CHAPTER 32

Movable Bed Friction Factors for Spectral Waves

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Abstract

This paper presents a summary of carefully conducted laboratory experiments on the attenuation of simulated spectral waves propagating over an 18-m-long bottom section covered by 10-cm-thick layers of 0.12- and 0.2-mm-diameter uniform quartz sands. The measured attenuation is used in conjunction with the theory for spectral wave attenuation developed by Madsen et al. (1988) to determine values of the movable bed friction factor for spectral waves. When the values of movable bed friction factors, f_{wr}, are plotted against a fluid-sediment interaction parameter, the ratio of skin friction Shields parameter for an equivalent monochromatic wave, ψ_{mr}', and the critical value of Shields parameter for initiation of sediment motion, ψ_c, a well-defined relationship between f_{wr} and ψ_{mr}'/ψ_c emerges. This relationship is independent of spectral shape (both finite-depth Jonswap and Neumann spectra were simulated) and is the same for the two sediment types tested. Although limited by the range of experimental conditions achieved in the laboratory study the results suggest a simple methodology for the evaluation of spectral wave attenuation from knowledge of sediment and wave characteristics.

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Introduction

In order to predict the attenuation by bottom friction for waves propagating in finite water depth over a movable bottom, it is necessary to estimate the value of the bottom roughness or alternatively the wave friction factor. For wave conditions exceeding the threshold of sediment movement, bottom roughness or wave friction factor is governed by wave-sediment interaction and is related to the characteristics of the wave-generated bedforms. Based on rather limited experimental evidence obtained for wave-sediment interaction, corresponding exclusively to simple periodic waves, predictive relationships for the roughness of movable beds subjected to wave agitation have been proposed, e.g., Grant and Madsen (1982). Neither theoretical nor experimental justification appeared to exist for the transfer of this "knowledge" to the more realistic description of waves in terms of their spectrum until Madsen et al. (1988) presented a theoretical model for the attenuation of spectral waves propagating over a rough bottom, described by its equivalent roughness, k_b. Based on this model Madsen and Rosengaus (1988) developed a methodology for the prediction of spectral wave attenuation by bottom friction. This methodology consisted of:
1) representing the spectral wave condition by a representative monochromatic wave having the same root-mean-square near-bottom orbital velocity, u_{br}, and excursion amplitude, A_{br}, as the spectral wave; 2) using the representative wave in conjunction with empirical relationships established for monochromatic waves to predict the geometry, relative height, η/A_{br}, and steepness, η/λ, of the wave-generated bottom bedforms; 3) obtaining the relative equivalent bottom roughness from an empirical relationship giving k_b/A_{br} as a function of bedform geometry.

The empirical relationships necessary for the implementation of the procedure outlined above were established through extensive and carefully conducted experiments with monochromatic waves (Madsen and Rosengaus, 1988) and the proposed methodology for the prediction of spectral wave attenuation by bottom friction was tested against experimental data. However, the experimental data on which the methodology was based were *limited* by having been obtained for a single sediment, a 0.2-mm quartz sand, and the test of the spectral wave attenuation methodology was *limited* to wave conditions exceeding only slightly the critical condition for initiation of sediment motion. For these reasons Madsen and Rosengaus (1988) emphasized that their proposed spectral wave attenuation methodology "should be regarded as preliminary until further experiments, covering a wider range of experimental parameters, can support its general validity."

The objective of the present study was to remove the limitations of the previous experimental investigation by: i) performing experiments with a different sediment (a 0.12-mm quartz sand); and ii) extending the range of spectral wave conditions tested to values well above those corresponding to initiation of sediment motion.

The philosophy behind the experimental methodology and procedures used in the present study is unchanged from that of Madsen and Rosengaus (1988). Simply stated, it is to measure the attenuation of water waves (periodic as well as simulated spectral waves) propagating over a long section of movable bed and from the measured attenuation of each incident wave component backfigure the appropriate value of the wave friction factor for each incident wave component using the theoretical model for spectral wave attenuation developed by Madsen et al. (1988). Attempts to relate the experimentally obtained values of friction factors through the corresponding equivalent bottom roughness to geometry of the bottom bedforms, i.e., using the Madsen and Rosengaus (1988) procedure in reverse, were unsuccessful. Therefore, in contrast to Madsen and Rosengaus (1988), the friction factor values obtained here are used directly in an attempt to establish a *friction factor relationship* without the intermediate steps of relating friction factors to equivalent roughness and then relate equivalent roughness to geometry of the wave-generated bottom bedforms.

Methodology and Procedures

The experimental set-up, methodology, and procedures used in this study are identical to those described by Madsen and Rosengaus (1988).

Experimental Set-up. Laboratory experiments were performed in a 0.75-m-wide, 0.90-m-deep, and 30-m-long wave flume in the R. M. Parsons Laboratory. This wave flume is equipped with a programmable piston-type wave maker and has a 1-on-10 sloping absorber beach, which in the present experiments was covered by 7.5-cm-thick horsehair mats to further decrease reflections. At the wavemaker a trapezoidal wooden ramp provided a smooth transition to a level 10 cm above the flume bottom. The region between the ramp--approximately 3 m from the wavemaker--to the start of the absorber beach--approximately 19 m from the wavemaker--was covered by a 10-cm-thick layer of very uniform 0.12- or 0.2-mm-diameter quartz sand. The water depth above the movable bed test section was 0.6 m for the majority of the experiments with a few experiments conducted at a depth of 0.5 m.

Waves. In order to mobilize the bed material it was necessary to generate waves of large amplitudes and wave length. The standard wave condition of Madsen and Rosengaus (1988) corresponded to a 6-cm-amplitude, 2.65-sec-period wave for which the depth-to-wavelength ratio $h/L = 0.1$ in the 0.6-m-deep section. In the present investigation the wavemaker motion was programmed to simulate the generation of an incident wave spectrum with a peak period corresponding to a depth to peak period wave length ratio of 0.1 in the test section and a root-mean-square wave height ranging from 8.4 cm to 13.4 cm. Both broad-banded Pierson-Neumann spectra and narrow-banded finite-depth Jonswap ($\gamma = 3.3$) spectra were simulated by five frequency components of equal energy.

Measurements. The incident and reflected wave spectra were determined from measurements of surface profiles obtained by two three-gauge arrays located 3 m and 18 m from the wavemaker, respectively. Although not used directly in the analysis of the experimental data obtained in the present study bottom bedform geometry was determined photographically by taking pictures of 1-m sections of the bed profile at four stations along the test section. These pictures, including horizontal and vertical scales, were then projected onto a digitizing tablet and analyzed. Using a vertically mounted laser to trace the bottom profile at different distances from the flume sidewalls it was shown that the trace along the sidewalls accurately represented the bedform geometry.

Procedures. Given the philosophy behind the experiment as outlined in the Introduction the procedure to be followed is quite simple. From the analysis of Madsen et al. (1988) we may write the energy conservation principle for each wave component when only bottom dissipation is considered

$$c_{gn}\rho g a_n \left[\frac{\partial a_n}{\partial x}\right]_{bf} = -E_{d,n} = -\tfrac{1}{4}\rho f_{wn} u_{br} u_{bn}^2 \qquad (1)$$

where ρ is the fluid density, g is gravity, a_n and c_{gn} are the amplitude and group velocity of the wave component of frequency ω_n, respectively, and for a finite number of wave frequency components

$$u_{br} = \sqrt{\sum_n u_{bn}^2} \qquad (2)$$

$$A_{br} = \sqrt{\sum_n (u_{bn}/\omega_n)^2} = u_{br}/\omega_r \qquad (3)$$

Thus, for a single wave component--whether it is alone or one of two or more--we have from Eqs. (1) through (3) and linear wave theory

$$\left[\frac{\partial a_n}{\partial x}\right]_{bf} = -f_{wn}\frac{u_{br}}{4gc_{gn}}\left[\frac{\omega_n}{\sinh k_n h}\right]^2 a_n \qquad (4)$$

In principle the experimental determination of a_n at two stations, Δx apart, suffices to determine a value of f_{wn} from Eq. (4). In practice, however, life is unfortunately not that simple.

All the experimental determinations of Δa_n involve the "small" difference between two "large" quantities. As an example we may take the standard periodic wave (a_n = 6 cm, $T_n = 2\pi/\omega_n$ = 2.65 sec, h = 0.60 m) and a friction factor of f_{wn} = 0.2 to obtain a change in amplitude of $\Delta a_n \simeq$ 7 mm over the 15-m-long test section. While this value of Δa_n is well above the accuracy with which the individual amplitude is determined experimentally--estimated to be of the order 0.5 mm--it should be recalled that component amplitudes of the order 2 to 3 cm are used in spectral simulations. In fact, the decision to simulate the wave spectra by five frequency components was arrived at precisely from consideration of desired accuracy. Nevertheless, we are in some cases attempting to measure differences of the order a mm or less, i.e., approaching the magnitude of uncertainty of each measurement used to determine the difference. For this reason alone it is essential that the experiments be conducted with great thoroughness and repeated several times in order to minimize the effect of experimental errors. In the present investigation all experimental determinations of wave attenuation for a given wave condition were repeated five to ten times.

As a result of the inherent experimental difficulties associated with an accurate determination of Δa_n the variability in friction factors for the individual frequency components, f_{wn}, obtained from Eq. (4) is quite large. However, the theory developed by Madsen et al. (1988) assumed the friction factors for the individual frequency components, f_{wn}, to be the same for all frequency components. For these reasons the wave attenuation data were used to obtain values of f_{wn} from Eq. (4) and these values were then used to obtain a single "representative" wave friction factor, f_{wr}, from

$$f_{wr} = \sum_n f_{wn} u_{bn}^2 / \left[\sum_n u_{bn}^2\right] \qquad (5)$$

In passing, it should be noted that the representative wave friction factor, defined by Eq. (5), will conserve total spectral dissipation when applied to each individual frequency component.

However, other effects in addition to those directly related to measurement accuracy play a role in these experiments. Owing to the requirement of long waves of substantial amplitude--to set the bottom sediment in motion--it can be expected that nonlinear effects come into play. To illustrate this, assume the change of amplitude, Δa_n, to have been determined experimentally over a distance Δx. Formally, we may then write

$$\frac{\partial a_n}{\partial x} \simeq \frac{\Delta a_n}{\Delta x} = m = m_{nl} + m_{sw} + \left[\frac{\partial a_n}{\partial x}\right]_{bf} \qquad (6)$$

in which m_{nl} denotes amplitude changes associated with non-linearity, m_{sw} the changes associated with dissipation along the side walls, and $(\partial a_n/\partial x)_{bf}$ is the quantity expressing the contribution of bottom friction, i.e., the quantity we are looking for. A priori we cannot neglect the unwanted contributions, m_{nl} and m_{sw}, in particular when the overall requirement of accuracy is kept in mind.

To evaluate the magnitude of the terms m_{nl} and m_{sw} in Eq. (6) a series of experiments was first conducted in which the incident wave characteristics at the two measurement stations were determined corresponding to flat-bed conditions, i.e., sufficient time following the start of the wavemaker was allowed for transients to die out and the bed was covered by thin steel plates to prevent bedform development. Experiments of this type were performed 5 to 10 times and the measured amplitude change in these experiments (denoted by subscript p)

$$\left[\frac{\Delta a_n}{\Delta x}\right]_p = m_p \simeq m_{nl} + m_{sw} + m_{fb} \qquad (7)$$

was taken to represent nonlinear and sidewall effects also in the movable bed runs. The bottom friction effect, m_{fb} in Eq. (7), was calculated theoretically--the bottom boundary layer remained laminar for flat bed conditions.

Following the preliminary runs the wave condition was maintained in the flume for sufficiently long time to ensure the bedforms on the bottom to be fully developed (in some cases this took several hours). Several wave measurements were then performed corresponding to fully developed bed conditions with bottom bedform geometry determined between individual wave measurements (to ensure that the bed indeed was fully developed). Combining the preliminary and the fully developed wave measurements produced experimental results for the isolated effect of wave attenuation

$$\left[\frac{\partial a_n}{\partial x}\right]_{bf} \simeq \left[\frac{\Delta a_n}{\Delta x}\right]_{bf} = m - m_p + m_{fb} \tag{8}$$

caused by the movable bed, which in turn were used in Eqs. (4) and (5) to obtain friction factors.

Results

The experimental values obtained for the representative spectral wave friction factor, f_{wr}, are presented as full symbols (with one standard deviation error bars indicated) in Figure 1 versus the representative value of a fluid-sediment interaction parameter

$$S_r = \frac{\psi_{mr}'}{\psi_c} \tag{9}$$

in which

$$\psi_{mr}' = \frac{\tau_{bm}'}{(s-1)\rho g d} \tag{10}$$

is the Shields parameter obtained from the maximum bottom shear stress predicted for the representative monochromatic wave defined by Eqs. (2) and (3) based on grain-size bottom roughness, i.e., for $k_b = d$, and ψ_c is the critical shear stress for initiation of sediment movement obtained from Shields criterion (Madsen and Grant, 1976). Thus the parameter S_r represents the extent to which threshold conditions are exceeded. For comparison the wave friction factors obtained from monochromatic wave experiments are included as open symbols in Figure 1.

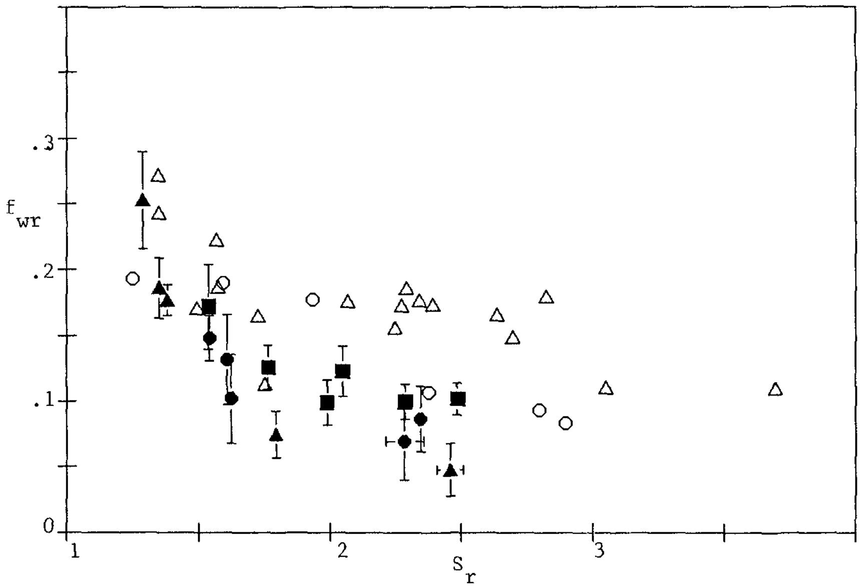

Figure 1. Movable bed friction factors for spectral waves (solid symbols) and periodic waves (open symbols). Triangles: 0.2-mm, Pierson-Neumann spectrum; circles: 0.12-mm, Pierson-Neumann spectrum; squares: 0.12-mm, finite-depth Jonswap spectrum

A striking feature of the spectral wave friction factors presented in Figure 1 is that they appear to form a unique relationship $f_{wr} = f_{wr}(S_r)$, despite the fact that the values were obtained for different sediments (0.2-mm- and 0.12-mm-diameter quartz sands) and for different spectral shapes (broad-banded Pierson-Neumann and narrow-banded finite-depth Jonswap spectra with $\gamma = 3.3$). While this finding is extremely fortunate from the point of view of practical application of the results it is of some concern that the spectral wave friction factors consistently fall considerably below the values of their monochromatic counterparts for $S_r > 1.5$.

The methodology for the determination of spectral wave friction factors proposed by Madsen and Rosengaus (1988) should result in a negligible difference between spectral and monochromatic wave friction factors so long as their associated bottom bedforms are similar. Examination of the height and length of bedforms generated by spectral and monochromatic waves reveals that the spectral bedforms have a slightly smaller height and a somewhat smaller steepness than their monochromatic counterparts. However, accounting for this difference in bedform geometry would at most account for a 20-percent decrease in the spectral wave friction factors relative to the monochromatic values when the bedform geometry is used in conjunction with the empirical relationship for bottom roughness proposed by Madsen and Rosengaus (1988). Referring to the results presented in Figure 1 it is evident that a larger-than-20-percent reduction of the monochromatic wave friction factors is called for to bring them in line with the spectral results. It is therefore concluded that the methodology for the evaluation of spectral wave friction factors proposed by Madsen and Rosengaus (1988) indeed is limited by the limited range of their experiments. In fact, it worked for them only because their spectral experiments were limited to values of $S_r < 1.5$, i.e., those represented in Figure 1 by the three points with $S_r \simeq 1.3$, which just happened to overlap with the monochromatic results.

In an attempt to explain the reduction of spectral wave friction factors the digitized records of the bottom profiles, obtained from photographs of the fluid-sediment interface along the glass side-walls of the flume, were carefully examined with particular emphasis on the characteristics of the ripple crests. In this manner it was found that ripple crests generated by spectral waves were somewhat more rounded than those generated by monochromatic waves. Clearly, the rounding of a sharp ripple crest only reduces the ripple height slightly (as observed) but results in a significant reduction of the strength of flow separation over the crest. It is hypothesized that the larger waves in a spectral simulation shave off the sharp ripple crests thereby causing the observed reduction in dissipation and friction factors for spectral waves. Since maximum near-bottom orbital velocities associated with spectral waves are approximately Rayleigh-distributed regardless of spectral shape, this hypothesis supports our finding that spectral dissipation is independent of spectral shape.

Summary and Conclusions

The spectral wave friction factors obtained from our experiments and presented in Figure 1 may be represented with a 20-percent error by the following movable bed friction factor relationship

$$f_{wr} = 0.29\ S_r^{-1.5} \qquad \text{for} \quad S_r > 1.2 \tag{11}$$

This expression may in turn be used with the spectral wave boundary layer theory of Madsen et al. (1988) to obtain a relationship, accurate to within 30 percent, for the movable bed roughness associated with spectral waves

$$k_b/A_{br} = 1.5S_r^{-2.5} \qquad \text{for} \qquad S_r > 1.2 \tag{12}$$

It is tentatively proposed that these relationships are generally valid for cohesionless sediments regardless of spectral shape. Having been burned once, it should be *emphasized* that experimental support of these expressions is limited to values of $S_r < 2.5$. However, some comfort in extrapolating the expressions beyond this limit may be found in the dependency of the relative roughness on S_r to the negative 2.5 power being identical to the S_r-dependency of the bottom bedform roughness of Grant and Madsen (1982) for large values of S_r.

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