CHAPTER 192

Behaviour Of Mobile Beds At High Shear Stress

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Introduction

Beds of granular material show various types of behaviour as the dimensionless shear stress or Shields ordinate, Y, is increased. This quantity is defined as $\tau/\rho g(S-1)d$, where τ is boundary shear stress, ρ is fluid density, g is gravitational acceleration, S is the ratio of solids density to fluid density and d is particle diameter. No movement occurs until Y exceeds a critical value. As Y is increased beyond the critical sand waves form, first increasing and then decreasing in steepness with successive increases in Y. Finally, in the high-shear-stress region, say $Y \geq 0.8$, the bed becomes plane (or exhibits antidunes in cases of critical or supercritical free-surface flow). This high-stress condition, sometimes called the upper plane-bed region, may be encountered for rivers in flood, large flows in estuaries, and closures or breaches of cofferdams or dykes. Because of the very high rates of sediment transport this type of flow has a disproportionate effect on both natural topographic features and engineering works. It is not easy to investigate this behaviour by traditional flume experiments, but the use of enclosed pressurised conducts can eliminate many of the experimental difficulties (Wilson, 1966).

Analysis

At high shear stress the bed load moves in a near-bed zone (the shear layer or sheet-flow layer), with thickness δ_s. Within this layer, the grains comprising the bed load (also called the contact load) are supported by intergranular contacts, which may be either continuous or sporadic. For this type of motion Bagnold (1956)

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demonstrated the existence of both an intergranular normal stress, σ_s, and an intergranular component of shear stress, τ_s. The latter is equal to $\sigma_s \tan \phi'$, where ϕ' is the dynamic equivalent of the friction angle used in soil mechanics. At the bottom of the shear layer all of the applied shear stress τ is resisted by the intergranular shear stress, which must equal $\tan \phi'$ times the normal intergranular stress at that location. This normal stress equals the submerged weight of the contact-load solids in the shear layer, which can be written $\rho\, g(S-1)\bar{C}\,\delta_s$ where $\bar{C}$ is the average volumetric concentration of bed-load solids within this layer. Rearrangement gives

$$\delta_s = \tau/[\rho g(S-1)\bar{C} \tan \phi'] \qquad (1)$$

showing that δ_s/d equals $Y/(\bar{C} \tan \phi')$. It was suggested earlier (Wilson, 1989) that for typical values of $\bar{C}$ and $\tan \phi'$ this relation gives δ_s/d as roughly 10Y. More recent analysis of concentration-profile data obtained in Saskatchewan (Daniel, 1965; Shook and Daniel, 1965) indicates that a value of 7.5 Y is better, i.e.

$$\delta_s/d \approx 7.5\ Y \qquad (2)$$

Previous analysis of the velocity distribution within the shear layer (Wilson, 1984, 1989) had demonstrated that the ratio of local velocity to shear velocity (U/U_*) can be expressed in terms of the relative height within the shear layer, y/δ_s. Thus at the top of the shear layer, where $y/\delta_s = 1.0$, the ratio U/U_* has a specific value which can be used as a match point for the logarithmic profile of the velocity in the main flow above the shear layer. On this basis the ratio of mean velocity to shear velocity can be calculated, leading to the overall friction relation. This analysis indicated that mobile beds at high shear stress are neither smooth boundaries nor rough ones, but obey their own frictional law analogous to the other cases but with characteristic length proportional to δ_s.

When this law is compared with the rough-boundary equation it is found that the roughness k equivalent to the mobile-bed friction law is about 0.5 δ_s, i.e. the value of k/d is not constant (as for the rough boundary law) but increases approximately linearly with Y. This conclusion was verified (Wilson, 1989) using the results of closed-conduct experiments carried out with sand (relative density S = 2.67) and nylon (S = 1.14). These are shown on Fig. 1, together with results of a more recent research program using particles of Bakelite (S = 1.56). The plot covers a considerable range of both variables, and although some scatter is present, the general increase of k/d with Y is very clear.

Up to this point it had been assumed in the analysis that the boundary shear stress τ was applied to all levels of the shear layer, i.e. the variation of total shear stress with height was ignored. However, for some of the experiments δ_s occupies a sizable fraction of the waterway height, implying a significant variation of τ within the shear layer. This variation depends directly on the ratio δ_s/R, (R is hydraulic radius). As the friction factor itself depends on R/k, and k is proportional to δ_s, no additional parameters were required for the analysis with variable τ, and the algorithm was re-written to incorporate this case.

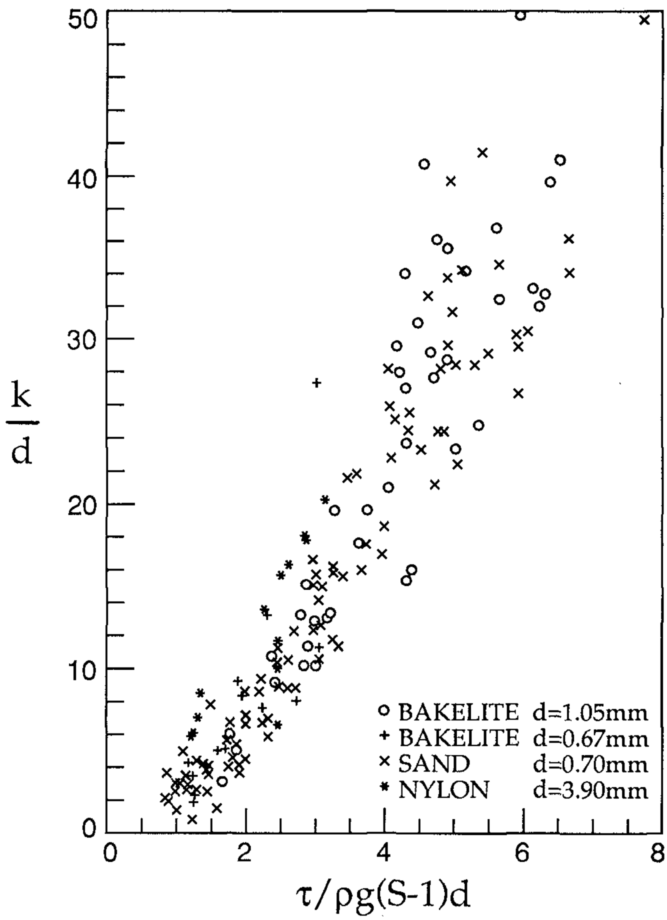

Fig. 1 Plot of Effective Roughness Ratio

By use of Eq. 1 it is readily seen that the ratio R/δ_s depends only on (S-1)/i, i.e.

$$\frac{R}{\delta_s} = \frac{(S-1)}{i} (\bar{C} \tan \phi') \approx 0.13 \frac{(S-1)}{i} \tag{3}$$

For constant τ, it was predicted that a plot of the velocity ratio $\bar{U}/U_*$ versus the logarithm of (S-1)/i should form a straight line. These axes can also be used to show the new calculations for variable τ. The resulting curve is plotted on Fig. 2, together with points for the data sets mentioned above.

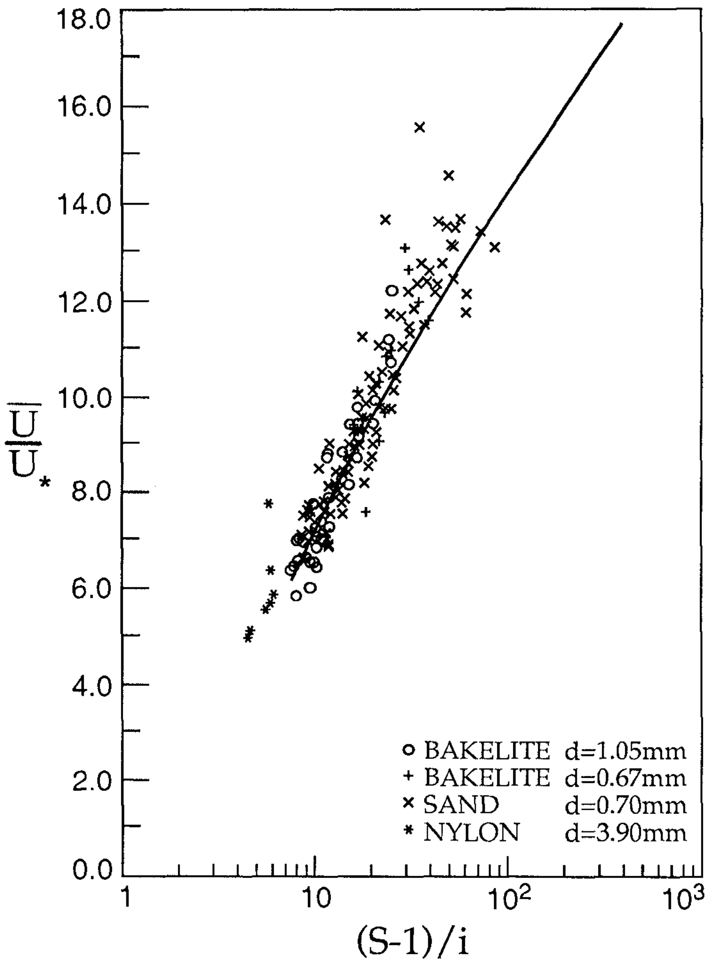

Fig. 2 Velocity-Ratio Plot

It is most encouraging to see the agreement between theory and experiment which is displayed on Fig. 2. From a practical point of view, however, the curve shown on this figure is not convenient to use, and it is preferable to approximate it by a simpler function such as a power law. If necessary, the curve can be divided into several regions, and a separate approximating power law found for each region.

In applying this approximating technique, it was found that a single power law was adequate to cover a broad range of conditions of practical interest i.e. subcritical flow with particles having S of 2.6 or more. For these conditions the approximating function was found to be

$$\frac{\bar{U}}{U_*} = 9.55 \left[\frac{(S-1)}{i}\right]^{0.11} \qquad (4)$$

or

$$\bar{U} = 9.55\ (S-1)^{0.11}\ i^{0.39}\ \sqrt{(gR)} \qquad (5)$$

This equation can also be re-written to give the Darcy-Weisbach friction factor, i.e.

$$f = 0.088 \left[\frac{i}{(S-1)}\right]^{0.22} \qquad (6)$$

The form of the approximating function given by Eq. 5 may be considered as a replacement for the Manning formula, for use with mobile beds at high shear stress ($Y \geq 0.8$). As friction does not depend on particle diameter for this type of behaviour, this equation involves no roughness coefficient (such as n in the Manning formula) and thus is exceptionally simple to apply. This point can best be illustrated by an example. Consider the flow over a mobile bed in an estuary, with, R of 6 m, and mean velocity of 4 m/s. With (S-1) taken as 1.65, Eq. 5 is readily solved for i, which is found to be approximately 5×10^{-4}, The shear stress τ, i.e. ρgRi, is about 30 Pa, and Y is 1.8, verifying that the flow is in the appropriate range. The thickness of the shear layer δ_s will be about 14mm and U_* about 0.17 m/s.

The values of τ (and U_*) found above can themselves be used to estimate the transport rates of bed-load and suspended load. This matter has been analysed by Wilson and Pugh (1988), who noted that the suspended load occurs as an addition to the bed-load particles considered previously. This addition can have only a secondary effect on the thickness of the sheet-flow layer that contains the bed load, and the same applies to the velocity at the top of this layer. Hence it is expected that suspended load will have, at most, a minor influence on the numerical coefficients of Eqs. 4 to 6.

Conclusion

Experimental evidence from flows in pressurised conduits has provided striking verification of the predictions from the analysis. Both analysis and experiment show that mobile-boundary flows at high shear stress do not obey the rough-wall friction law, (as had been imagined for many years). Instead, they follow their own law, with frictional length scale proportional to the shear stress itself. In this case equations based on rough-wall friction, such as the Manning formula, do not apply and the friction factor depends on the ratio i/(S-1). A power-law approximation to this function gives a simple friction equation which can replace the Manning formula for high-shear-stress mobile-boundary flow.

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