CHAPTER 151

Effects of Short-crested Waves on the Scouring Around the Breakwater

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The aim of this research is to investigate qualitatively the influence of short-crested waves on the scouring around the breakwater through mainly some laboratory studies. At first, we succeed to observe clearly in laboratory some sedimentary bed forms such as troughs, holes, triangle bars and longitudinal bars under short-crested wave actions. Then we elucidate the associstion of their formation mechanism with theory of short-crested waves, also indicate its effect on scouring at the toe of breakwaters. In addition, it is shown that the breaking wave height of short-crested waves is certainly higher than that of two-dimensional standing waves. Finally we demonstrate some facts of failure of breakwater caused by short-crested wave breaking basing on some field results.

1. Introduction

As the breakwater is built in the sand bed region the incident waves and their reflected waves are combined into Short-crested waves system. The vortex motion and maxro-tubulence structure under the wave system can scour deep holes and result in subsidence of the breakwater. This phenomen is well known to the practicing engineers.

A theorietical solution presented by Fuchs (1952), based upon linear theory, had been applied to short-crested wave system. Later, the extensive work by Chappelear (1961) had improved the accurocy of Fuch's approximation, but with the difficulty of immediate application due to very length formulation. A more sophisticated second order theory had been proposed by Hamada (1965) by means of the secondary interactions. A third order approximation to the short-crested waves was derived by Hsu (1978). In addition, that breaking wave height of short-crested waves is certainly higher than that of two dimensional progressive waves was shown by Halliwell and Machen (1981). The breaking wave height obtained from the third approximation of Hsu & Siloverster (1982) matchs well with that result by Halliwall (1981). The aim of this paper is to investigate qualtatively influence of short-crested waves on the scoaning around the breakwater through mainly the movable model tests studies.

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2. Summary of short-crested wave theory

2.1 Basic properities of short-crested waves

When waves approach and are reflected from vertical breakwater, they creat a surface undulation of a diamond pattern, known as short-crested waves, as shown in Fig. 1. With a coordinate system of x being in the direction of advancing the combined crest, Y tranversally to the celerity vector of the combined wave and Z positive upward from the still water surface, Fuchs (1952) derived the frist order approximation as:

$$\phi = \frac{ag \cos h\ r\ (d+y)}{m\ C_S \cos h(rd)} \cos m\ (X - C_S\ t) \cos nZ \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}1)$$

$$m = \frac{2\pi}{L} \qquad n = \frac{2\pi}{L'} \qquad r^2 = m^2 + n^2$$

$$Y_S = -\left[\frac{1}{g} \frac{\partial \phi}{\partial t} \right]_{y=0} = a \sin m\ (X - C_S t) \cos nZ \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}2)$$

$$C_S^2 = \frac{gr}{m^2} \tan h\ (r \bullet d)$$

$$= \frac{gL}{2\pi} \sqrt{1 + \left(\frac{L}{L'} \right)^2} \tanh \left[\frac{2\pi d}{L} \sqrt{1 + \left(\frac{L}{L'} \right)^2}\ \right] \ldots\ldots\ldots\ldots\ldots \quad (2\text{-}3)$$

where ϕ, Y_S and C_S are velocity potential, surface profile and wave velocity repectively.

$$C_S = \frac{gT}{2\pi} \sqrt{1 + \left(\frac{L}{L'} \right)^2} \tanh \left[\left(\frac{2\pi d}{L}\right) \sqrt{1 + \left(\frac{L}{L'} \right)^2}\ \right] \ldots\ldots\ldots\ldots\ldots \quad (2\text{-}4)$$

because $\sqrt{1 + \left(\frac{L}{L'} \right)} \geq 1$ then

$$C_S \geq \left(\frac{gT}{2}\right) \tanh \frac{2\pi d}{L} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}5)$$

Eq.(2-5) Show that the combined celerity of short-crested wave is larger than wave celerity of incident waves.

The water partical velocities in X,Y and Z direction under the short-crested waves are as follows:

$$u = \frac{\partial \phi}{\partial X} = \frac{ga}{C_S} \quad \frac{\cosh\ [r(Y+d)]}{\cosh\ (rd)} \quad \sin m\ (X - C_S t) \cos nZ$$

$$v = \frac{\partial \phi}{\partial Y} = -\frac{gar}{mC_S} \quad \frac{\sinh\ [r\ (Y+d)]}{\cosh\ (rd)} \quad \cos m\ (X - C_S t) \cos nZ$$

$$w = \frac{\partial \phi}{\partial Z} = \frac{gan}{mC_S} \quad \frac{\cosh\ [r(Y+d)]}{\cosh\ (rd)} \quad \cos m\ (X - C_S t) \sin nZ \ldots\ldots \quad (2\text{-}6)$$

The orbital of water partical is :

$$X = \left(\frac{H}{2} \right) \frac{\cos \left(\frac{2\pi}{L}\right)(Y+d) \sqrt{1+\left(\frac{L}{L'}\right)^2}}{\sqrt{1+\left(\frac{L}{L'}\right)^2} \sinh \left(\frac{2\pi d}{L} \right) \sqrt{1+\left(\frac{L}{L'} \right)^2}} \left[\cos \left(\frac{2\pi Z}{L'}\right) \sin\left(\frac{2\pi X}{L}\right)\right]$$

$$Y = \left(\frac{H}{2}\right) \frac{\sinh\left(\frac{2\pi}{L}\right)(Y+d)\sqrt{1+\left(\frac{L}{L'}\right)^2}}{\sinh\left(\frac{2\pi}{L}\right)\sqrt{1+\left(\frac{L}{L'}\right)^2}} \left[\cos\left(\frac{2\pi Z}{L'}\right)\cos\left(\frac{2\pi X}{L}\right)\right]$$

$$Z = \left(\frac{H}{2}\right)\left(\frac{L}{L'}\right) \frac{\cosh\left(\frac{2\pi}{L}\right)(Y+d)\sqrt{1+\left(\frac{L}{L'}\right)}}{\sqrt{1+\left(\frac{L}{L'}\right)^2}\ \sinh\left(\frac{2\pi d}{L}\right)\sqrt{1+\left(\frac{L}{L'}\right)^2}} \left[\sin\left(\frac{2\pi Z}{L'}\right)\cos\left(\frac{2\pi X}{L}\right)\right] \ldots (2\text{-}7)$$

The water particles follow elliptical paths which are angled to the horizontal.

$$\tan \beta = \frac{n \tan (nZ)}{r \tanh r(Y+d)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}8)$$

Furthermore, also derived the free surface profile to the second-order approximetion as follow:

$$\Phi = A \cosh r\ (d+Y) \cos m(X - C_s t) \cos n\ Z + B \cosh 2r\ (d+Y) \bullet$$
$$\sin 2\ m(X-C_s t) \cos 2nZ + D \cosh 2m\ (d+Y) \sin 2\ m(X-C_s t)$$

$$A = \frac{am}{r}\ \frac{\cosh r\ (d+Y)}{\sinh r\ d}\ \cos n\ Z$$

$$B = -\frac{3}{16}\ \frac{A^2\ r^2}{C_s\ m \sin h^2 r\ d}$$

$$D = \frac{A}{8\ m\ C_s}\ \frac{4\ n^2 \cosh^2 rd - 3r^2}{2 \cosh 2\ md - (m \sinh 2\ md/r \tanh rd)} \quad \ldots\ldots\ldots\ldots \quad (2\text{-}9)$$

$$Y_s = a \sin m\ (X-C_s t)\bullet\cos nZ + \frac{a^2\ r^2}{4 \sinh 2\ rd}\ (2 \sinh^2 rd - 1$$

$$+ \frac{3 \cosh 2\ rd}{\sinh^2 rd}) \bullet \cos 2\ m\ (X-C_s t) \bullet \cos 2\ n\ Z$$

$$+ \frac{a^2(m^2-n^2 \cosh 2\ rd)}{4r \sinh 2rd} \bullet \cos 2\ nZ + \frac{a^2}{4\ r \sinh 2\ rd}$$

$$\left[\frac{(3r^2-4\ n^2 \cosh^2 rd)\cosh 2md}{\cosh 2\ md - (m \sinh 2\ md/2\ r \tanh rd)} - m^2 \cosh^2 rd \right.$$

$$\left. + 3\ r^2 \sinh^2 rd + n^2 \cosh^2 rd \right] \bullet \cos 2m(X-C_s t) \quad \ldots\ldots\ldots \quad (2\text{-}10)$$

2.2 Water partical motions under the short-crested waves system.

Basing on the Fuths theory, Silvester (1974) sketched the schematic diagram of water particle path under short-crested wave action, as Fig. 2. From this figure, the water particle motions can be described as

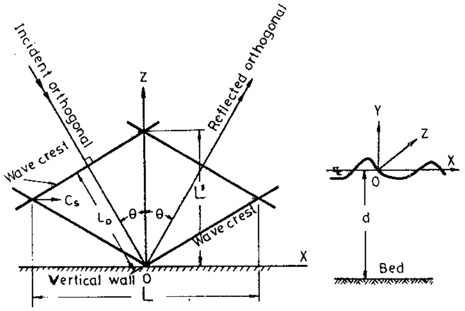

Fig. 1 Short-crested wave formed in front of a vertical wall.

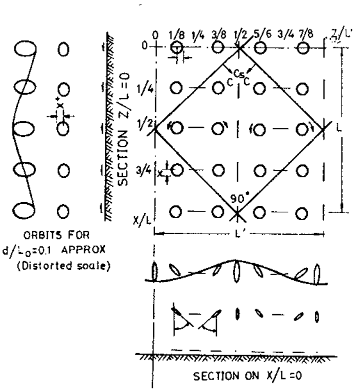

Fig. 2 Water particle orbits and mass transport under the short-crested wave. (Silverster, 1974)

follows.

(1) The wave profile along the centreline of the crests, at Z/L' = o, 1/2 , 1. etc., is more likely to be that of a solitary wave.

(2) Half-way between these crest alignment, at Z/L'=1/4, 3/4, etc., the orbital motion are rectilinear and are transverse to wave propagation.

(3) Between these again, at Z/L' = 1/8, 3/8, 5/8 , etc., the water particles follow elliptical paths which are angled to the horizontal.

The influence of these complex water motions on a sedimentary bed have been examined by. Tanalca and ozasa (1972), and Silvester (1974).

2.3 The breaking wave height wave height of short-crested wave.

There are two different theorm on the breaking wave height of short-crested wave so far. One is solitery wave theorm, another is progressive wave theorm.

(A) Solitary wave theorm:

Halliwell and Machen (1981), basing upon work of Milies (1977) , derived the breaking wave height of short-crested as follows:

when

$$C_s/C_1 \div \sec\theta_1$$

$$C = \sqrt{g(\eta+d)} \quad \cdots\cdots (2\text{-}11)$$

$$\frac{d+\eta_3}{d+\eta_1} = \sec\theta_1 \quad \cdots\cdots (2\text{-}12)$$

$$\frac{\eta_3}{\eta_1} = \left(\frac{d}{\eta_1}+1\right)\sec^2\theta_1 - \frac{d}{\eta_1} \quad \cdots\cdots (2\text{-}13)$$

$$\therefore \quad \eta_3 \geq \eta_1 \quad \cdots\cdots (2\text{-}14)$$

Let the breaking wave condition be Umax/C_s = 1, since $C_s > C$, there are greater Umax than that of incideut progressive wave with celectiy C. That is why the breaking wave height of short-crested wave is higher than that of progressive wave.

(B) The interaction of progressiv wave theorem.

Hsu & Silverster (1982), basing the third order approximation and taking the $U_{max}/C_s=1$ as a condition of breaking wave, got the breaking wave height which is larger than that of progressive wave and depend on the angle of interaction of two progressive wave.

3. Laboratory experiments

In order to investegate the effects of short-crested wave on the scouring around the breakwater, the movable bed model expeniments were performed in a wave basin with 60 m length, 43 m width and 1 m depth. A vertical breakwater was built in wave basin for producing a reflected wave train . The scouring at the toe of the breakweter has been since before one of the serious problems confronting the Taichung engineers. The laboratory studies are therefore based on the model test of Taichung Harbour located in Mid-western Taiwan. Fig. 3 depictes a schematic sketch of the experimental hydraulic model employed.

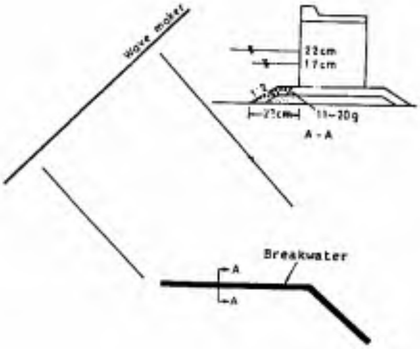

Fig. 3 Schematic diagram of the experimental model.

Fig. 4 Short-crested wave formed in front of the breakwater.

The ratios of model employed were that the breakwater , wave period, wave velocity., and wave pressure were 1/64 and the rock weight was $1/64^3$. The sand used for movable bed was directly taken from the natural beach in the vicinity of Taichung Harbour and its mean median diameter D_{50} was 0.22 mm.

It takes about one hour of time to complete a run of experiaments. Measurements performed were mainly included wave height distribution, breaking wave height and overall bed form. The bed forms were simultaneously obtained by photographing and level measuring. In order to investigete the scouring, when performing the experiments, we paid particularly attention to the situation as the short-crested waves are breaking.

4. Results and discussions

The runs of model test, the results of laboratory studies and the field data survey are shown in Table 1, 2 and 3. The short-crested wave formed in front of the breakwater was shown in Fig. 4 and Fig. 5. A typical result of bed forms under the action of short-crested waves are depicted in Fig 6 and 7. It can be clearly seen from this figure that the sedimentary bed is scoured by the breaking short-crested waves. As shown in Fig. 8 and Fig. 9, concerning the bed form in detail near the breakwater, deep holes and triangle bar are generally formed at the place of wave breaking. In Fig. 10 the sand ripples appear in the front of the breakwater and in Fig 11, the longitudinal bar and trough appear around the corner of the breakwater. If the breaking short-crested wave height is even greater, the impinging wave breaking will produce a large scouring hole just at the toe of the breakwater, as shown in Fig. 12.

(1) The brearking wave height resulting from the short-crested wave is greater than that of incident wave and this result matchs that of Hsu & Silvester. (1982), as shown in Fig. 13 and 14.

(2) The scouring in front of breakwater under the short-crested wave is strong and serious. The characteristics of the scouring as follows.
   (a) at Z/L'=0,1/2 , 1..., the sand hed were scoured and the sediments were transported in the X dinection.
   (b) at Z/L'=1/4, 3/4, ... there are the back and forth motion of water particles along the Z direction. There are no sediment transported in the X direction
   (c) at Z/L'=1/8, 3/8, 5/8, 7/8...etc., the swirling motion of water particles have much contribution in expediting the sediment movement. The moving sediments are transported into the adjoining regions.

(3) The mechanism of various sedimentary bed forms may be explained as follws:
   (a) at Z/L'= 0, 1/2, 1, etc, the sediments are transportad along the short-crested wave progessing. The greater the wave height is, the more the transportation is. Then, in this condition, the trough is formed along these lines (Z/L'=0, 1/2, 1 etc), as shown in fig. 1.
   (b) at Z/L'=0, 1/4, 1, .. only a few sediments are transported along transverive and longitadinal direction. In this characteristic, the longitudinal bar was formed.
   (c) When the breaking waves take place, the sand bed is formed

Table 1: The wave conditions of model test

Run	Wave period (sec)	Incident angle (θ_i)	Water depth (m)	Wave height (m)
1	1.25	35°	0.22	0.098
2	1.25	45°	0.17	0.098
3	1.25	45°	0.22	0.098
4	1.25	45°	0.17	0.124
5	1.25	30°	0.165	0.111
6	1.25	60°	0.21	0.123
7	1.25	45°	0.21	0.075
8	1.00	45°	0.205	0.110

Table 2: The breaking wave heights obtained from the results of experimental tests

Run	Wave period (sec)	Incident angle θ_i	Water depth d_{sb}	Breaking wave height	Wave length	$\frac{d_{sb}}{gT^2}$	$\frac{H_{sb}}{ds}$	$\frac{H_{bs}}{L_1}$	$\frac{d_{sb}}{L_1}$
1	1.25	35°	0.22	0.3093	1.663	0.0144	1.41	0.186	0.132
2	1.25	45°	0.17	0.258	1.495	0.011	1.518	0.173	0.114
3	1.25	45°	0.22	0.3232	1.663	0.0144	1.47	0.194	0.132
4	1.25	45°	0.17	0.297	1.495	0.011	1.74	0.199	0.114
5	1.25	30°	0.165	0.2317	1.476	0.0107	1.404	0.157	0.112
6	1.25	60°	0.21	0.2575	1.632	0.0137	1.226	0.157	0.129
7	1.25	45°	0.21	0.3003	1.632	0.0137	1.43	0.184	0.129
8	1.00	45°	0.205	0.2582	1.222	0.021	1.26	0.211	0.168

Table 3: The breaking wave heights of short-crested wave obtained from the field data survey, Taichung Harbor Bureau, 1973.

Wave period (sec) T	Incident angle θ_i	Water depth d_{sb}	Breaking wave height H_{sb}	Wave length L_1	$\frac{d_{sb}}{gT}$	$\frac{H_{sb}}{ds}$	$\frac{H_{bs}}{L_1}$	$\frac{d_{sb}}{L_1}$
10		13	18	156	0.01	1.38	0.115	0.083

Fig. 5 Short-crested wave breaking in front of the breakwater.

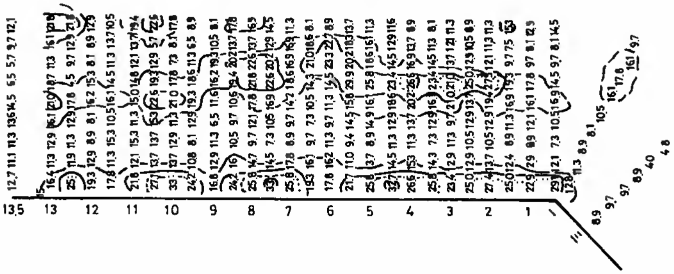

Fig. 6 Short-crested wave height distrobution in front of breakwater. Incident wave period = 1.25 sec, incident angle = 45°, incident wave height = 0.098m, water depth in front of breakwater = 22 cm.

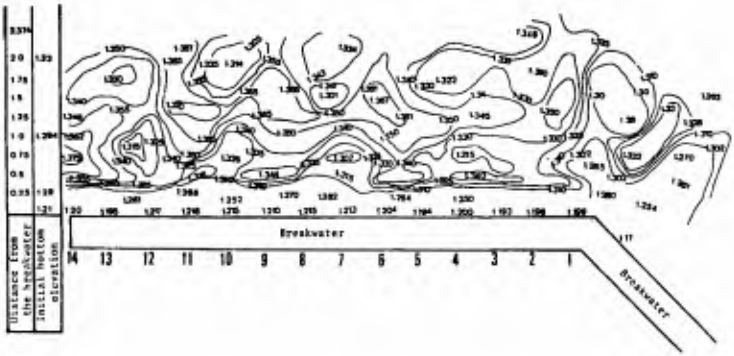

Fig. 7 Contours of bottom topography under short-crested wave actions.
incident wave period=1.25sec., incident angle=45°.
incident wave height=0.098m, water depth at the breakwater= 22cm.

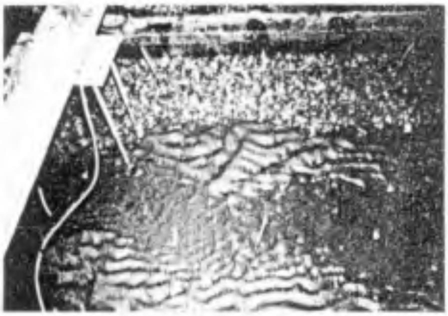

Fig. 8 Triangle sand bar and deep hole in the front of the breakwater.

Fig. 9 Triangle sand bar under the short-crested wave breaking.

Fig. 10 The sand ripples under the short-crested wave which did not break.

Fig. 11 The longitudinal bar and trough around the corner of the breakwater.

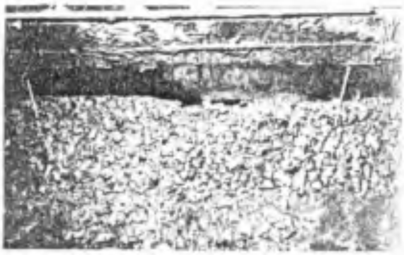

Fig. 12 Scouring hole at the toe of the breakwater.

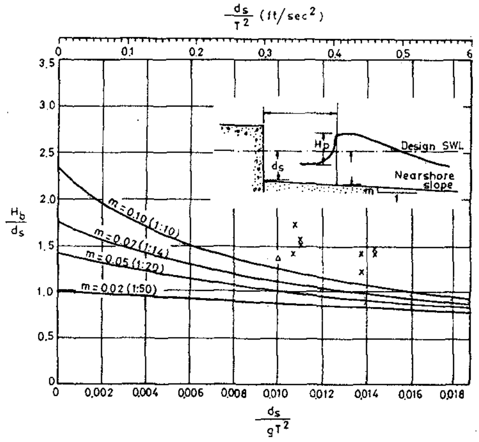

Fig. 13 Compare the breaking wave height of short-crested wave with that from Shore Protection Manual (1977).

-: the results by Weggel

X: the lab results by authors

Δ: the field data

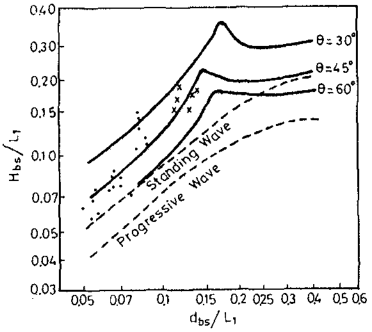

Fig. 14 Compare the breaking wave height of short-crested wave with that of standing wave, progressive wave, the numerical solution by Hsu & Silvester and the lab results by Halliwell.

• : the lab results by Halliwell

- : the numberical solution by Hsu & Silvester

X : the lab results by authors

into deep hole under the breaking waves, and the sand bed, by the deep hole, was formed triangle bar.

(d) Furthermore, for comparision, the sea bottom contours around the corner of the breakwater obtaned from the field data survey is depicted in Fig. 15, revealing the same tendancy as that of leboratory.

5. Conclusions and Remarks

From the laboratory experiments and field observation, the scouring effect, resulted from the short-crested wave system, on the foundation of breakwater can be drawn:

(1) Oblique incident waves and their reflected waves are formed into short-crested waves system in front of the breakwater. This short-crested waves progress along the breakwater and gradually swell up. Swelling up along the breakwater is greater than that of breaking wave height limit, the short-crested wave are broken. The breaking short-crested wave height is larger than that of progressive wave, and belong to plunging breaking wave. This breaking waves result in serious scouring around the breakwater.

(2) The characteristics of water particle motions under the short-crested wave system enforced the sediment transport and form into special bed forms such as troughs, holes, triangle bars and longitadinal bars.

(3) While the breaking waves happen, the vortex motions penetrate to the bed in the vicinity of the breakwater scour the foundation of the breakwater, and result in subsidence of the breakwater.

(4) It's suggested that when the breakwater is designed or built, the influence of short-crested wave should be considered.

(5) However, in this paper, only the qualitative results were reported.

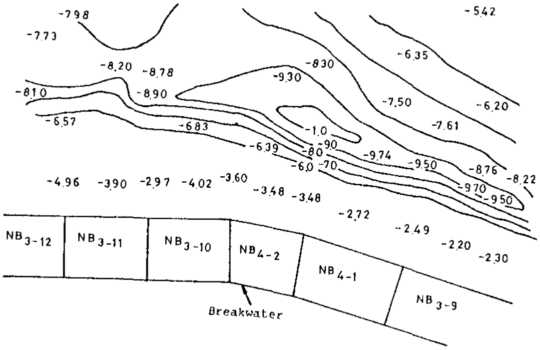

Fig. 15 Bottom contours around the corner of the breakwater for field case.

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