CHAPTER 159

Grain-sorting over ripples induced by sea waves

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Abstract

A predictive theory for the formation of ripples under sea waves is presented for a sandy bottom characterized by a large grain-size distribution. The theory is based on a linear stability analysis. The conditions for the amplification or decay of bottom perturbations are determined and the experimentally observed sorting effect due to the presence of the bedforms is modelled. A comparison between the experimental data available in literature and the present results is attempted.

Introduction

In the past, the dynamics of sediment in nearshore regions was extensively studied in terms of a uniform material. However, coastal sediment typically has a wide range of grain sizes and the presence of mixtures has a large influence on coastal morphodynamics. Indeed, the grain sorting process which is typical of the transport of mixtures may inhibit or enhance sediment transport in areas characterized by low shear stress. For this reason, the last decade saw a major change in thinking, and important problems involving mixtures were at least formulated correctly and a fair number of them were solved as well, at least for sediment in transport in rivers, (Parker, 1991).

In accordance with this viewpoint, a predictive theory is presented in the present paper for ripple formation under sea waves in the case of a cohesionless bottom made up of a mixture. Following Blondeaux (1990), the theory is based on a linear stability analysis of a flat bottom subject to a viscous oscillatory flow. The aim of the work is twofold: first, to determine the conditions for the decay or the amplification of a bottom perturbation and second to study the grain-sorting

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process which leads the coarser fraction to accumulate along the crests of the perturbation while the finer fraction tends to move towards the troughs (Mei, 1989).

The procedure used in the rest of the paper is the following: in the next section, the problem is formulated by focusing our attention on the equation forcing the sediment mass balance and the relationship between sediment transport rate and fluid flow. Indeed, the above equations need particular care when written for a mixture. As discussed more deeply in the following, the continuity equation is written introducing: i) an active layer, scaled on the largest grain sizes and characterized by a vertical uniform size density (Hirano, 1971), which corresponds to the reservoir of material directly available for transport, ii) a bottom layer underneath the active layer, the sediment of which can be entrained by the flow only because of bottom erosion. In the bottom layer, the size density may have a vertical structure. The sediment transport rate is evaluated by means of a formula valid for a uniform sediment times the size density evaluated in the active layer. The sheltering effects exerted by the large grains on small ones are taken into account by introducing a "hiding" factor in the sediment transport rate formula (Parker, 1991).

In §3 an approximate analytical solution for the bed evolution is obtained taking into account that the time scale of the bed time development is much longer than the period of fluid oscillations, i.e. the period of sea waves. Finally in §4 the quantitative results are presented along with a qualitative comparison with the experimental data available in literature.

Formulation of the problem

As pointed out in the introduction, a significant feature of coastal areas is the wide range of sediment sizes found there. Let us then consider an initially cohesionless sea bottom formed by a sand mixture uniformly distributed in space. To deal with the statistics of the mixture, the logarithmic sedimentological scale ϕ is used, defined by

$$d^*/l^* = 2^{-\phi} \tag{1}$$

where the grain size d^* is made dimensionless by means of a characteristic length scale of the problem l^* which will be defined in the following. Hence, the characteristics of the sand mixture are specified in terms of the size distribution $p_f(\phi)$ or the size density $p(\phi)$. The function $p_f(\phi)$ is defined such that fraction $p_f(\phi)$ of a sample is finer than size ϕ while $p(\phi)$ is its derivative, i.e. $p(\phi) = dp_f(\phi)/d\phi$.

There are other statistical parameters that summarize the characteristics of the mixture: the geometric mean size d^*_m and the geometric standard deviation σ_g given by

$$d_m^*/l^* = 2^{-\phi_m} \qquad \sigma_g = 2^\sigma \tag{2}$$

where

$$\phi_m = \int_{-\infty}^{+\infty} \phi p(\phi) d\phi \qquad \sigma^2 = \int_{-\infty}^{+\infty} (\phi - \phi_m)^2 p(\phi) d\phi \tag{3}$$

A Cartesian orthogonal coordinate system (x^*, y^*, z^*) is then defined with the x^* and z^* axis lying on the sea bottom and the y^* axis directed upward. Because of the presence of a two-dimensional surface gravity wave, let us assume that the fluid close to the bottom but outside the viscous boundary layer oscillates in time with velocity

$$(u^*, v^*, w^*) = (-\frac{1}{2} U_0^* e^{i\omega^* t^*} + c.c., 0, 0) \tag{4}$$

where t^* is time, $\omega^* = 2\pi/T^*$ the angular frequency of the sea wave, U_0^* the amplitude of the irrotational velocity oscillations evaluated close to the bottom, c.c. denotes the complex conjugate of a complex number and (u^*, v^*, w^*) are the velocity components according to the Cartesian coordinate system (x^*, y^*, z^*). If the bottom is flat and all the grain sizes in the sand mixture are much smaller than the characteristic thickness of the bottom boundary layer δ^*, the fluid motion is described by the well known Stokes' solution and the sediment moves to and fro. The thickness δ^*, which can be defined as $\sqrt{2\nu/\omega^*}$ (ν being the kinematic viscosity of sea water), can then be assumed as the length scale of the problem, i.e. $l^* = \delta^*$.

The study of the time development of a two-dimensional bottom perturbation in the form

$$y^* = \eta^*(x^*, t^*) = \epsilon^* C_1(t) e^{i\alpha^* x^*} + c.c. \tag{5}$$

is posed by the vorticity equation, the flow and sediment continuity equations plus a relationship between sediment flow rate q^* and flow properties, along with boundary conditions which force the matching of the flow with the irrotational motion outside the bottom boundary layer and the no-slip condition at $y^* = \eta^*$. Because of the presence of the sand mixture, the sediment continuity equation needs to be discussed in detail. Assuming all grains to have the same density, the statement of mass balance for each grain size can be reduced to a similar one for volume balance

$$-\frac{\partial(q^* p_s)}{\partial x^*} = (1-n)\frac{\partial}{\partial t^*}\int_{-\infty}^{\eta^*} p dy^* \tag{6}$$

where it is assumed that the volume transport of bedload per unit time per unit width of grain in the size range $(\phi, \phi + d\phi)$ is provided by the relationship describing the sediment transport rate for a uniform material $(q^*(\phi))$ time the size density $p_s(\phi)$ at the surface. As discussed in Parker (1991), to simplify the problem it is assumed that near the surface there is an active layer characterized by a thickness L_a^* which corresponds to the reservoir of material directly available for transport and where the grains are well mixed. It follows that the size density p_s within it has no vertical structure even though it can have a streamwise and time structure. Below the active layer lies the substratum material with size density p_b. This may vary arbitrarily in x^* and y^* but cannot change directly in time because it is not directly subject to movement. Material can be exchanged between the substratum and the active layer through the intermediary of bed aggradation or erosion as outlined below.

By applying Leibnitz's rule, the following result is obtained from (6)

$$-\frac{\partial(p_s q^*)}{\partial x^*} = (1-n)\left\{p_i\frac{\partial \eta^*}{\partial t^*} + \frac{\partial}{\partial t^*}(L_a^* p_s)\right\} \tag{7}$$

The value of p_i can be specified in the case of a degrading bed as p_b, since substratum is incorporated into the surface layer as the bed elevation drops. In the case of an aggrading bed p_i can be assumed equal to p_s since the surface material is transferred directly to the substratum.

$$p_i = \begin{cases} p_s & \text{if} \quad \partial(\eta^* - L_a^*)/\partial t > 0 \\ p_b & \text{if} \quad \partial(\eta^* - L_a^*)/\partial t < 0 \end{cases} \tag{8}$$

Finally, it should be pointed out that L_a^* can be assumed to scale with some large size, e.g. d_{90}^*.

Let us then define the following dimensionless variables

$$(x,y) = (x^*, y^*)/\delta^* \qquad t = t^*\omega^* \qquad \epsilon = \epsilon^*/\delta^* \qquad \alpha = \alpha^*\delta^* \qquad L_a = L_a^*/\delta^*$$

$$\eta = \eta^*/\delta^* \qquad \psi = \psi^*/U_0^*\delta^* \qquad q = \frac{q^*}{[(\rho_s - \rho)g d_{mo}^{*3}]^{1/2}}$$

where g is the gravity, ψ^* is the stream function such that $u^* = \partial\psi^*/\partial y^*$, $v^* = -\partial\psi^*/\partial x^*$ and d_{mo}^* the mean size of the initial uniform mixture characterized by a size density $p_o(\phi)$.

The governing differential problem then reads:

$$\frac{2}{R_\delta}\frac{\partial}{\partial t}(\nabla^2\psi) + \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi) = \frac{1}{R_\delta}(\nabla^4\psi) \tag{9}$$

$$\frac{\partial\psi}{\partial y} \to -\frac{1}{2}e^{it} + c.c. \qquad \frac{\partial\psi}{\partial x} \to 0 \quad \text{for } y \to \infty \tag{10}$$

$$\frac{\partial\psi}{\partial y} = 0 \qquad \frac{\partial\psi}{\partial x} = 0 \quad \text{for } y = \eta \tag{11}$$

where the flow Reynolds number R_δ is defined as follows:

$$R_\delta = \frac{U_o^* \delta^*}{\nu}$$

As pointed out previously, in order to close the above formulation, we need a relationship between sediment flow rate q^* and flow properties. Presently, a modified version of the Grass-Ayoub (1982) formula is used. The differences consist in the introduction of the effect of gravity related to the bed elevation and in the introduction of the "hiding" factor $(\frac{d^*}{d^*_{mo}})^r$. This formula, even though simple and possibly rough, appears to contain the main physical ingredients controlling the process of transport:

$$q = a(\frac{2}{R_{do}})^b(\frac{d^*}{d^*_{mo}})^{-0.03-b} \cdot |(\frac{d^*}{d^*_{mo}})^r v_t - \frac{\beta R_{do}}{2F^2_{do}}\frac{\partial\eta}{\partial x}|^{b-1} \cdot$$
$$\cdot \; ((\frac{d^*}{d^*_{mo}})^r v_t - \frac{\beta R_{do}}{2F^2_{do}}\frac{\partial\eta}{\partial x}) \tag{12}$$

where a = 1.23; b = 4.28; β = 0.15 and v_t is the fluid velocity evaluated at $y^* = d^*/2$ parallel to the bed profile. It can be seen that values of r close to one as discussed by Parker (1991) correspond to the condition for equal-mobility.

The particle Reynolds number R_{do} and the particle Froude number F_{do} are defined in terms of the geometric mean size d^*_{mo} to the initial grain size distribution:

$$R_{do} = \frac{U_0^* d^*_{mo}}{\nu} \;; \quad F_{do} = \frac{U_0^*}{[(s-1)g d^*_{mo}]^{1/2}} \;; \quad s = \frac{\rho_s}{\rho} \;;$$

Since the bottom waviness is assumed to be of a small amplitude the quantity ϵ can be assumed much smaller than one and the solution to the problem can be expanded in power series of ϵ in the form:

$$\psi = \psi_0(y,t) + \epsilon C_1(t)\phi_1(y,t)e^{i\alpha x} + c.c. + 0(\epsilon^2) \tag{13}$$

$$q = q_o(\phi,t) + \epsilon C_1(t) q_1(\phi,t)e^{i\alpha x} + c.c. + 0(\epsilon^2) \tag{14}$$

$$p = p_o(\phi) + \epsilon C_1(t) p_1(\phi,t)e^{i\alpha x} + c.c. + 0(\epsilon^2) \tag{15}$$

By substituting (13) $\div$ (15) into (9) $\div$ (12) and equating similar powers of ϵ, at order one a problem is found which can easily be solved. As pointed out previously, the flow is described by the well-known Stokes' solution and the sediment moves to and fro. Because the moving material is still considered as belonging to the bottom, the size density p_0 does not change with time. Moreover, p_0 does not depend on x because of the supposed uniformity of the problem.

At order ϵ the problem reads:

$$\frac{2}{R_\delta}\frac{\partial}{\partial t}\left(\frac{\partial^2\phi_1}{\partial y^2} - \alpha^2\phi_1\right) + i\alpha\left[\frac{\partial\psi_0}{\partial y}\left(\frac{\partial^2\phi_1}{\partial y^2} - \alpha^2\phi_1\right) - \frac{\partial^3\psi_0}{\partial y^3}\phi_1\right] = \frac{1}{R_\delta}\left[\frac{\partial^4\phi_1}{\partial y^4} - 2\alpha^2\frac{\partial^2\phi_1}{\partial y^2} + \alpha^4\phi_1\right], \quad (16)$$

$$\frac{\partial\phi_1}{\partial y} \to 0, \qquad \phi_1 \to 0, \quad y \to \infty \quad (17)$$

$$\frac{\partial\phi_1}{\partial y} = -\frac{\partial^2\psi_0}{\partial y^2}, \qquad \phi_1 = 0, \quad y = 0 \quad (18)$$

$$-\frac{2F_{do}}{R_{do}}(1-n)\frac{dC_1(t)}{dt} = i\alpha q_1(t)C_1(t), \quad (19)$$

$$q_1 = ab\left(\frac{2}{R_{do}}\right)^b\left(\frac{d^*}{d^*_{mo}}\right)^{-0.03-b}\left|\frac{\partial\psi_o}{\partial y}\left(\frac{d^*}{d^*_{mo}}\right)^r\right|^{b-1} \cdot \left\{\left[\frac{\partial\psi_1}{\partial y} + C(t)e^{i\alpha x}\frac{\partial^2\psi_o}{\partial y^2}\right]_{y=\frac{d}{2}}\left(\frac{d^*}{d^*_{mo}}\right)^r - \frac{i\alpha\beta R_{do}}{2F_{do}^2}C(t)e^{i\alpha x}\left(\frac{d^*}{d^{*2}_{mo}}\right)^2\right\} \quad (20)$$

It should be pointed out that at order ϵ, the term proportional to the time derivative of $C_1(t)$ in the vorticity equation has been ignored. From a physical point of view, this corresponds to ignoring the influence of the variation of bottom elevation on fluid motion. From a mathematical point of view, this assumption is justified by the small value usually attained by the dimensionless parameter

$$Q = 0.615((s-1)/s)F_{do}^{-1.45}(R_{do}/R_\delta)^{4.28}/(1-n) \sim \frac{dC_1(t)}{dt}/C_1(t). \quad (21)$$

Solution

Because of the assumption $dC_1/dt \ll C_1(t)$, flow development has been decoupled from the sediment motion. Equations (16) ÷ (20) and boundary conditions (17) ÷ (18) can then be solved with the same procedure used in Blondeaux (1990) to which the reader should refer for details. Once the stream function is known, the bottom time development can be obtained from the sediment continuity equation and the sediment flow rate formula, which at order ϵ provides

$$(L_0p_1 + p_0)\frac{dC_1}{dt} = -L_0C_1\frac{\partial p_1}{\partial t} - \frac{i\alpha R_{d0}}{2F_{d0}(1-n)}[q_0p_1 + p_0q_1]C_1 \quad (22)$$

By integrating (22) over the whole range of ϕ and forcing that

$$\int_{-\infty}^{+\infty} p_0(\phi)d\phi = 1 \qquad \int_{-\infty}^{+\infty} p_1(\phi,t)d\phi = 0 \tag{23}$$

an equation for $C_1(t)$ is obtained which allows the time development of the ripple amplitude to be determined once the function $p_1(\phi,t)$ is known,

$$\frac{dC_1}{dt} = -\frac{i\alpha R_{d0}}{2F_{d0}(1-n)}\left[\int_{-\infty}^{+\infty}(q_0p_1 + p_0q_1)d\phi\right]C_1 \tag{24}$$

The perturbation of the grain size distribution $p_1(\phi,t)$, can be obtained by substituting (22) for (24):

$$\begin{aligned}\frac{\partial p_1}{\partial t} &= -\frac{i\alpha R_{d0}}{2F_{d0}(1-n)L_0}\{[q_0p_1 + p_0q_1] - \\ &- (L_0p_1 + p_0)\int_{-\infty}^{+\infty}(q_0p_1 + p_0q_1)d\phi\}\end{aligned} \tag{25}$$

The solution to the problem posed by equations (24) and (25) cannot be found in closed form. However, an asymptotic solution can be determined by taking advantage of the small values usually attained by the quantity Q. Indeed, it turns out that:

$$\frac{dC_1}{dt} = -i\alpha Q\left[\int_{-\infty}^{+\infty}(\tilde{q}_0p_1 + p_0\tilde{q}_1)d\phi\right]C_1 \tag{26}$$

$$\frac{\partial p_1}{\partial t} = -\frac{i\alpha Q}{L_0}\left[\tilde{q}_0p_1 + p_0\tilde{q}_1 - (L_0p_1 + p_0)\int_{-\infty}^{+\infty}(\tilde{q}_0p_1 + p_0\tilde{q}_1)d\phi\right] \tag{27}$$

where the quantities $\tilde{q}_0$, $\tilde{q}_1$ (defined below) along with p_0, p_1, L_0 are expected to be quantities of order one:

$$(\tilde{q}_o, \tilde{q}_1) = \frac{R_{do}}{2F_{do}(1-n)Q}(q_o, q_1) \tag{28}$$

The functions $C_1(t)$ and $\phi_1(\phi,t)$ can then be expanded in power series of Q:

$$\begin{aligned} C_1(t) &= C_{10} + QC_{11}(t) + 0(Q^2); && (29)\\ p_1(\phi,t) &= p_{10}(\phi) + Qp_{11}(\phi,t) + 0(Q^2) && (30)\end{aligned}$$

The functions C_{10} and p_{10} turn out to be time independent. The constant C_{10} can be fixed equal to one without loss of generality while the function p_{10} depends on the initial conditions. By substituting (29) and (30) for (26) and (27) it can be seen that

$$
\begin{aligned}
\frac{dC_{11}}{dt} &= -i\alpha\left[\int_{-\infty}^{+\infty}\tilde{q}_0 p_{10} + p_0\tilde{q}_1 d\phi\right] = f(t) \\
\frac{\partial p_{11}}{\partial t} &= -\frac{i\alpha}{L_0}\{\tilde{q}_0 p_{10} + p_0\tilde{q}_1 - \qquad (31)\\
&- (L_0 p_{10} + p_0)\int_{-\infty}^{+\infty}(\tilde{q}_0 p_{10} + p_0\tilde{q}_1)d\phi\} \qquad (32)
\end{aligned}
$$

The right hand side of (31) is a periodic complex function of time denoted $f(t)$. The growth or decay of the perturbation amplitude, i.e. ripple formation, is thus controlled by the sign of the real part of the time average of $f(t)$. The time average of the imaginary part is related to ripple migration and turns out to vanish because of the symmetry of the problem. The oscillating parts of $f(t)$ with vanishing time average describe the time variation of the perturbation profile during a wave cycle. More precisely the real part describes oscillations of the amplitude of the bottom perturbation while the imaginary part controls the small longitudinal oscillations of the ripple profile around its average position. The value of the time average of the real part of $f(t)$ is negative or positive depending on the values attained by the flow and sediment parameters α, R_δ, R_{do}, F_{do}, p_o.

Discussion of the results

As in Blondeaux (1990), two contributions to $\bar{f} = \frac{1}{2\pi}\int_o^{2\pi} f(t)dt$ can be identified. The former is associated with the steady component of the fluid velocity and is usually destabilizing since the steady drift close to the bed tends to carry sediments from the troughs towards the crests of the perturbation, thus causing its growth. The latter contribution is due to the component of the gravity along the bed profile which has a stabilizing effect. In fact, gravity opposes the tendency of the flow to carry sediments from the troughs towards the crests of the perturbation, thus causing the decay of the latter. Assuming that $p_o(\phi)$ is a normal distribution, so that the standard deviation σ_o identifies the grain size distribution once R_{do} is fixed, the behaviour of the perturbation is controlled by a balance between the two effects described above which depends on the values of α, R_δ, R_{do}, F_{do} and σ_o.

In figure 1 the value of $\bar{f}_r$ is plotted versus the wave number of the disturbance α for fixed values of R_δ, R_{do} and σ_o and for different values of F_{do}. It appears that a critical value F_{dc} of F_{do} exists such that: for F_{do} less than F_{dc} the bottom perturbations characterized by any value of the wave number α decay; for F_{do} larger than F_{dc} disturbances characterized by values of α falling within a restricted range experience an average amplification during a cycle.

The qualitative behaviour of the results does not change when different values of σ_o are considered. However, in figure 2 it can be seen that the critical value of F_{do} increases when increasing values of σ_o are considered: a bottom made up of a mixture is more stable than one composed of well sorted material. Similar results are obtained for different values of R_{do}.

In figure 3 the critical wave number of the bottom perturbation α_c is plotted versus R_δ for fixed values of R_{do} and σ_o.

A comparison between the present theoretical findings and experimental data by Blondeaux et al. (1988) is shown in figure 4 where the ratio between the amplitude of fluid displacement a^* and the critical ripple wavelength l^* is plotted versus R_δ. The theoretical predictions are shown considering sediments characterized by a specific weight $\rho_s/\rho = 2.65$; $R_{do} = 10$; $\sigma_o = 0.05$. The experimental data refer to a well sorted silt characterized by a mean diameter equal to 0.124 millimeters, a standard deviation equal to 0.02 millimeters and a specific weight equal to $2.65 t/m^3$. The experimental conditions are such that R_δ ranges between 20 and 85 and the Reynolds number of the sediments falls within the range (5,15). The agreement seems satisfactory even though the lack of experimental data in literature concerning mixtures characterized by large values of the standard deviation σ_o^* does not make a good test for the present theory possible.

The tendency of the process to pile up larger sediments towards the crests is described by means of the integration of equation (32). Also in this case the right hand side of (32) turns out to be a periodic function and the process is thus controlled by the time average of $\frac{\partial p_{11}}{\partial t}$. If initial perturbations are absent in the grain size distribution, i.e. $p_{10}(\phi) = 0$, the time average of $\frac{\partial p_{11}}{\partial t}$ shows that smaller grains are shifted towards the troughs and larger grains towards the crests, as experimentally observed (Mei, 1989). Indeed, figure 5 shows that the time average of p_{11} is real and positive for small values of ϕ and negative for large values of ϕ. It is worthwhile pointing out that values of r close to one have been used in obtaining the results shown in figure 5. These values of r are suggested by experimental measurements by Parker (1990) and correspond to equal mobility of all grains, i.e. the bias toward fine material in the bedload relation is almost counteracted by the mean of the hiding function $(\frac{d^*}{d^*_{mo}})^1$.

Figure 6 where the time average of p_{11} is plotted for the same values of the parameters as in figure 5 but for $r = 0$ shows that the hiding effects exerted by large grains on small ones is essential in describing the sorting process. In fact, when hiding effects are ignored, small grains tend to pile up near the crests and large grains towards the troughs.

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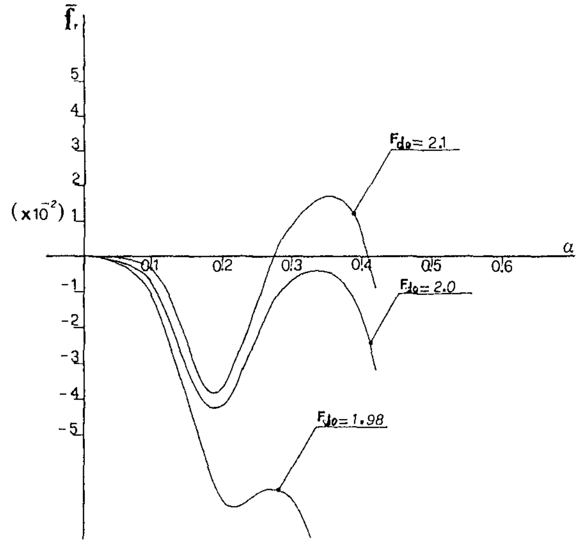

Figure 1. Time averaged amplification factor $\tilde{f}_r$ versus α for different values of F_{do}. ($R_{do} = 10, \sigma_o = 1, R_\delta = 50$).

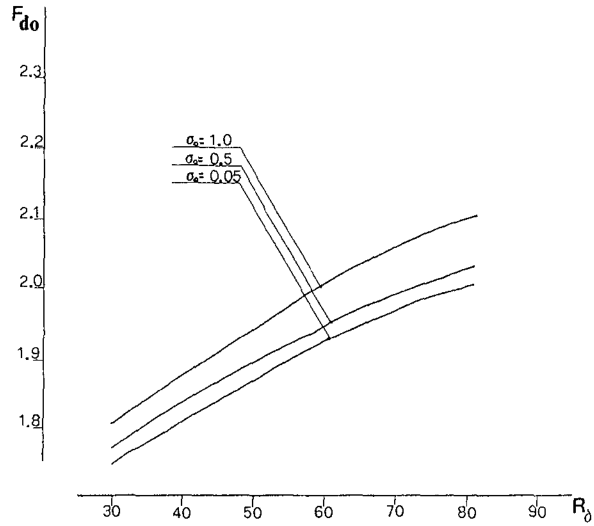

Figure 2. Critical value of the sediment Froude number versus R_δ for different values of σ_o. ($R_{do} = 10$).

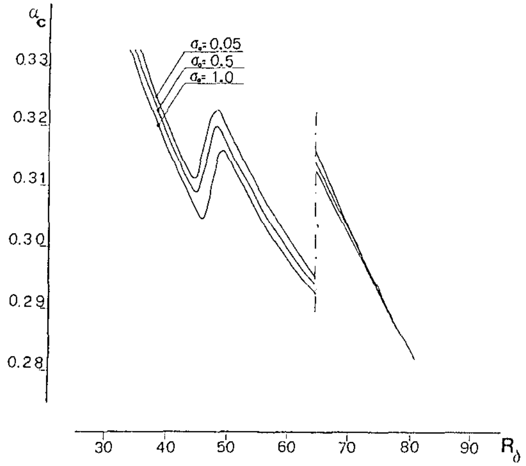

Figure 3. Critical wave number of the bottom perturbation versus R_δ for different values of σ_o. ($R_{do} = 10$).

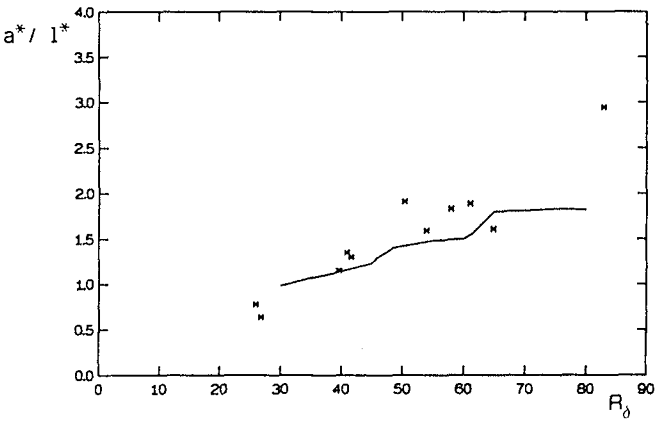

Figure 4. Comparison between experimental and theoretical dimensionless ripple wavelengths.

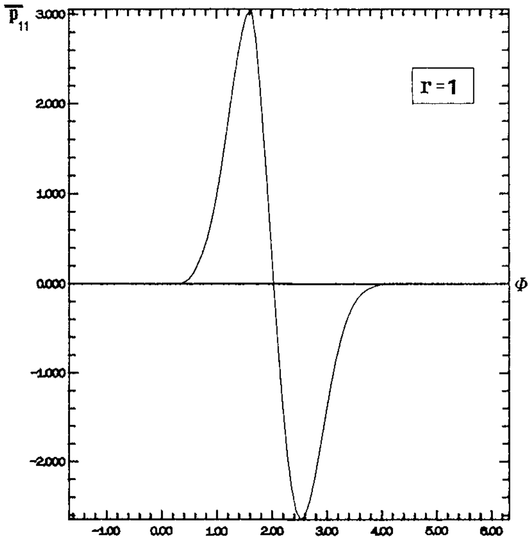

Figure 5. Perturbed density distribution function (averaged over a cycle) versus the ϕ-scale considering the "hiding" factor. $(F_{do} = 2.3, R_{do} = 10, R_\delta = 50, \sigma_o = 0.5)$.

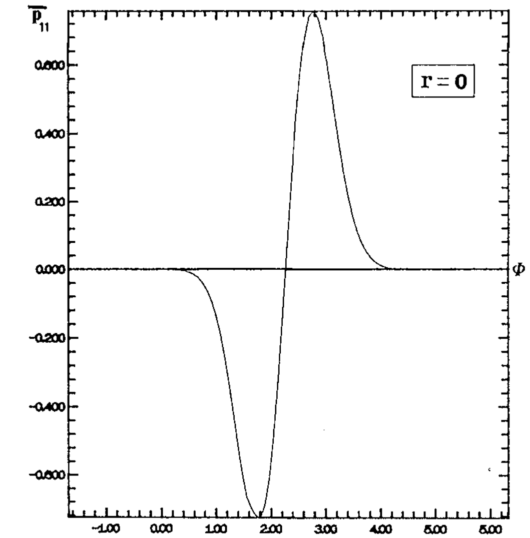

Figure 6. Perturbed density distribution function (averaged over a cycle) versus the ϕ-scale without the "hiding" factor. $(F_{do} = 2.3, R_{ab} = 10, R_\delta = 50, \sigma_o = 0.5)$.