagated on a horizontal smooth bed. He found a strong, forward drift-current along the bed, and a weak, backward one induced under the water surface. The difference between Bagnold's observation and the well-known ‘Stokes drift’ created a sensation. Longuet-Higgins (1953) solved it theoretically by taking into account the existence of the Stokes layer. Russell & Osorio (1958) measured the velocities of the forward drift-currents at the outside edge of the Stokes layer for horizontal, up-sloping and down-sloping beds. Their data collapse well on the theoretical curve obtained by Longuet-Higgins. The authors (1988) also carried out laboratory experiments to investigate the drift-currents for the case when two-dimensional regular waves climb up a sloping

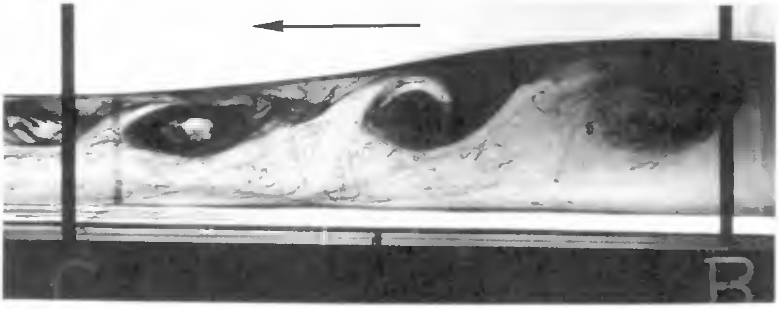

Figure 1. Offshore vortex train.

bed. At that time, they found a row of vortices forming under the waves. As mentioned previously, the fluid motion in the offshore zone has been treated generally as irrotational one. Therefore, this is a fresh phenomenon in the fluid dynamics as well as the coastal engineering. It was called the offshore vortex train. Figure 1 shows one example of the offshore vortices visualized by using dye of aniline blue. The arrow indicates the direction of wave propagation. The interval between B and C was 0.5 m. The mean water depth was 14.0 cm at position B. The bed slope was 1/23.5 and the breaking point was 2.68 m shoreward from position C. The vortices with the clockwise rotation are seen obviously. They move slowly in the offshore direction along the water surface and increase their intervals by the vortex merging. After reaching a location of the offshore side, however, they decay rapidly.

In this paper, universal expressions about the formation region of the offshore vortices, the horizontal length scale of the vortices and the velocity of the vortex movement are given through visual experiments for three bed slopes.

Experimental set-up and procedure

Figure 2 shows schematically an experimental apparatus. The wave tank was 12 m long, 0.4 m deep and 0.15 m wide. It was made of transparent acrylic plates and equipped with a sloping flat bed. The grades of tan θ = 1/37.0, 1/23.5 and 1/12.3 were used as a bed slope. Two-dimensional regular waves were made by oscillating a flap. The wave period T was varied from 0.517 s to 2.59 s. The wavelength in deep water L_0, being calculated from $gT^2/2\pi$, was in the range from 0.417 m to 10.5 m, where g is the gravitational acceleration. The local wave height H was measured by using a capacitance-type wave gauge, and the local wave velocity C by putting a measuring point between two wave gauges. The wave height in deep

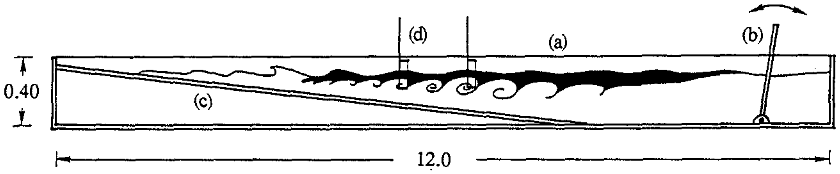

Figure 2. Schematic diagram of experimental set-up (dimensions in m). (a) water tank; (b) wave generator; (c) sloping bed; (d) wave gauges.

water H_0 was evaluated by dividing the shoaling coefficient into H. The value of H_0 ranged from 0.93 cm to 6.58 cm. The local wavelength L was calculated from L = C T.

Drift-currents induced in the offshore side from the breaking point were investigated by means of flow visualization. Dye of water-soluble aniline blue was used as a tracer. Granules of the dye were scattered on the water surface of the offshore zone. Whether the vortex train formed or not was confirmed through visual experiments. The behavior of vortices was videotaped to evaluate their characteristic quantities, i. e., the horizontal length scale of the vortices l and the velocity of vortex movement u. The length scale, which is defined by the horizontal distance from center to center, was determined by sampling some dozen of pictures in which the vortices arrange well over a distance of 0.5 m, and by averaging the read vortex intervals over the number of the samples. About ten sampled pictures were used for the evaluation of each value of u. The value of u was obtained by dividing a given distance by the time elapsed while a vortex moved through the distance, and by averaging the velocities measured in this way over the sampling number.

Universal expression for vortex formation

Figure 3 shows a relation between the formation of the offshore vortex train and the types of breaking waves. The open circles and solid ones indicate its formation and non-formation, respectively. The two solid lines arc border lines between three types of breakers, which are given by Gaughan and Komar (1975). It is seen from this figure that the division of the formation region is given by the dash-dot line, i. e., $H_0/L_0 = 4.2 \times 10^{-2}$, and that it is independent of the breaker typcs.

The formation conditions for the offshore vortices can be determined by using the variables of h, L, H, T, C, g and tanθ, where h is the mean water depth. Two variables can be eliminated by considering into account the dispersion relation of water waves and the relation of C = L/T. If C and g are chosen as the two variables and the dimensional consideration is performed, a non-dimensional expression for the formation region is given by

$$f\ (\ H/L,\ h/L,\ \tan\theta\ ). \qquad (1)$$

In figures 4(a) to (c), the regions of the vortex formation and

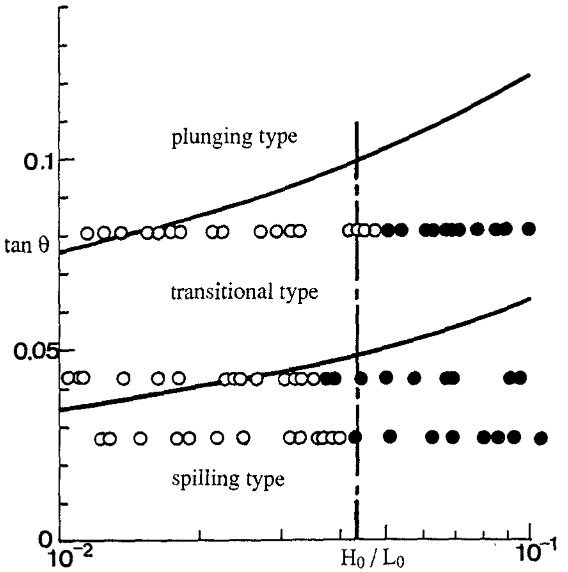

Figure 3. Relation between the formation of offshore vortex train and the types of breaking waves.

non-formation are indicated by the open circles and solid ones, respectively. The solid curves mean the limiting wave steepness. These figures make it clear that the offshore vortices form in a shallow region offshoreward from the breaking point in the case when the wave steepness is relatively small.

Let us try to represent the three formation regions shown in figures 4(a) to (c) universally. For this purpose, at least, a coordinate system should be chosen to give an universal expression for the limiting wave steepness. Various empirical relations between the wavelength L_b, wave height H_b and water depth h_b at the breaking point have been proposed by many researchers (e.g. McCowan 1894 and Miche 1944). Galvin (1972) gives a relation including the bed slope,

$$H_b/L_b = 0.72\ (1 + 6.4 \tan\theta\ )\ h_b/L_b\ . \qquad (2)$$

In figure 5, the data obtained in this study are plotted to confirm the validity of Galvin's relation. The measured values for $\tan\theta = 1/10$ are also included in this figure. The linear line shows Galvin's relation and the curved line is obtained by applying the

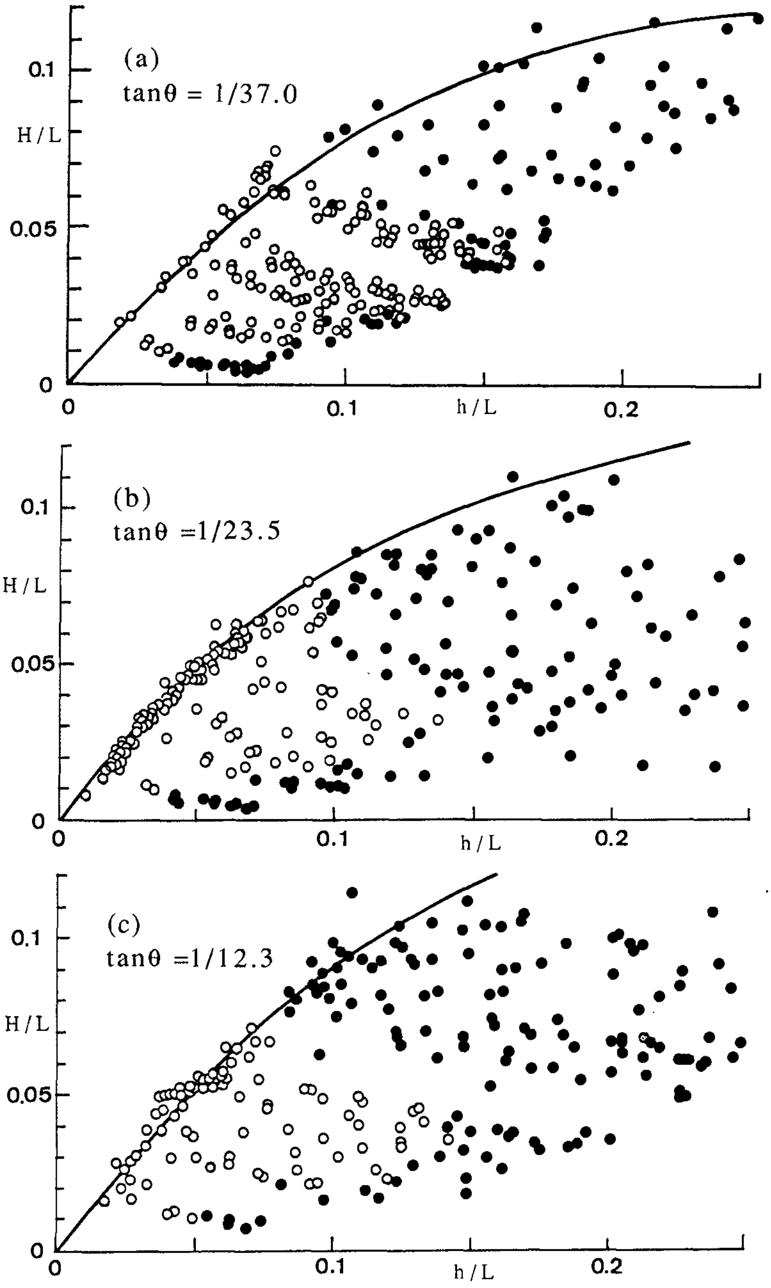

Figure 4. Formation regions of vortices for three bed slopes.

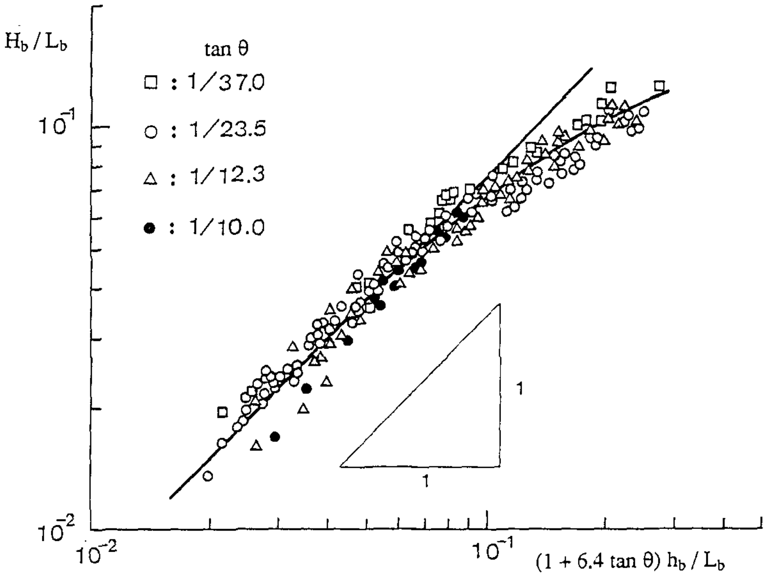

Figure 5. Universal expression for characteristic quantities of breaking waves on various bed slopes.

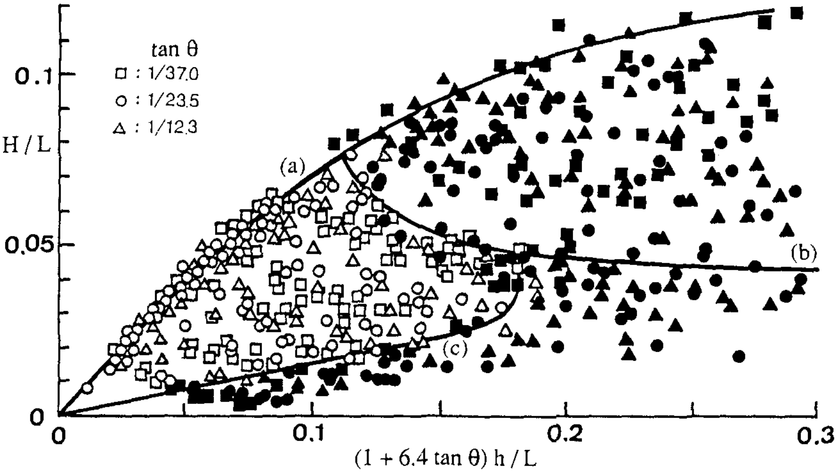

Figure 6. Universal expression for the vortex formation.

least squares method to these data. Though the data agree well with relation (2) at $H_b/L_b \le 0.05$, the disagreement between them becomes remarkable in the range of $H_b/L_b > 0.05$. However, it is seen that the use of this coordinate system can express universally the limit of wave steepness over the wide range of H_b/L_b.

In figure 6, the data shown in figure 4 are rearranged in Galvin's coordinate system. The three formation regions are unified in the region bounded by three solid lines (a), (b) and (c). The offshore vortices form when waves with $H_0/L_0 \le 4.2 \times 10^{-2}$ propagate on a sloping bed. The condition is expressed by the region under line (b). The onshore border of the vortex formation is the breaking point, and that of the offshore side is given by line (c).

Quantitative properties

1. Horizontal length scale of offshore vortices

The horizontal interval between offshore vortices l can be described completely by using the seven characteristic quantities of h, L, H, C, T, g and $\tan\theta$. As discussed in the previous section, however, two dimensional quantities can be removed from these seven quantities. If C and g are eliminated, a dimensionless form of l is given by

$$l/h = f\ (H/L,\ h/L,\ \tan\theta). \qquad (3)$$

Figures 7(a) to (c) show relations between l/h and H/L for each value of $\tan\theta$. It is seen that l/h is proportional to $(H/L)^{-1}$. Though the value of h/L was varied from 0.015 to 0.15 in this study, no systematic dependence of l/h on h/L was recognized. By approximating these data with the solid lines and plotting the values of $(l/h)\ /\ (H/L)^{-1}$ against $\tan\theta$, the relation

$$l/h = 1.2 \times 10^{-2}\ (\tan\theta)^{-1/3}\ (H/L)^{-1} \qquad (4)$$

is obtained empirically from figure 8. The comparison of the experimental data and equation (4) is shown by figure 9. It gives a good agreement. Another important thing is that l/h is of order unity. This means that the length scale of the offshore vortices is controlled mainly by the local water depth.

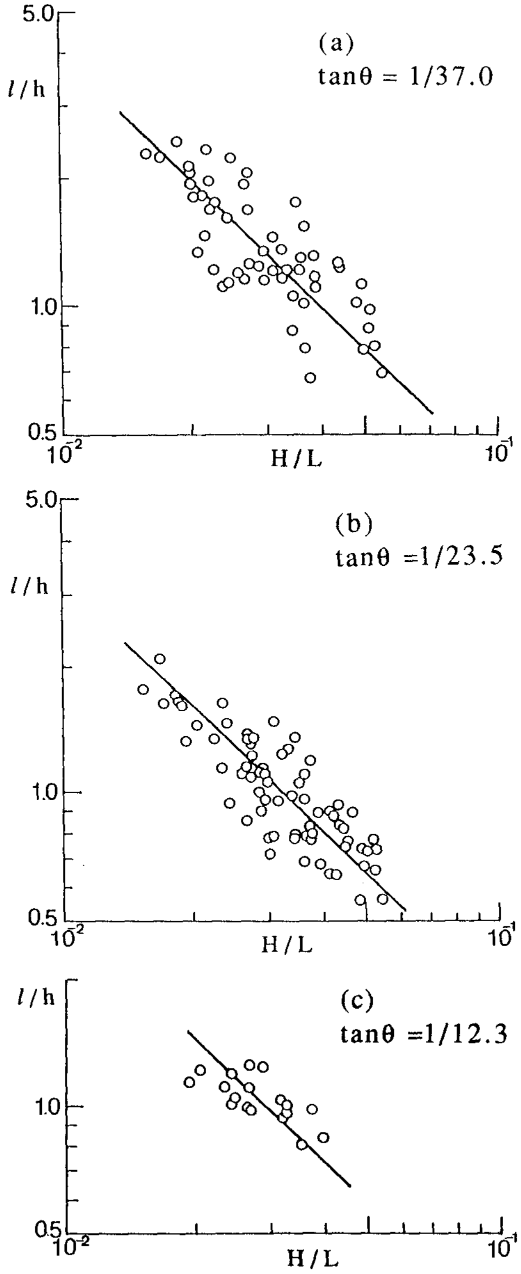

Figure 7. Relations between l/h and H/L.

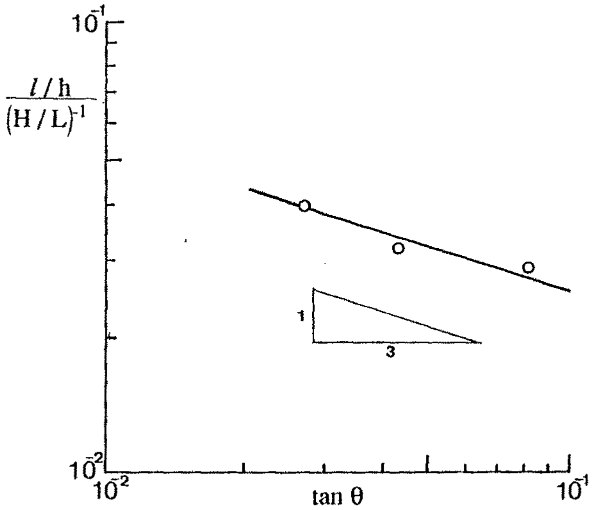

Figure 8. Dependence of l/h on tanθ.

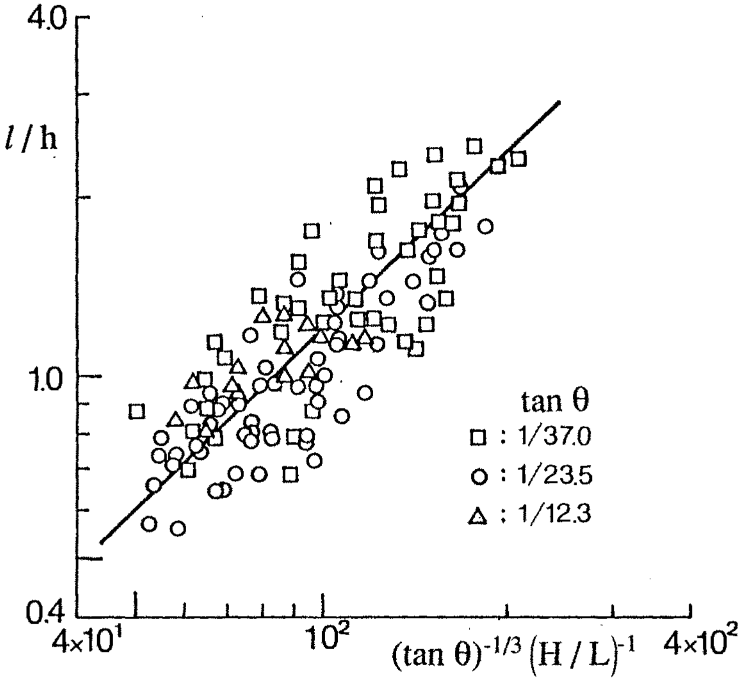

Figure 9. Universal expression for vortex intervals.

2. Velocity of vortex movement

The offshore vortices move slowly in the offshore direction. As discussed above, their velocities u also depend on h, L, H, T and $\tan\theta$. The dimensional consideration deduces

$$u/(L/T) = f\ (h/L,\ H/L,\ \tan\theta). \tag{5}$$

Since u and L/T are of order 10^{-1} cm/s and 10^{2} cm/s, respectively, the value of u/(L/T) becomes extremely small. Therefore, L/T seems to be unsuitable as a quantity to non-dimenisonalyze u. Another representative quantity is the velocity of offshore drift-current in the offshore zone. Longuet-Higgins derived its velocity profile induced by the wave propagation on a horizontal bed, i, e.,

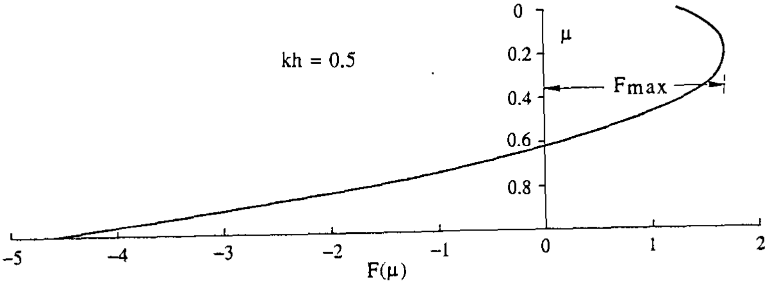

Figure 10. Longuet-Higgins' solution at kh = 0.5

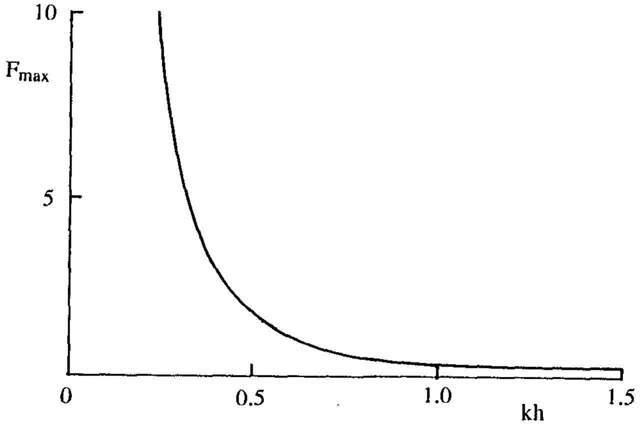

Figure 11. Dependence of Fmax on kh.

$$U(z) = (\pi H/L)^2\ C\ F(z/h), \tag{6}$$

where

$$F(\mu) = \frac{1}{4\ \sinh^2 kh}\Bigg[2\ \cosh\left\{2kh(\mu - 1)\right\} + 3 + kh\ \sinh\left\{2\ kh\left(3\mu^2 - 4\mu + 1\right)\right\} + 3\left(\frac{\sinh\ 2kh}{2kh} + \frac{3}{2}\right)\left(\mu^2 - 1\right)\Bigg], \tag{7}$$

z-axis is taken in the direction downward from the mean water level and k is the wave number. Figure 10 shows the profile of $F(\mu)$ at $kh = 0.5$. The positive value of $F(\mu)$ means that $U(z)$ has the velocity in the offshore direction. The profile given by equation (7) takes a positive maximum value F_{max} as shown in figure 10. Figure 11 shows the relation between F_{max} and kh which was obtained numerically. Here, let us use the maximum offshore velocity calculated from

$$U_{max} = (\pi\ H/L)^2\ C\ F_{max} \tag{8}$$

as a characteristic quantity controlling u, though it is obtained for a horizontal flat bed. In this case, a dimensionless form is given by

$$u/U_{max} = f\ (h/L,\ H/L,\ \tan\theta). \tag{9}$$

Figures 12 (a) to (c) show the relations between u/U_{max} and h/L for three bed slopes. The tendency that u/U_{max} increases in proportion to h/L is recognized from these figures, but no systematic dependence of u/U_{max} on H/L is seen. The relation between the proportional factors, which are obtained by approximating the data with the solid lines, and $\tan\theta$ is shown in figure 13. It is seen that the proportional factors depend on $(\tan\theta)^{2/3}$. From these results, an empirical relation for the velocity of vortex movement,

$$u/U_{max} = 3.6 \times 10^1\ (\tan\theta)^{2/3}\ (h/L) \tag{10}$$

is obtained. The comparison of the experimental data and equation (10) is made in figure 14. A good agreement is recognized.

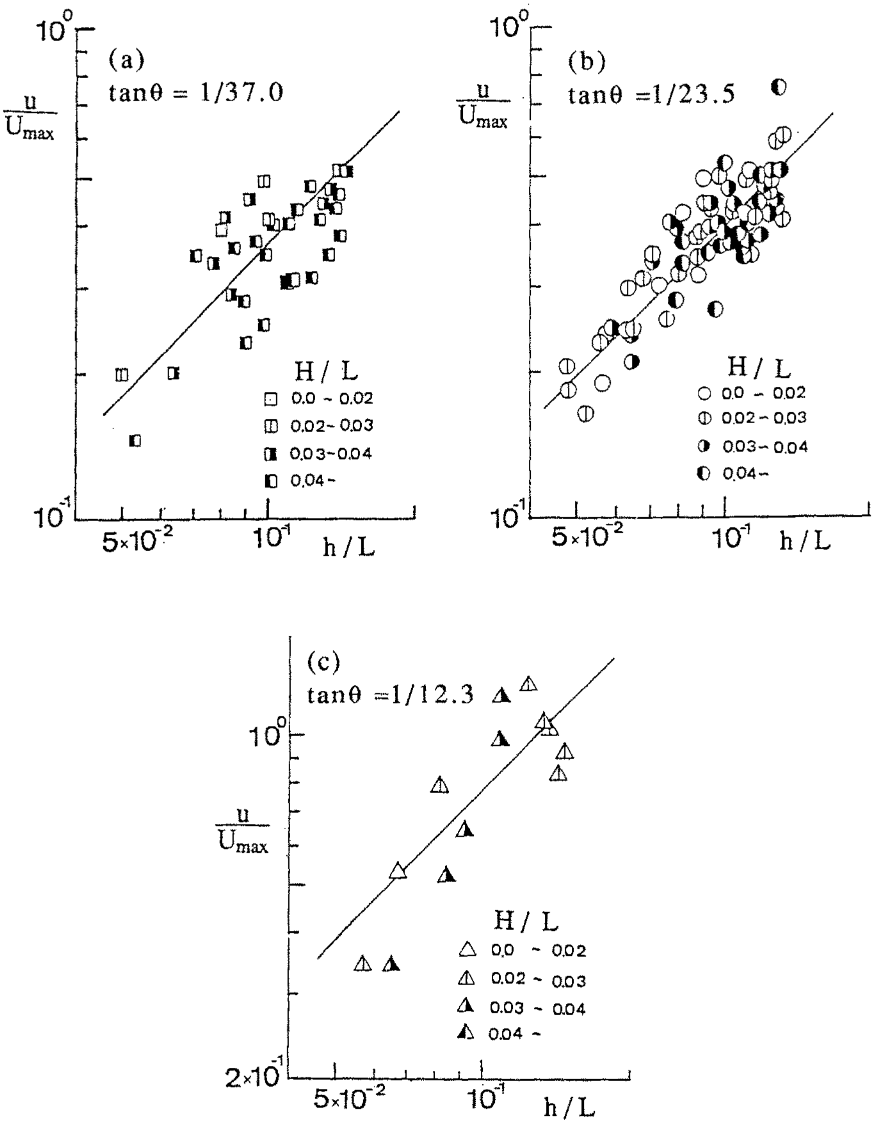

Figure 12. Relations between u/U_{max} and h/L.

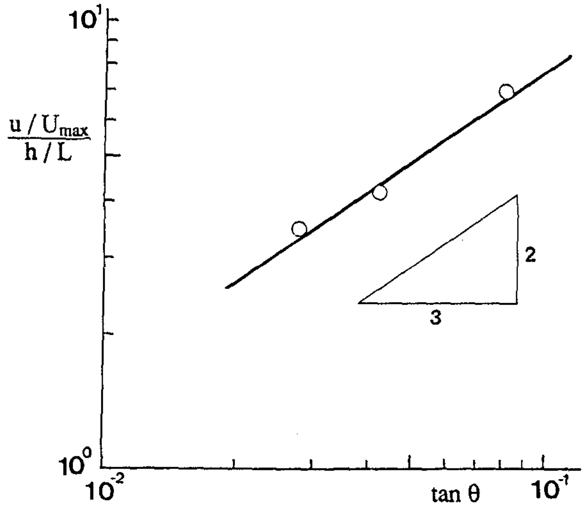

Figure 13. Dependence of u/U_{max} on $\tan\theta$.

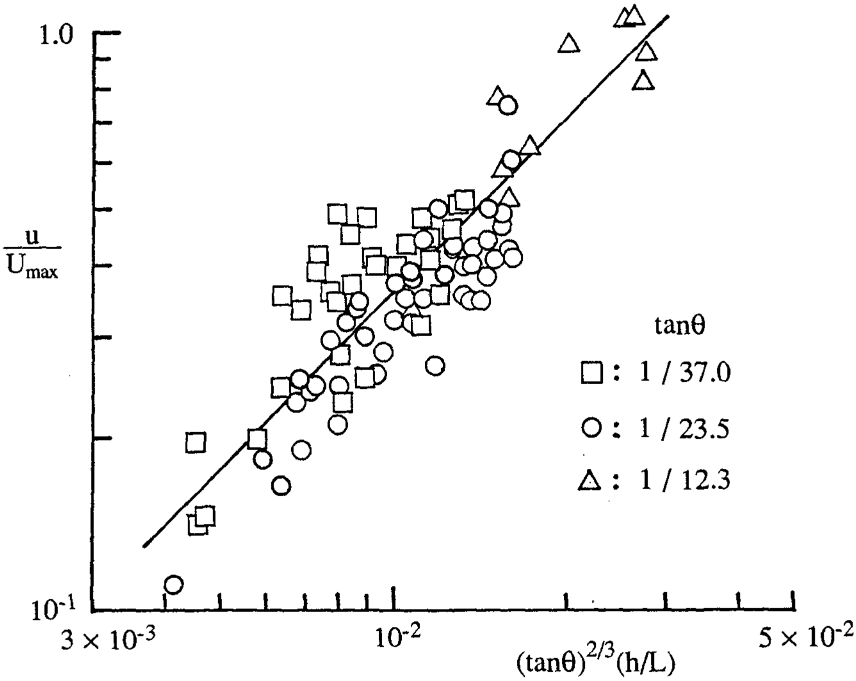

Figure 14. Universal expression for vortex velocities.

Conclusions

It was found that a row of vortices, which was called the offshore vortex train, forms along the water surface of an offshore zone when two-dimensional regular waves whose steepness in deep water is smaller than 4.2×10^{-2} run up on a sloping bed. It seems to occur owing to the shear instability between the onshore drift-current and the offshore one. The instability starts near the breaking point. Moving in the offshore direction, the vortices merge each other and increase their intervals in the size of water depth. After reaching a location of the offshore side, they begin to decay because of the decrease of the strain rate between the two drift-currents. The universal expressions were also obtained empirically for the formation of the offshore vortices, their horizontal length scales and the velocities of vortex movement.

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